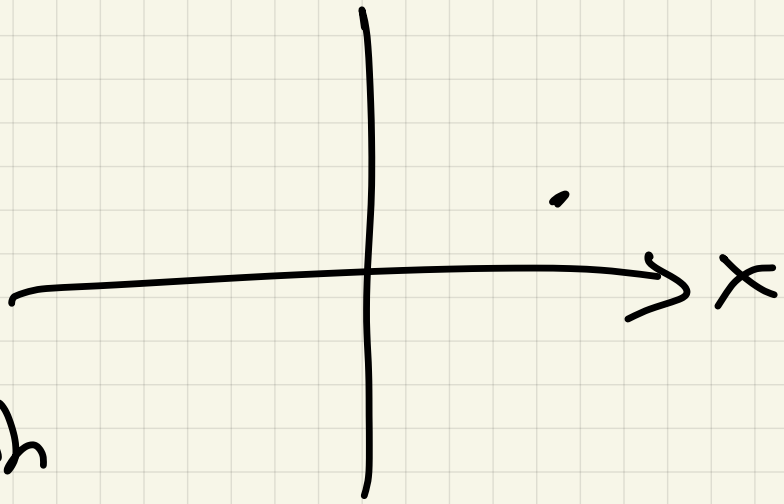
9/15/Precalc

Quiz 6 avg 65
med 80

1. $2y - x^2 + 16 = 0$

(a) x-axis symmetry



(x, y) on graph

$\Leftrightarrow$

$(x, -y)$ on graph

$$2(-y) - x^2 + 16 = 0$$

not same formula

$(0, -8)$ on graph

$(0, 8)$ not on graph

NO

(b) $(x,y)=(4,0)$ on graph.

check:

$$2(0)-4^2+16=0 \quad ??$$

(c) y-ints:

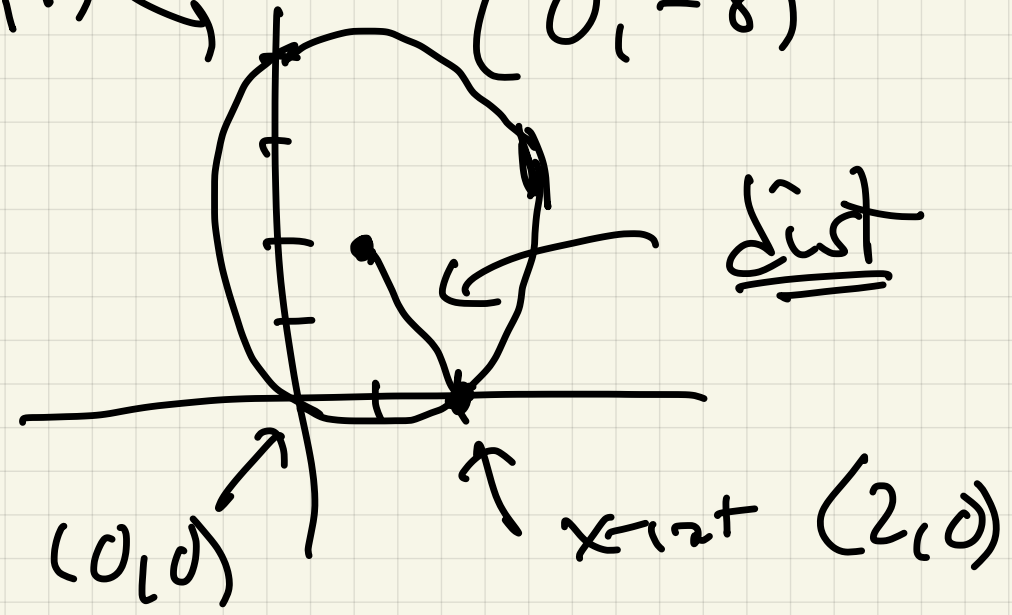
Set $x=0$ $\quad 2y-x^2+16=0$

$$2y+16=0 \Rightarrow$$

$$2y=-16 \rightarrow y=\frac{-16}{2}=-8$$

$(0,4)$ $\quad (0,-8)$

2



(a) $(1, 2)$

(b) dist from $(1,2)$ to $(2,0)$

$$dist = \sqrt{(1-2)^2 + (2-0)^2}$$

$$= \sqrt{1 + 4} = \sqrt{5}$$

(c) $(x-1)^2 + (y-2)^2 = (\sqrt{5})^2 = 5$

Last time Quadratic equation

$$ax^2 + bx + c = 0, \quad a \neq 0$$

(A) $b = 0$: $2x^2 - 17 = 0$

$$2x^2 = 17$$

$$x^2 = 17/2$$

$$x = \pm\sqrt{17/2}$$

(B) If you can factor it's also easy:

$$x^2 - 7x + 12 = 0$$

$$(x-3)(x-4) = 0$$

$x = 3, 4$ so

(C) Usually not easy

Ex 0 $x^2 - 2x - 9 = 0$

Trick $x^2 - 2x + 1 = 9 + 1$

$$(x-1)^2 = 10$$

$$x - 1 = \pm\sqrt{10}$$

$$\boxed{x = 1 \pm \sqrt{10}}$$

factoring $(x-1-\sqrt{10})(x-1+\sqrt{10})$

In general: $b \neq 0$ a number

x^2+bx is not a square,

BUT

$x^2+bx+\left(\frac{b}{2}\right)^2$ is a square

$\Downarrow$

$\left(x+\frac{b}{2}\right)^2$

adding this extra term is called "completing the square"

