

10/22/Precalc

Exam 2 → Thurs

Oct 29

Review sheet

Ch 2, 3.1-3.2

Quiz Monday 3.2

Quiz 16

1) $f(x) = \sqrt[3]{x-5}$ $g(x) = \underline{x^3+5}$

$$f(g(x)) = f(x^3+5) =$$

$$\sqrt[3]{(x^3+5)-5}$$

$$\sqrt[3]{x^3} = x$$

$$g(f(x)) = g(\sqrt[3]{x-5}) =$$

$$(\sqrt[3]{x-5})^3 + 5$$

$$x - \cancel{5} + \cancel{5} = x \quad \checkmark$$

2

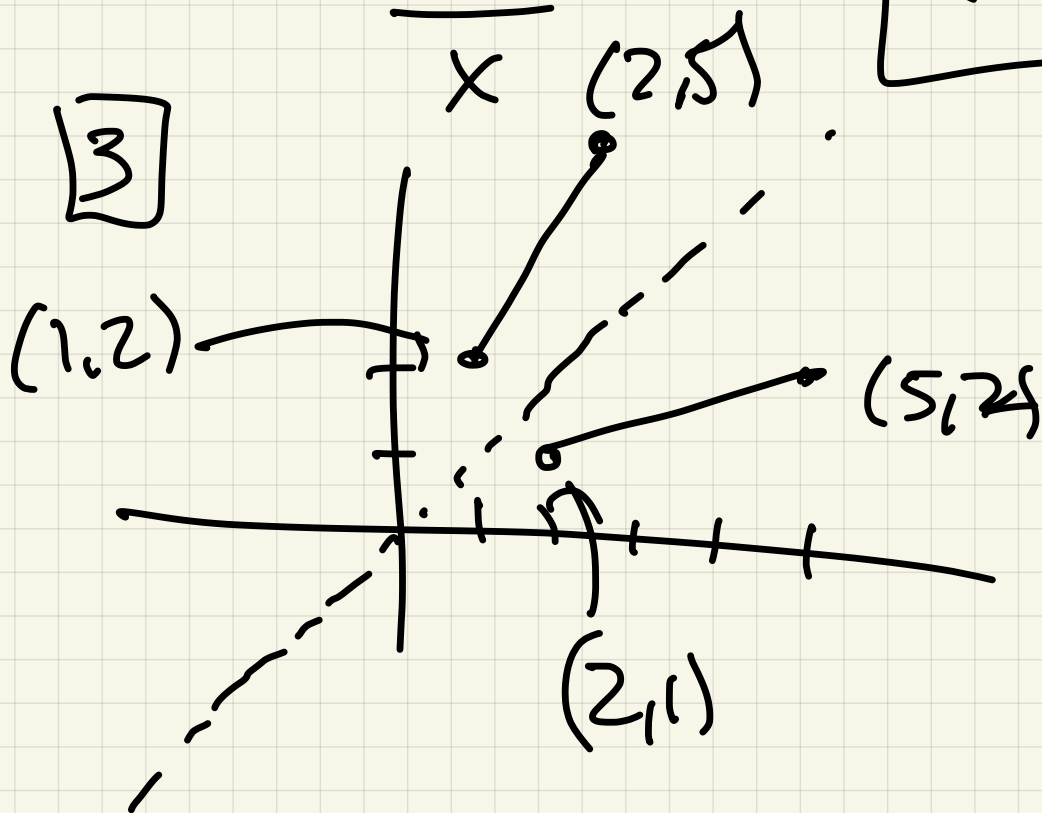
$$g(f(x)) = \frac{2}{4-x}$$

$$x = f^{-1}(y) = \frac{2}{4-y} \rightarrow$$

$$(4-y)x = 2$$

$$4-y = \frac{2}{x} \Rightarrow 4 = \frac{2}{x} + y \Rightarrow$$

