9/22/ Precalc

Last time

1.7 Linear inequalities

$|u| < 2$ $|u| \leq 2$

$|u| > 2$ $|u| \geq 2$

1.8 Other inequalities

Polynomial inequalities

① Rewrite to get $p(x) \begin{matrix} > \\ \geq \\ < \\ \leq \end{matrix} 0$

② Solve $p(x) = 0$

③ Use test intervals

Ex1 $x^4 > 5x^2$

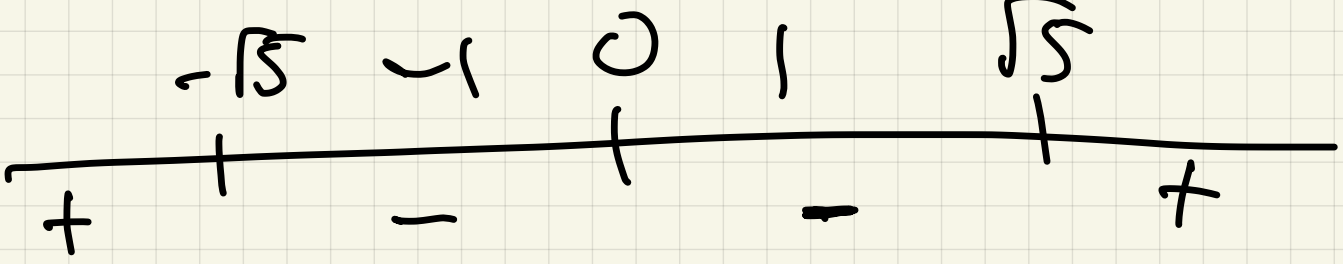
ZERO

$x^4 - 5x^2 > 0$

$x^2(x^2 - 5) = 0 \Rightarrow x = 0$

$x = \pm\sqrt{5}$

$x^2(x$



So soln: $(-\infty, -\sqrt{5}) \cup (\sqrt{5}, \infty)$

(b) $x^4 - 5x^2 \geq 0$ ↑ $0, \pm\sqrt{5}$

$(-\infty, -\sqrt{5}] \cup \{0\} \cup [\sqrt{5}, \infty)$

(c) $x^2 + 8x > 7$

$\boxed{x^2 + 8x - 7 > 0}$

$x^2 + 8x - 7 = 0$ Q.F.