(a) $x^2+8x-25=0$

$\Downarrow$

$b=8$

$\left(\frac{b}{2}\right)^2=16$

$$x^2+8x+16 = 25+16$$

$$(x+4)^2 = 41$$

$$x+4 = \pm\sqrt{41}$$

$$x = -4 \pm \sqrt{41}$$

(b) $x^2+8x+20=0$

$$x^2+8x = -20$$

$$x^2+8x+16 = -20+16$$

$$(x+4)^2 = -4$$

no solution

(c) $7x^2+5x-2=0$

$$x^2+\tfrac{5}{7}x-\tfrac{2}{7}=0$$

//

$$x^2 + \frac{5}{7}x + \left(\frac{5}{14}\right)^2 = \frac{2}{7} + \left(\frac{5}{14}\right)^2$$

$b = \frac{5}{7}$ → square

$$\frac{b}{2} = \frac{5}{14}$$

$$\left(x + \frac{5}{14}\right)^2 = \frac{2}{7} + \frac{25}{196}$$

$$= \frac{2 \cdot 28}{7 \cdot 28} + \frac{25}{196}$$

$14^2 \times 7 \cdot \underline{14 \cdot 2}$

$$= \frac{56 + 25}{196} = \frac{81}{196}$$

$$\left(x + \frac{5}{14}\right)^2 = \frac{81}{196}$$

$$x + \frac{5}{14} = \pm\sqrt{\frac{81}{196}} = \pm\frac{9}{14}$$

$$x = -\frac{5}{14} \pm \frac{9}{14} = -1, \frac{2}{7}$$

$$7x^2 + 5x - 2 = 0$$

$$(7x - 2)(x + 1)$$

Ex 9 A Calc2 application

write with a completed square

(a) $$\frac{1}{x^2 + 2x + 10} = \frac{1}{x^2 + 2x + 1 + 9}$$

$$= \frac{1}{(x+1)^2 + 9}$$

(b) $$\frac{1}{\sqrt{6x - x^2}} = \frac{1}{\sqrt{-(x^2 - 6x)}} =$$

$$\frac{1}{\sqrt{-(x^2-6x+9)+9}} =$$

$$\frac{1}{\sqrt{9-(x-3)^2}}$$

Quadratic formula

We saw that completing square can solve any quadratic equation

Can automate:

SOLVE $ax^2+bx+c=0$

$\div a$

$$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

$$x^2 + \left(\frac{b}{a}\right)x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

$\left(\frac{b}{2a}\right)$ Square

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$

$$\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} \Downarrow$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Exs (a) $x^2 - 2x - 8 = 0$

$a = 1,\ b = -2,\ c = -8$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-8)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{36}}{2}$$

$$= \frac{2 \pm 6}{2} = 1 \pm 3$$

$$= 4, -2$$

$(x-4)(x+2) = 0$

$\Downarrow$

$4, -2$

(b) $3x^2 - x - 2 = 0$

$\Downarrow$

Solutions $a=3,\ b=-1,\ c=-2$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-2)}}{2(3)}$$

$$= \frac{1 \pm \sqrt{25}}{6}$$

$$= \frac{1 \pm 5}{6} = \frac{1+5}{6} = 1$$

$$\frac{1-5}{6} = \frac{-4}{6} = -\frac{2}{3}$$

but

$3x^2 - x - 2$

$=$

$(3x+2)(x-1) = 0$

$x = -2/3 \qquad x = 1$

(c) $1 \cdot x^2 + 2x + 4 = 0$

$$x = \frac{-2 \pm \sqrt{4 - 4(4)}}{2}$$

$$= \frac{-2 \pm \sqrt{-12}}{2} \quad \times$$

no solution

(d) $x^2 + 6x + 9 = 0$

$$x = \frac{-6 \pm \sqrt{36 - 4(1)(9)}}{2(1)} =$$

$$\frac{-6 \pm \sqrt{0}}{2} = \frac{-6 \pm 0}{2} = -3$$

Def: $ax^2 + bx + c = 0$

The discriminant is

$d = b^2 - 4ac$

① $d > 0$ 2 solutions

② $d = 0$ 1 solution

③ $d < 0$ 0 solutions

Skip 1.5

1.6 Other equations

Ex (a) $x^4 - 25x^2 = 0$

$x^2(x^2 - 25)$

$x^2(x-5)(x+5) = 0$

$x = 0, 5, -5$

(b) $x^3 + x^2 - 9x - 9 = 0$

$x^2(x+1) - 9(x+1) = 0$

$(x^2 - 9)(x+1) = 0$

$(x-3)(x+3)(x+1) = 0$

$x = 3, -3, -1$