$$\frac{4x-2}{x} = \boxed{4 - \frac{2}{x} = y}$$



Last time

Polynomials

Functions

$$y = f(x) = \boxed{a_d x^d} + \dots + a_0$$

(A) Leading coefficient test

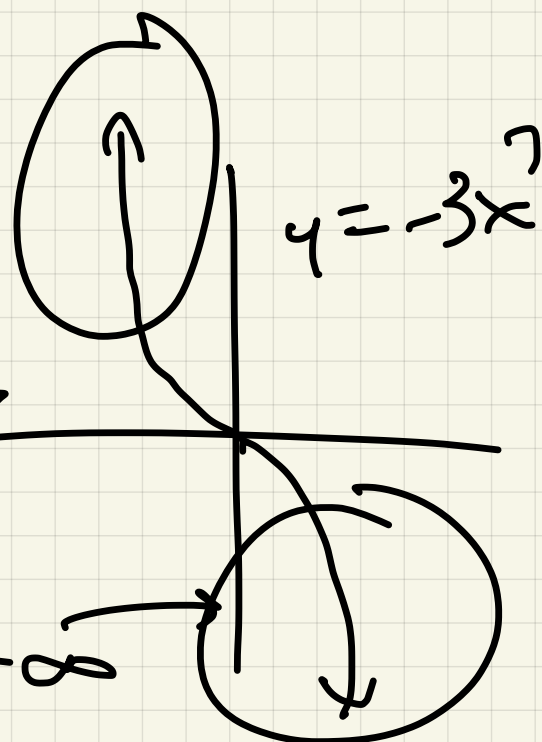
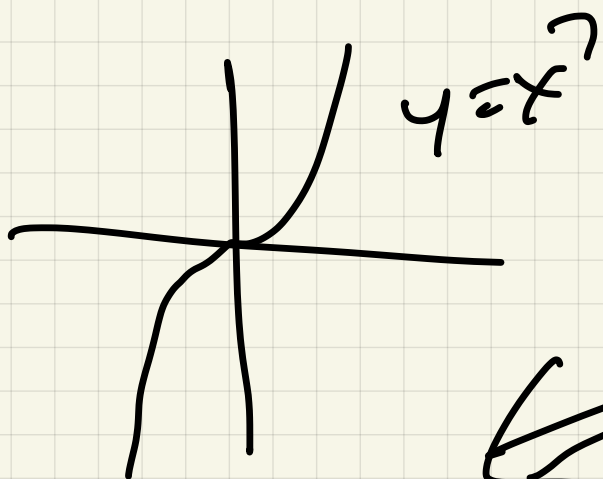
Long term behavior (as $x \rightarrow +\infty$ or $x \rightarrow -\infty$) are same as $y = a_d x^d$

Ex Determine behavior of

graph as $x \rightarrow \infty$ or $x \rightarrow -\infty$

(a) $y = f(x) = \boxed{-3x^7 + 15x^5 - 100x^2}$

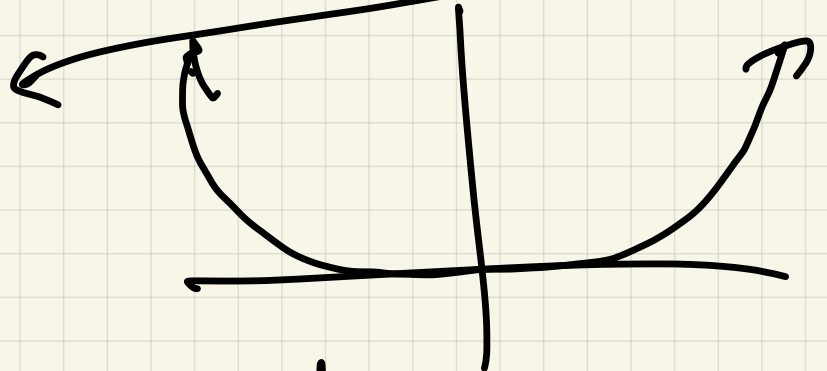
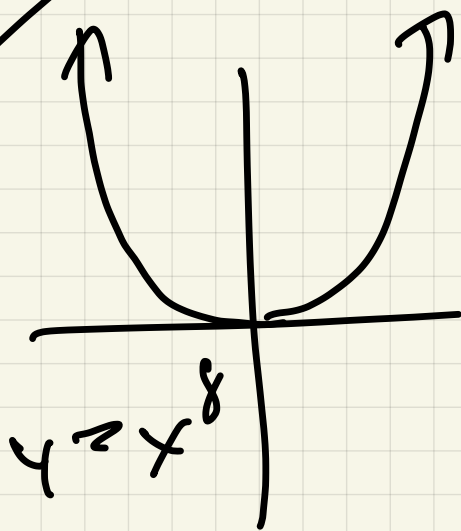
focus on $-3x^7 = y$