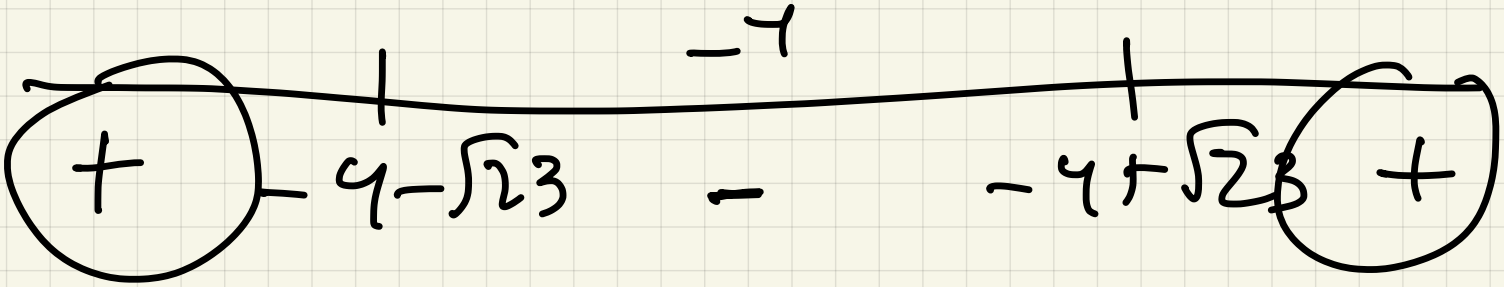
$a = 1$
$b = 8$
$c = -7$

$$x = \frac{-8 \pm \sqrt{64 - 4(1)(-7)}}{2(1)}$$

$$= \frac{-8 \pm \sqrt{64 + 28}}{2} =$$

$$= \frac{-8 \pm \sqrt{92}}{2} = \frac{-8 \pm 2\sqrt{23}}{2} =$$

$-4 \pm \sqrt{23}$



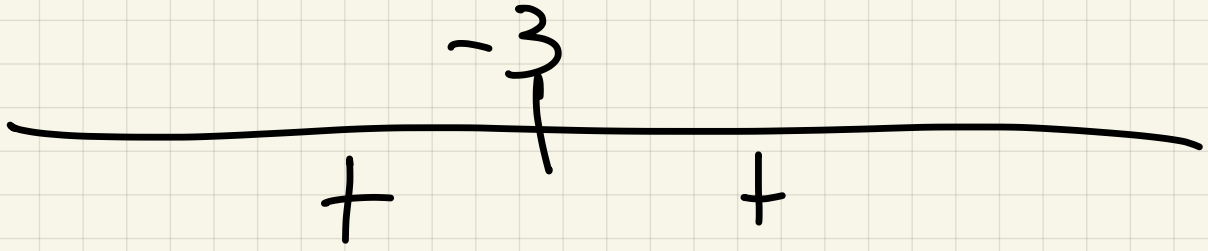
$(-\infty, -4-\sqrt{23}) \cup (-4+\sqrt{23}, \infty)$

(d) $-x^2-6x-9 \geq 0$

$x^2+6x+9 \leq 0$

$(x+3)^2 = 0$

$x = -3$



solution: $\{-3\}$

(e) $x^2+6x+9 > 0$

solution: $(-\infty, -3) \cup (-3, \infty)$

Rational inequalities

Ex 2 (a) $\frac{3x-7}{x+5} < 4$

Temptation: multiply by $x+5$

Wrong: $x+5<0$ is possible

Safe solution: rewrite with 0 on one side:

$$\frac{3x-7}{x+5} < 4 \Rightarrow \frac{3x-7}{x+5} - 4 < 0$$

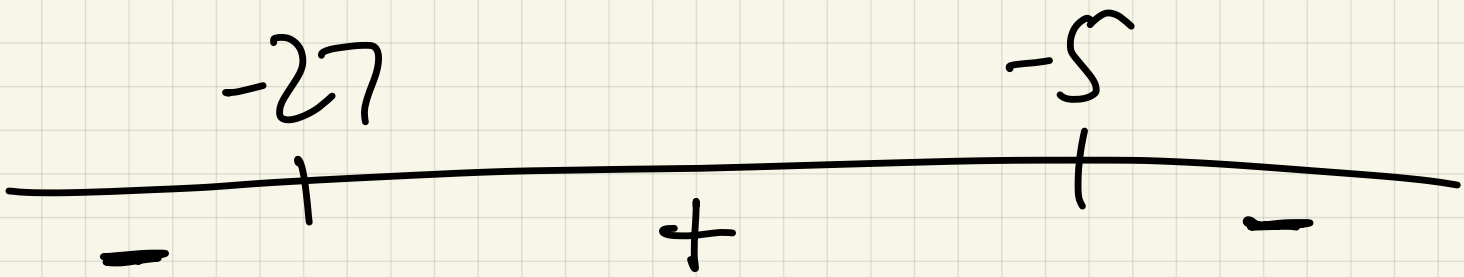
$$\frac{3x-7}{x+5} - \frac{4(x+5)}{x+5} < 0$$

(a) $\frac{-x-27}{x+5} < 0$

Set top/bottom $= 0$

top $-x-27=0 \Rightarrow x=-27$

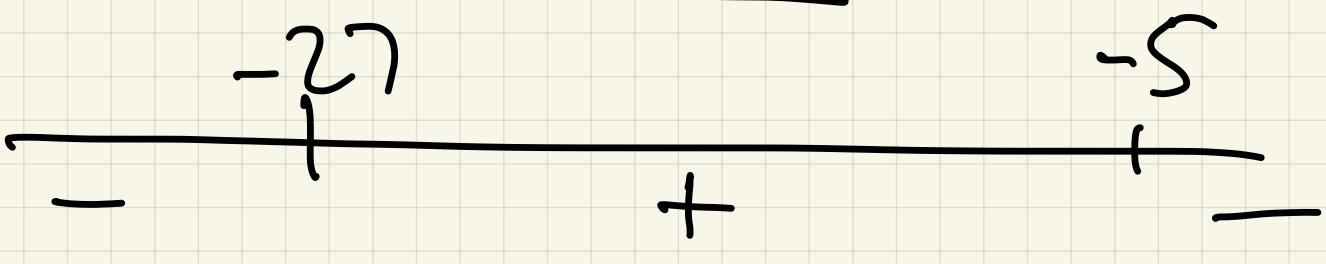
bottom $x+5=0 \Rightarrow x=-5$



Solutions: $(-\infty, -27) \cup (-5, \infty)$

(b) $\frac{3x-7}{x+5} \leq 4$

Process identical:



. Looks like

$$(-\infty, -27] \cup [-5, \infty)$$

but $x = -5$ DNE

so $(-\infty, -27] \cup (-5, \infty)$

(c) $$\frac{x-2}{x+3} \geq -3$$

$$\frac{x-2}{x+3} + 3 \geq 0$$

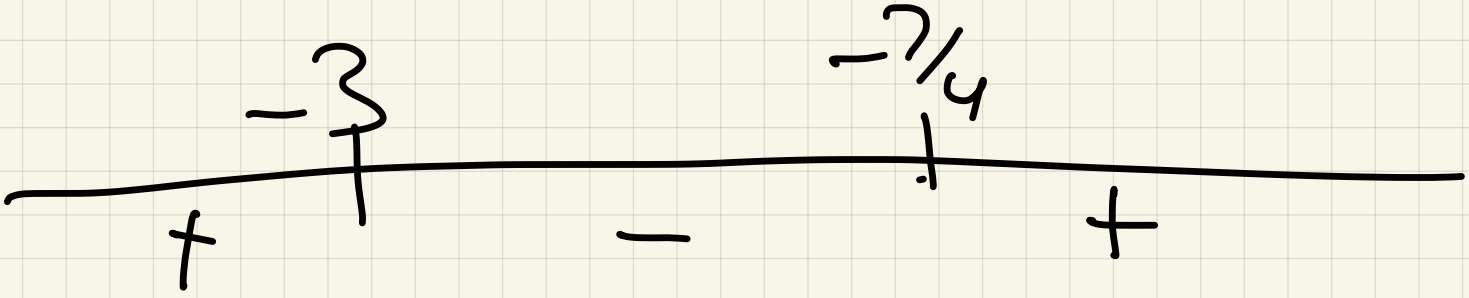
$$\frac{x-2}{x+3} + \frac{3(x+3)}{x+3} \geq 0$$

