9/1/ Precalc: P3 – P4

# Factoring Polynomials

Hg 9/1.

factor with integer coefficients

Ex1 Can factor out "obvious"

(a) = $x^5(x+1)^2$

already factored

(b) $2x^2+44x$

$2x$ $2x$ common to both

$2x(x+22)$

(c) $x(x-5)+7(x-5)$

$(x-5)[x+7]$ "Grouping"

(d) $\boxed{x^3-3x^2-4x+12}$

$x^2(x-3) - 4(x-3)$

$=$

$(x-3)(x^2-4)$

$A^2-B^2$

$=$

$(A-B)(A+B)$

$(x-3)(x-2)(x+2)$

**Difference of squares!**

$u^2-v^2 = (u-v)(u+v)$

Ex2 (a) $5-45x^2$ $\quad 1^2-(3x)^2$

$=$

$5(1-9x^2) =$

$$5(1-3x)(1+3x)$$

(b) $$81x^4-16$$

$4$

$$(9x^2)^2-4^2$$

$$=$$

$$(9x^2-4)(9x^2+4)$$

$$((3x)^2-2^2)(9x^2+4)$$

$$(3x-2)(3x+2)(9x^2+4)$$

(c) $$(x-1)^2-(y+1)^2$$

$$\cancel{A^2-B^2} \quad u^2-v^2$$

$$((x-1)-(y+1))\cdot((x-1)+(y+1))$$

$$=(x-y-2)(x+y)$$

## Difference of cubes:

$$u^3 - v^3 = (u-v)(u^2+uv+v^2)$$

Ex 3 $\quad x^3 - 8 = (x-2)(x^2+2x+4)$

## Typical Calculus/DE examples

How does $x^2 + ax + b$    $a, b$ ints

$\qquad\qquad = (x+A)(x+B)$    $A, B$ ints?

find such ints $A$ & $B$?, $\begin{cases} AB = b \\ A+B = a \end{cases}$

Ex 4 $\quad x^2 + 7x + 10$

$(x+2)(x+5)$

(a)

| | |
|---|---|
| 1 | 10 |
| 2 | 5 |
| 5 | 2 |
| 10 | 1 |

$x^2 - 7x + 10$

(b) $(x-2)(x-5)$

(c) $x^2 - x - 6$

//

| | |
|---|---|
| $-1$ | $6$ |
| $-2$ | $3$ |
| $2$ | $-3$ |
| $1$ | $-6$ |

$(x-3)(x+2)$

(d) $x^2 + 5x - 6$

4

$(x+6)(x-1)$

(e) $x^2 + 8x + 16$

| | |
|---|---|
| $1$ | $16$ |
| $-1$ | $-16$ |
| $2$ | $8$ |
| $-2$ | $-8$ |
| $4$ | $4$ |
| $-4$ | $-4$ |