$y \rightarrow -\infty$ as $x \rightarrow +\infty$
 $y \rightarrow +\infty$ as $x \rightarrow -\infty$

Book: falls to right
lifts to left

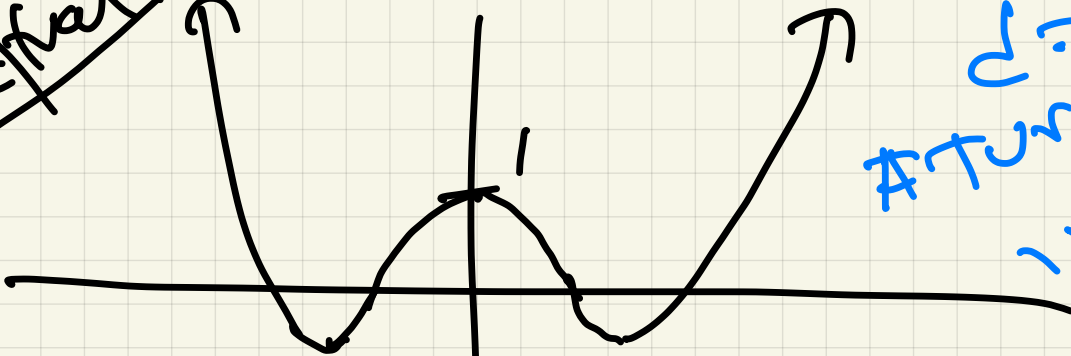
$$(b) \quad y = f(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{120} + \frac{x^8}{40320}$$