$$\frac{4x+7}{x+3} \geq 0$$

$4x+7=0 \Rightarrow 4x=-7$

$x=-7/4$

$x+3=0 \Rightarrow x=-3$



$(-\infty,-3] \cup [-7/4,\infty)$

At most ‖

$(-\infty,-3) \cup [-7/4,\infty)$

Ch2: Graphing functions

2.1 Linear equations in 2 variables

Linear equation in x and y has form

$$Ax + By = C$$

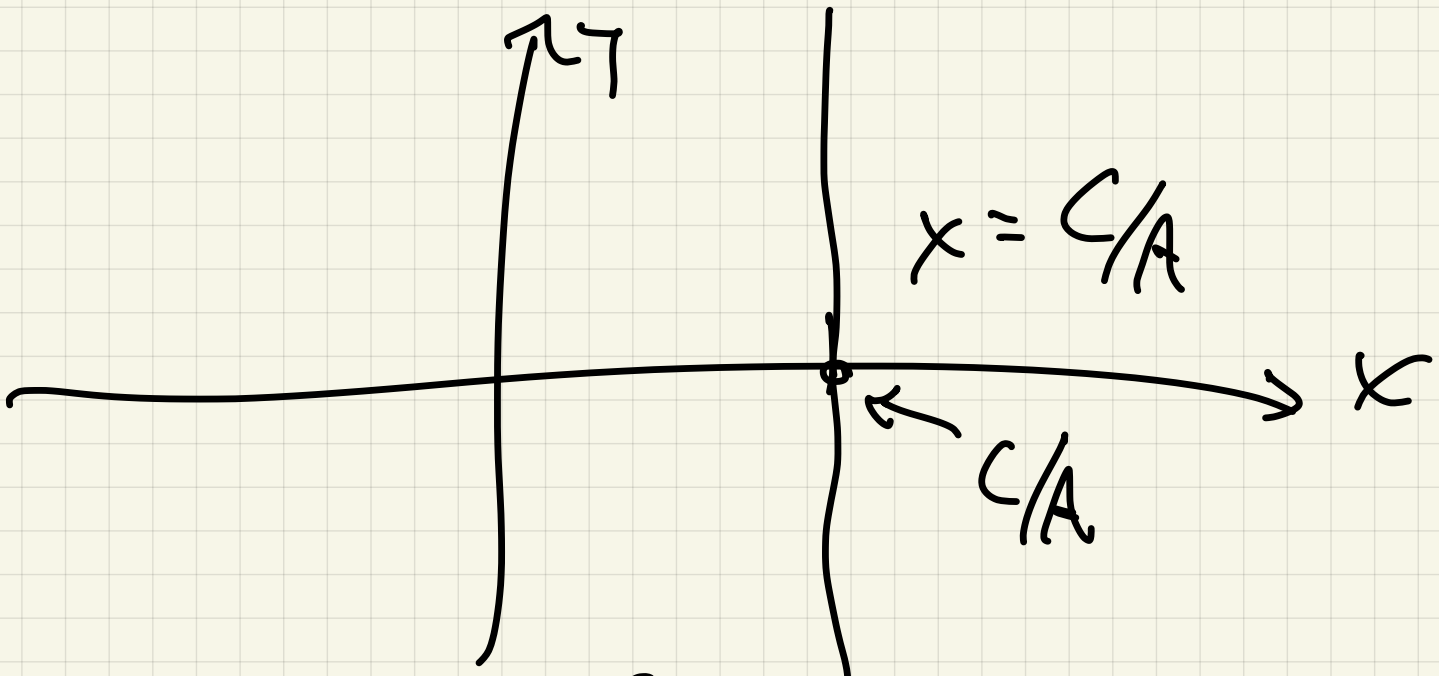
A, B, C reals

A, B not both zero

Case 1 If $B = 0$, then $A \neq 0$,

$$Ax = C \Rightarrow x = C/A$$

$x = C/A$ a vertical line



Case 2 If $B \neq 0$,

Can solve for y:

$$Ax + By = C \Rightarrow$$

$$By = C - Ax \Rightarrow$$

$$y = \frac{C}{B} - \frac{A}{B}x =$$

$$= \underbrace{-A/B}_{m}x + \underbrace{C/B}_{b}$$

$$\boxed{y = mx + b}$$

Called <u>slope - intercept form</u>

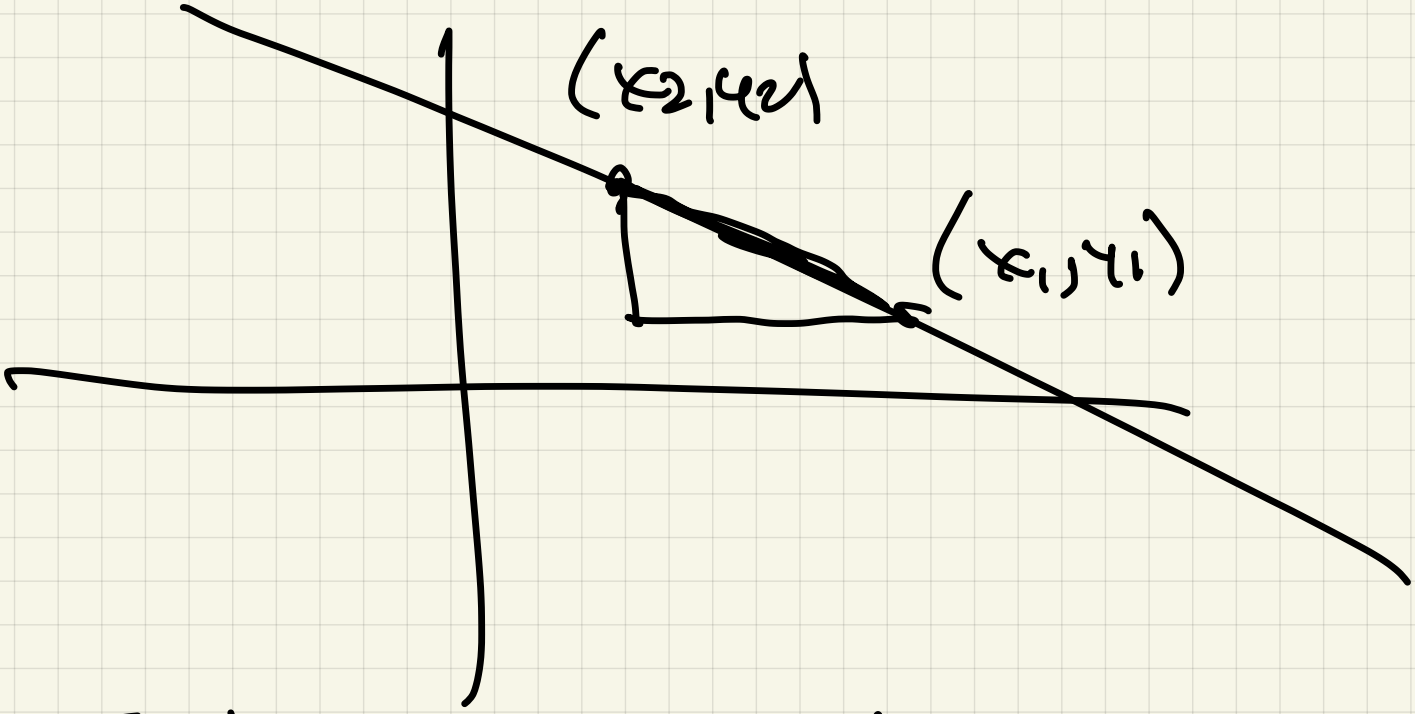
because

$(0, b)$ = y-intercept and

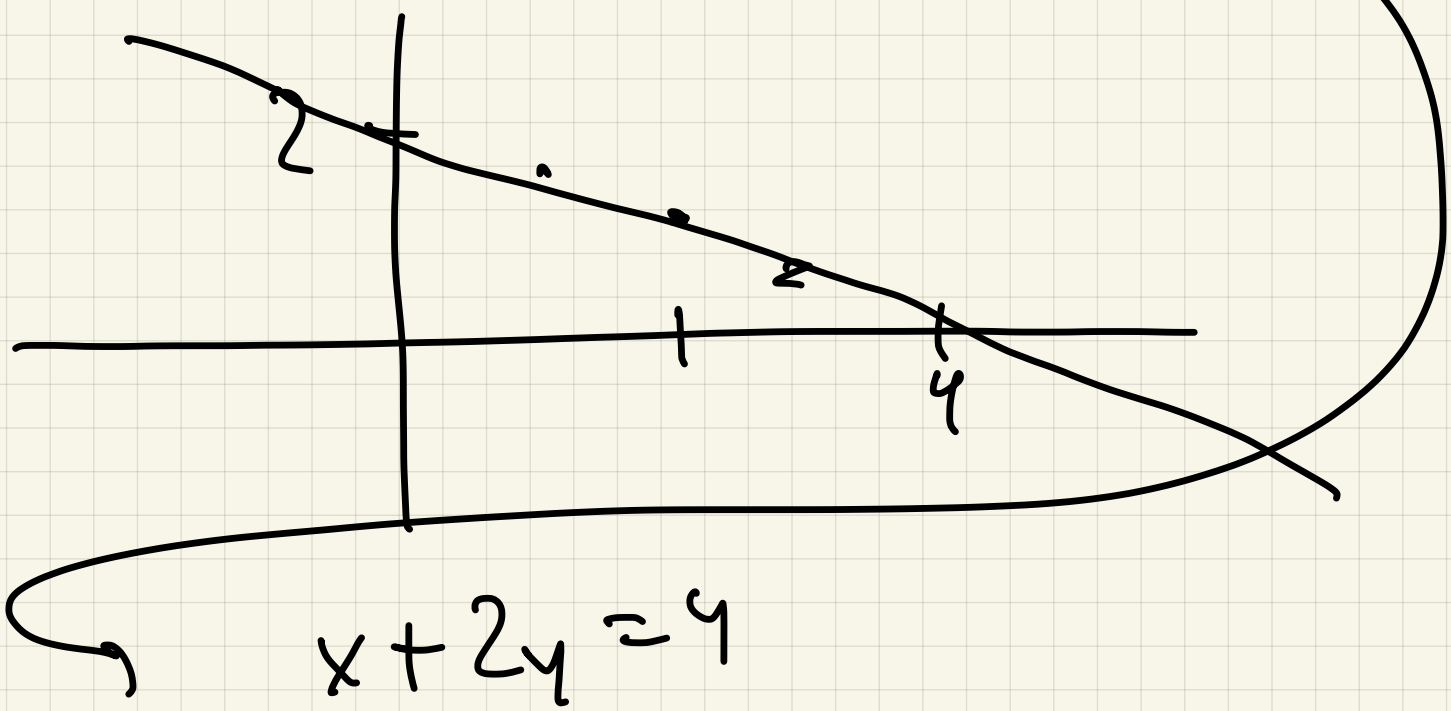
m = slope of line.