$(x+4)(x+4)$

(f) $x^2 - 8x + 16$

$(x-4)^2$

(g) $x^3 - 9x$

$x(x^2-9)$

$x(x-3)(x+3)$

P5 <u>Rational</u> expressions

$R(x) = \dfrac{p(x)}{q(x)}$

<u>Domains</u>: set of

$\{x : R(x)$ is defined$\}$

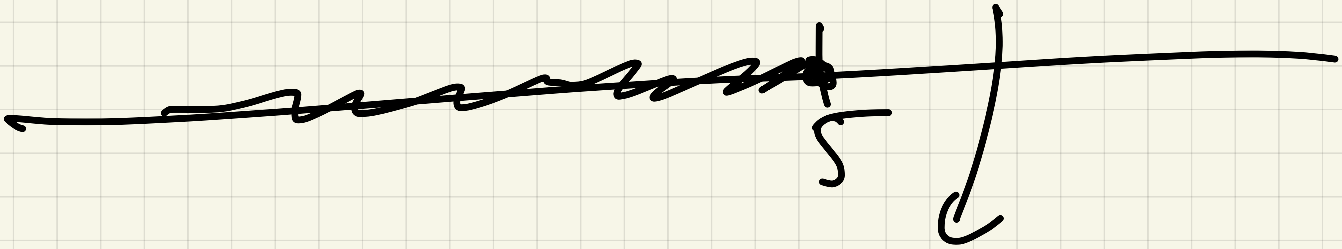
(a) $R(x) = x^2 - 4x$

Domain $= (-\infty, \infty) = \mathbb{R}$

(b) $R(x) = \sqrt{5-x}$

Domain: NEED $5-x \geq 0$

$5-x \geq 0 \Rightarrow 5 \geq x$




$(-\infty, 5]$

(c) $R(x) = \sqrt{5-x^2}$

NEED $5-x^2 \geq 0$

$5 \geq x^2$

$-\sqrt{5} \leq x \leq \sqrt{5}$

$[-\sqrt{5}, \sqrt{5}]$

(d) $R(x) = \sqrt{x^2-100}$

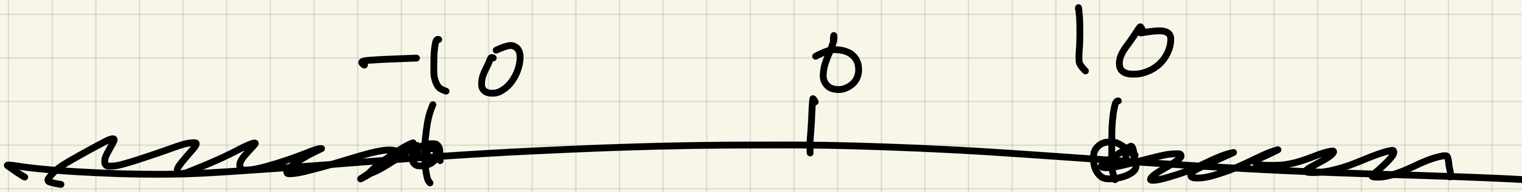
$x^2-100 \geq 0$

$x^2 \geq 100$

$x \geq 10$ or $x \leq -10$

"union"

$(-\infty, -10] \cup [10, \infty)$




(e) $R(x) = \dfrac{x+5}{x-10}$

$x - 10 \neq 0$

So Domain = $x : x \neq 10$

$(-\infty, 10) \cup (10, \infty)$

(f) $R(x) = \dfrac{\sqrt{x-8}}{x+5}$

Need: $\underbrace{x - 8 \geq 0}_{x \geq 8}$, $x \neq -5$

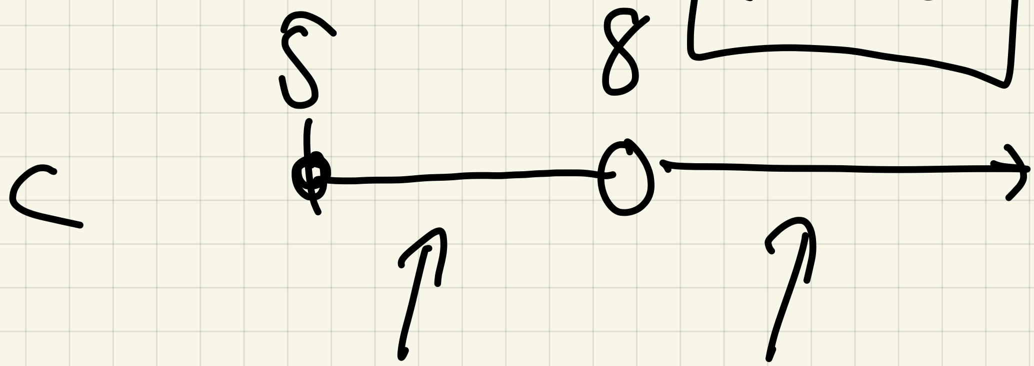
$[8, \infty)$

(9) $\dfrac{\sqrt{x-5}}{x-8}$

$x-5 \geq 0$

$\boxed{x \geq 5}$

$\boxed{x \neq 8}$




$[5,8) \cup (8,\infty)$

<u>Simplification</u>: can cancel when appropriate

<u>Ex2</u> (a)

$$f(x) = \frac{x^2+5x}{x^2-25} = \frac{x\cancel{(x+5)}}{(x-5)\cancel{(x+5)}}$$

$$= \frac{x}{x-5}, \quad x \neq -5$$

(h) $R(x) = \dfrac{3x^2 - x - 2}{5 - 4x - x^2} =$

$$\frac{3x^2 - x - 2}{-(x^2 + 4x - 5)} = \frac{3x^2 - x - 2}{-(x+5)(x-1)}$$

earlier

$3x^2 - x - 2$

$(3x + A)(x + B) = (3x+2)(x-1)$

| | |
|---|---|
| 2 | −1 |
| −2 | 1 |
| 1 | −2 |
| −1 | 2 |

$$\frac{(3x+2)\cancel{(x-1)}}{-(x+5)\cancel{(x-1)}} = -\frac{(3x+2)}{x+5}$$

$x \neq 1$

$$R(x) = \frac{x^2+4x-5}{(x^2-25)(x^2-1)} =$$

$$\frac{\cancel{(x+5)}\cancel{(x-1)}}{(x-5)\cancel{(x+5)}\cancel{(x-1)}(x+1)} =$$

$$\frac{1}{(x-5)(x+1)} \qquad x \neq -5, 1$$