$$y = \frac{1}{40320} x^8$$

$y \rightarrow \infty$ as $x \rightarrow \infty$
 $x \rightarrow -\infty$

~~Actual:~~



$d = 8$
#Turning pt
1, 5, 3

[B] Zeros of polynomial function

Defn: The number $x=a$
is a zero for function $f(x)$
if $f(a) = 0$

Note: a is a zero for $f(x)$

$\Downarrow$

$$f(a) = 0$$

$\Downarrow$

$(a, 0)$ is x-intercept for

graph of $y = f(x)$

$\Downarrow$

$x=a$ is solution to $f(x) = 0$

$\Downarrow$

$(x-a)$ is factor of $f(x)$
polynomial

Ex 3 Find all zeros of

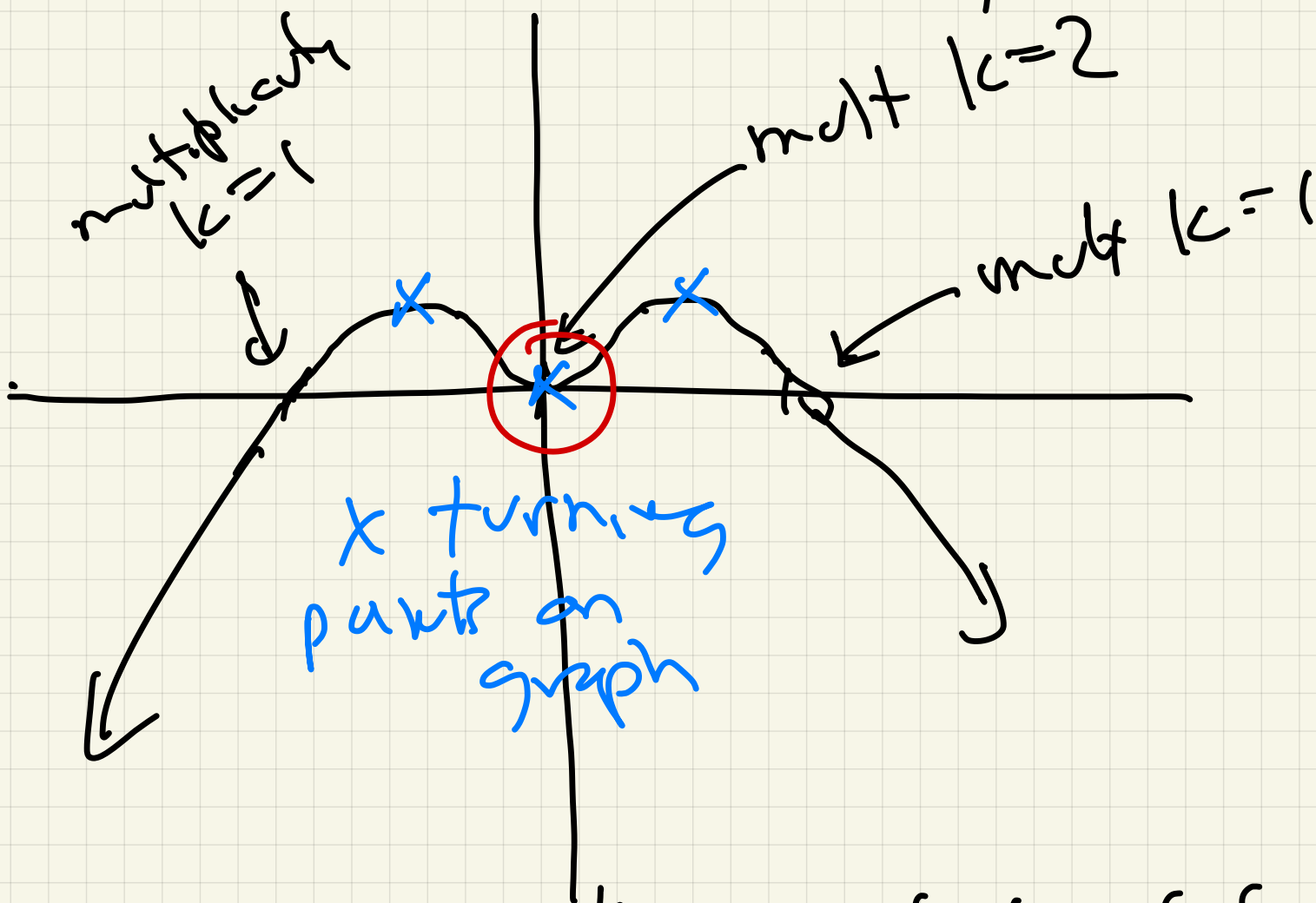
$$f(x) = -2x^4 + 2x^2$$

$$= -2x^2(x^2 - 1)$$

$$-2x^{(2)}(x-1)^{(1)}(x+1)^{(1)} = 0$$

↑

$$x = 0, \pm 1$$



Defn: If $(x-a)^k$ is a factor of $f(x)$ but $(x-a)^{k+1}$ is not, then a is a zero of multiplicity k