<u>Defn</u> If (x_1, y_1) + (x_2, y_2) are on the line L, then L has slope

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \left(\frac{\Delta y}{\Delta x}\right)$$



Ex1: $x + 2y = 4$



$x + 2y = 4$

$2y = 4 - x$

$y = 2 - x/2$

$y = \underset{m}{-\frac{1}{2}x} + \underset{b}{2}$

Using this, can sketch any line:

Ex1 Sketch:

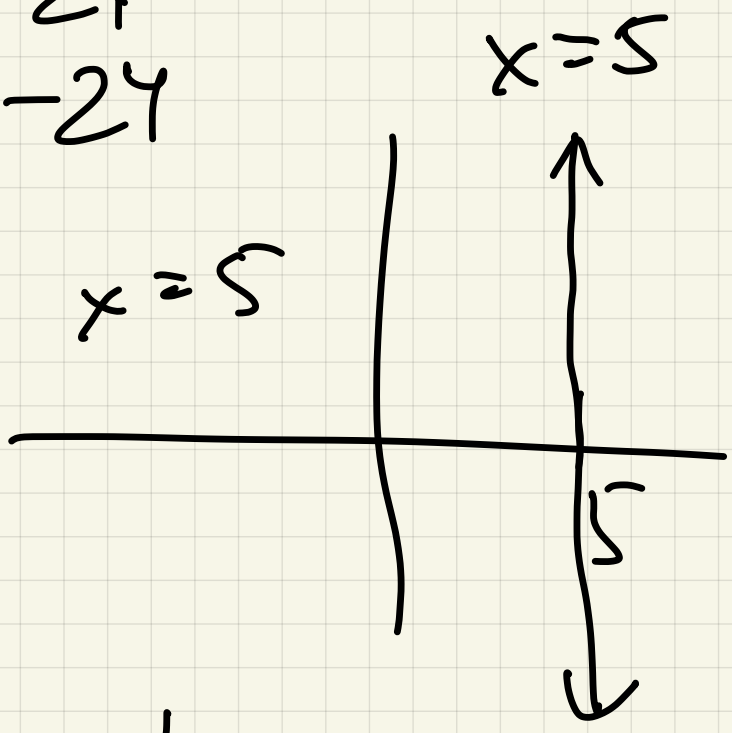