① k odd $\Rightarrow$ graph crosses ~~x-axis~~ at $x=a$

② k even $\Rightarrow$ graph touches
x-axis at $x=a$
but does not cross

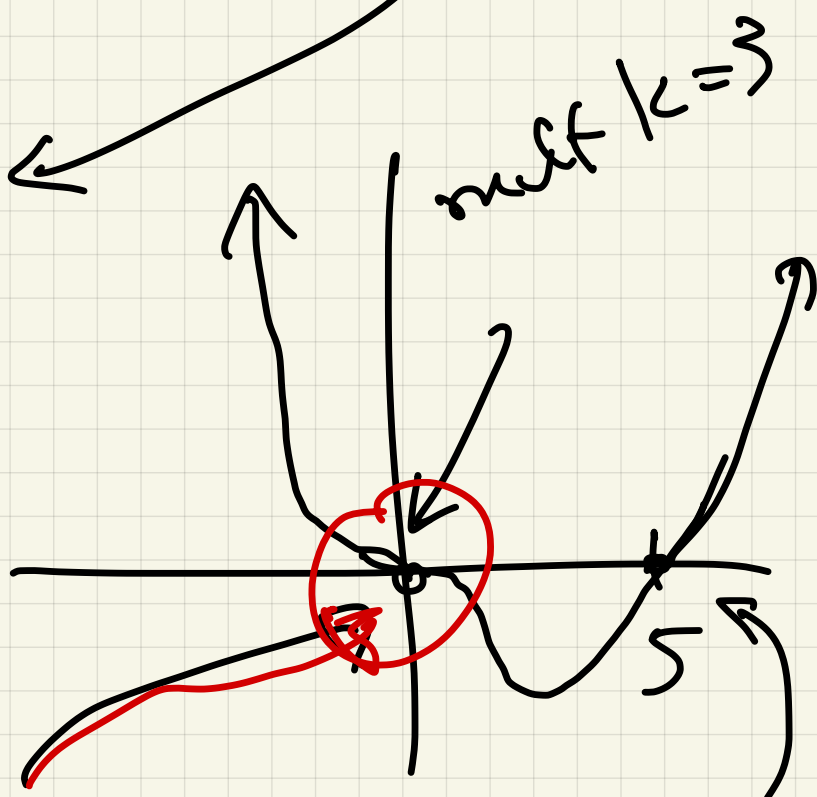
③ Graph has at most
 $d-1$ turning points
 $d = \text{degree of } f(x)$

Ex 4

$$y = x^4 - 5x^3$$

$$= x^3(x-5)$$

zero	mult
0	3
5	1

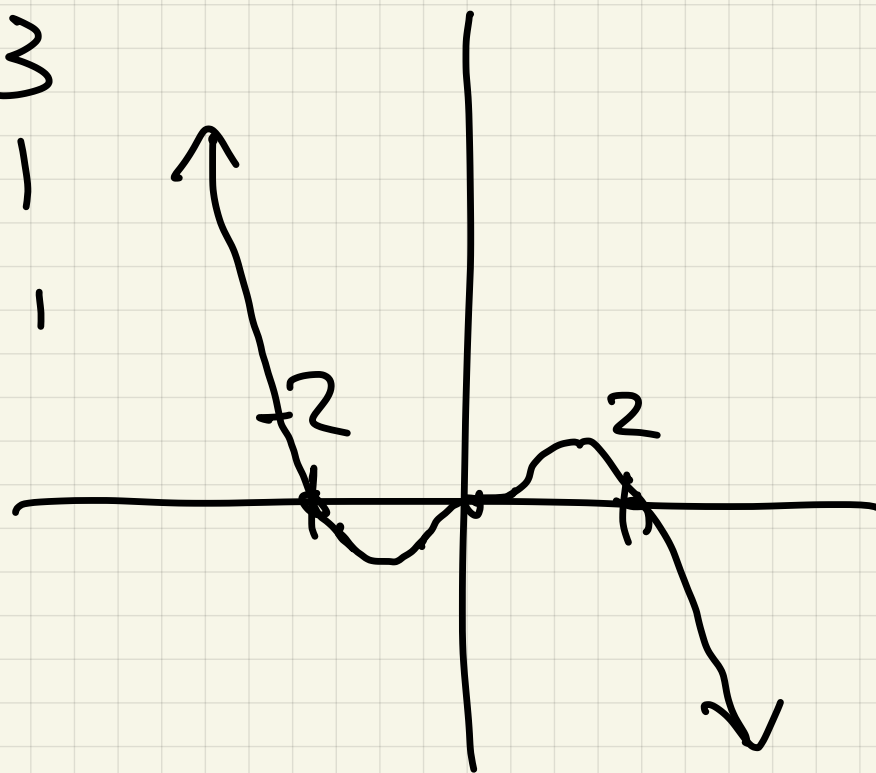


mult $k=1$

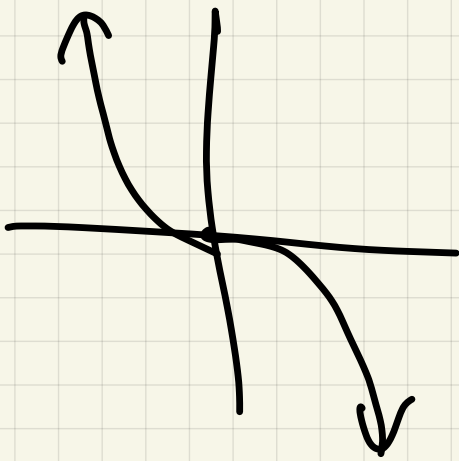
If mult > 1 ,
graph is tangent
to x-axis where
it touches