a) $3x = 15$

b) $4y = 80$

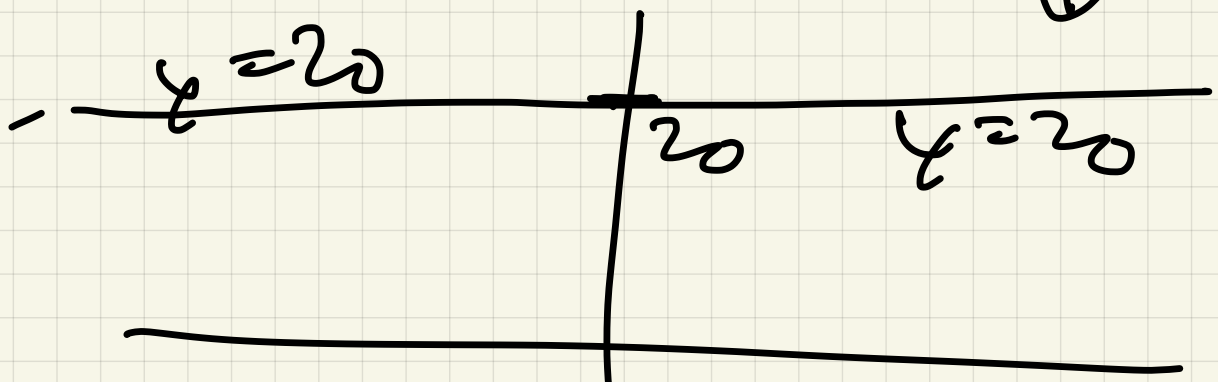
c) $2x - 3y = 24$

d) $2x + 3y = -24$

(a) $3x = 15 \Rightarrow x = 5$



(b) $4y = 80$



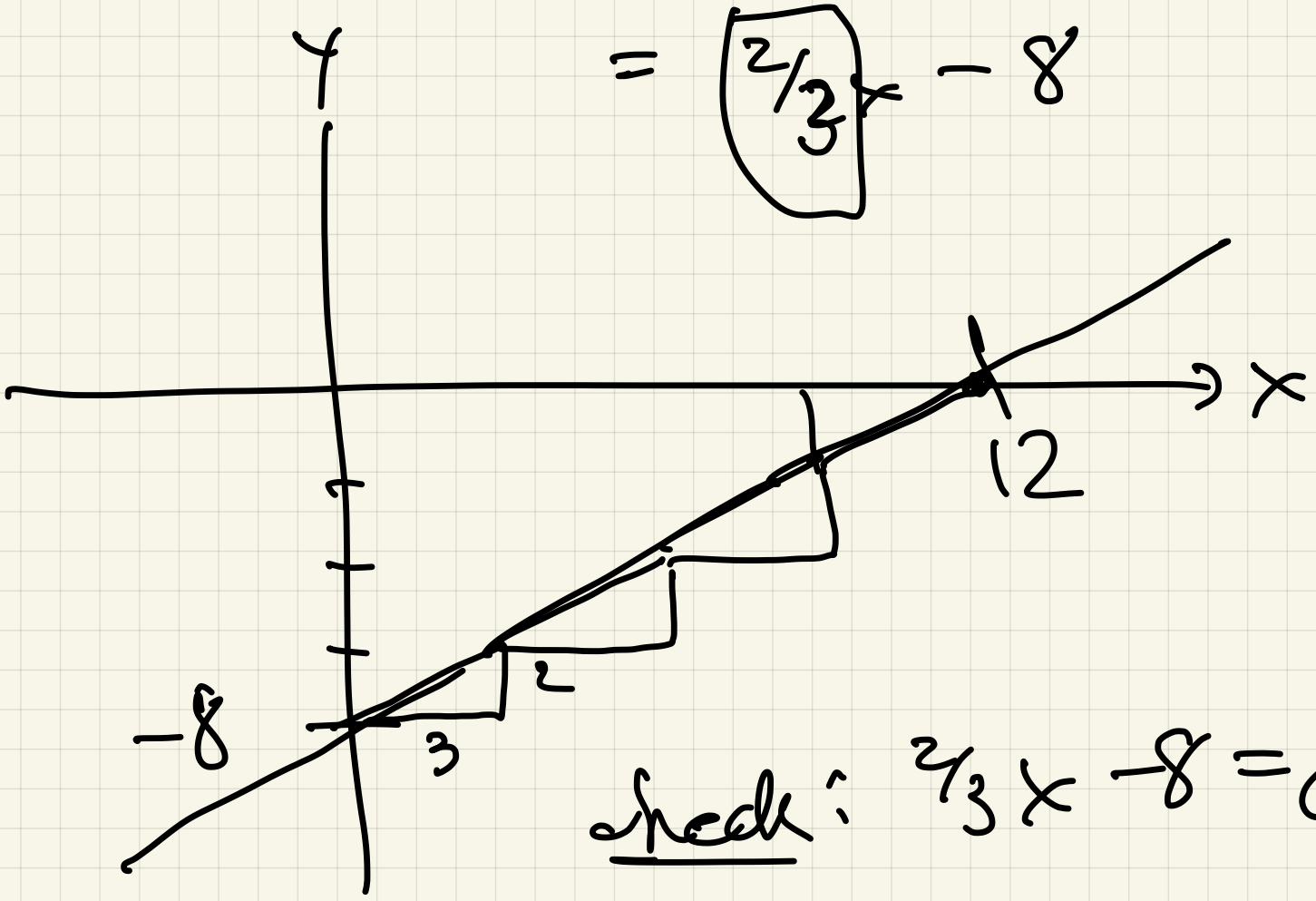
(c) $2x - 3y = 24$

$$-3y = 24 - 2x$$

$$= -2x + 24$$

$$y = -\tfrac{2}{3}x + \tfrac{24}{-3}$$

$$= \boxed{\tfrac{2}{3}x - 8}$$