(b) $y = -x^5 + 4x^3$
 $= -x^3(x^2 - 4)$
 $= -x^{(3)}(x-2)^{(1)}(x+2)^{(1)}$

zero	mult
0	3
2	1
-2	1



$$y = -x^5$$



~~(c)~~
y =

$$(c) \quad y = -x^5 + 8x^2$$

$$= -x^2(\underline{x^3 - 8})$$

$$= -x^2(x-2)(\underline{x^2 + 2x + 4})$$

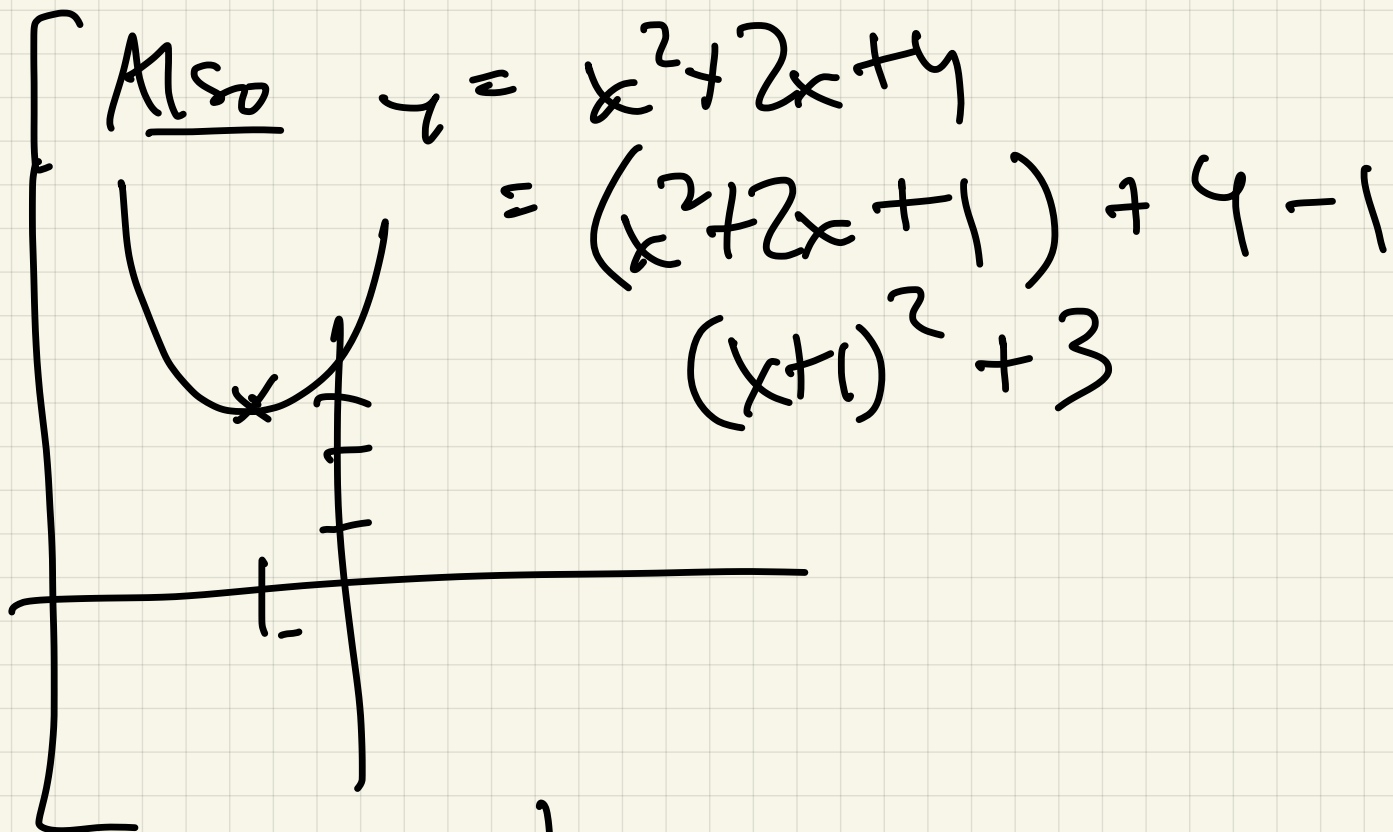
$$(x^3 - a^3) = (x-a)(x^2 + ax + a^2)$$

§ 1.4

discriminant: $b^2 - 4ac$

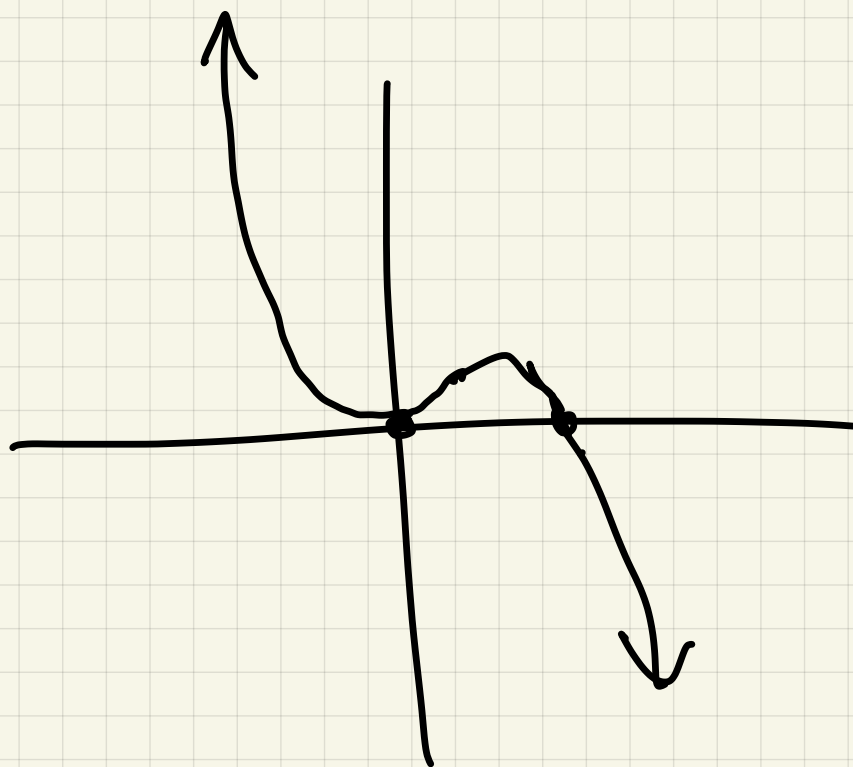
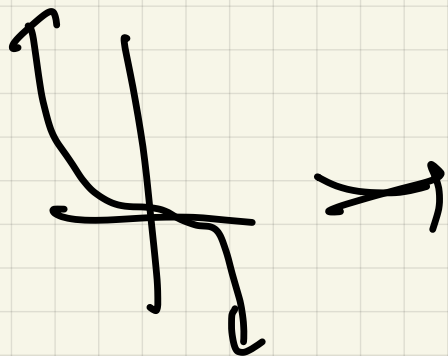
$$4 - 4(1)(4) = -12 < 0$$

$\therefore$ no factoring



zeros	mult
0	2
2	1

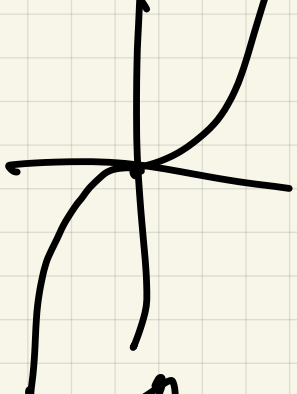
$$y = -x^3$$

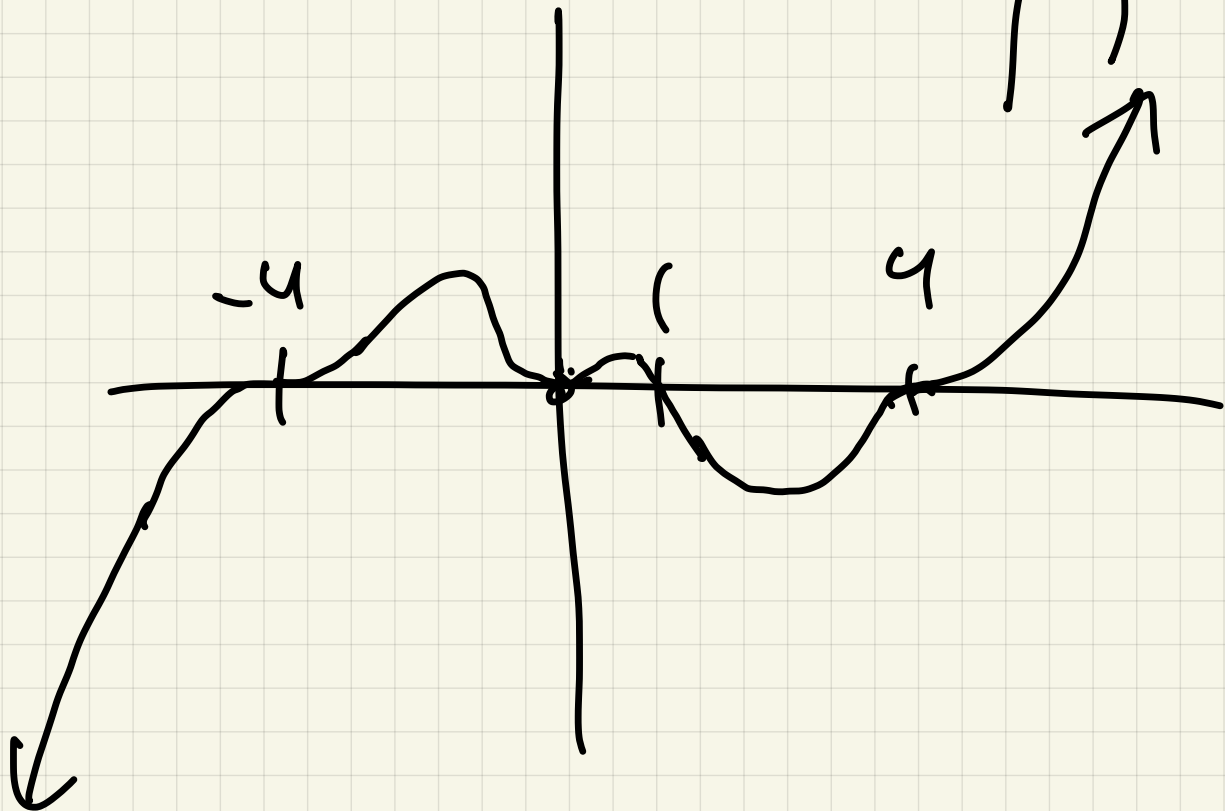


$$(d) \quad y = 2(\underline{x+4})^3 \underline{x}^4 (\underline{x-1})^1 (\underline{x-4})^3$$

zeros	mult
-4	(3)
0	4
1	1
4	3

long term??

$$y = 2x''$$




$$(e) \quad y = -3(\underline{x+4})^1 \underline{x}^3 (\underline{x-2})^1 (\underline{x-4})^1 (\underline{x-6})^1$$

2nd	mult
-4	1
0	3
2	1
4	1
6	1
8	1

long form

$$y = -3x^8$$