check: $\tfrac{2}{3}x - 8 = 0$

$\tfrac{2}{3}x = 8$

$x = \tfrac{3}{2}(8) = 12$

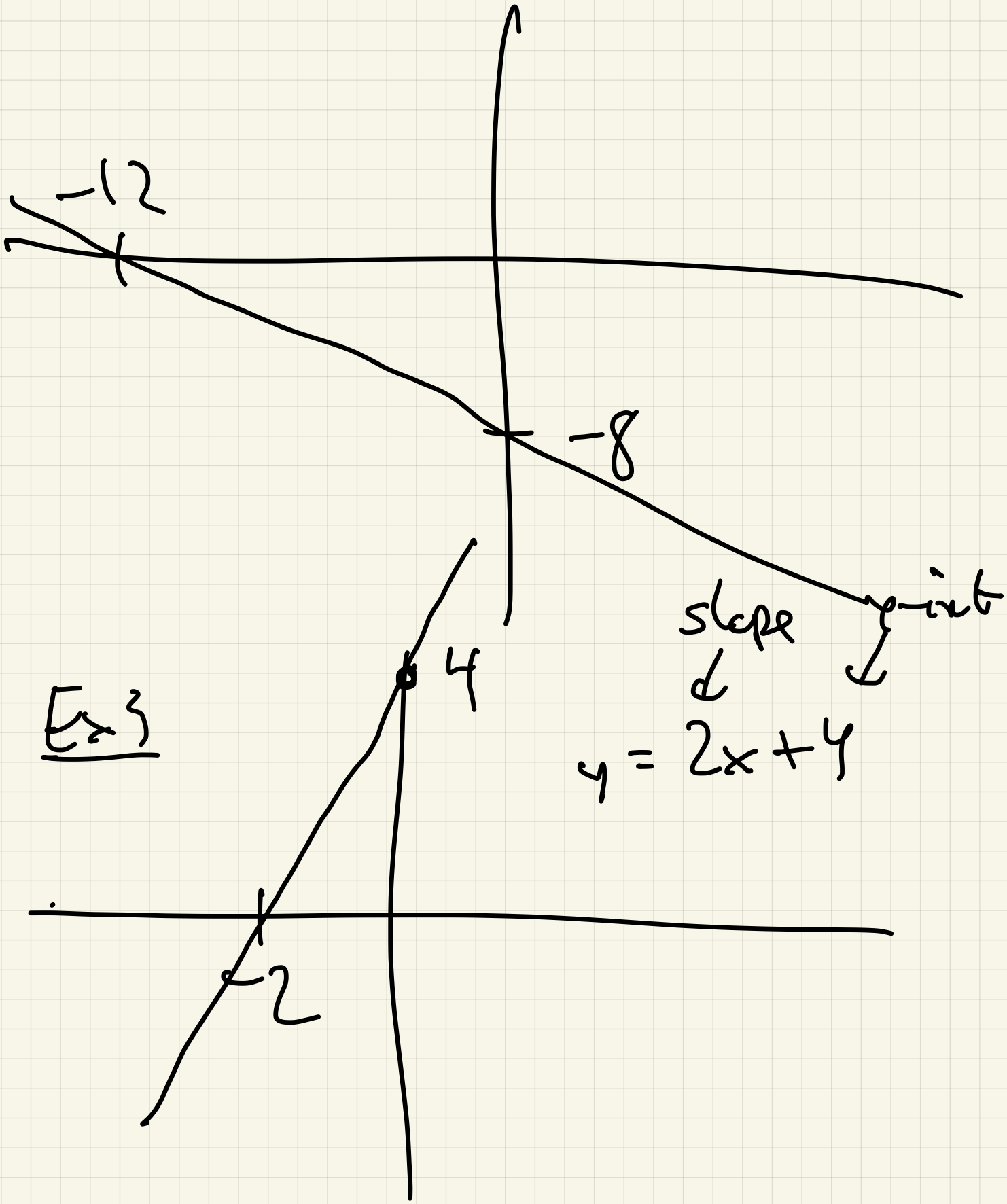
(d) $2x + 3y = -24$

$3y = -2x - 24$

$y = -\tfrac{2}{3}x - 8$

$b = -8$

$m = -\tfrac{2}{3}$


-12
-8
slope
y-int
4
Ex 3
y = 2x + 4
-2

Point-slope form

If a line L has slope m and (x_1, y_1) on L, then L has equation

$$y - y_1 = m(x - x_1)$$