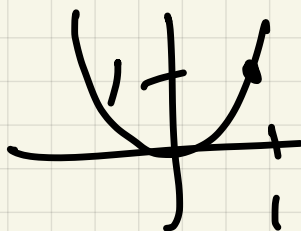


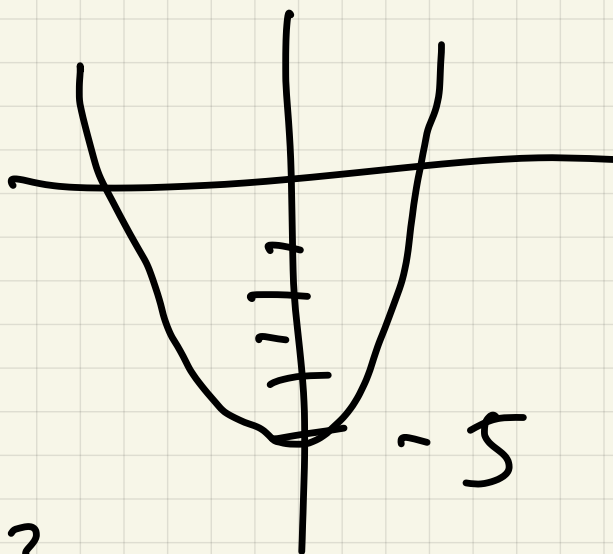
10/15 / Precalc

Unit 13

① (a) $y = x^2$

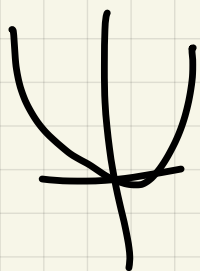


(b) $y = x^2 - 5$
 ~~$y =$~~ $(x-5)^2$

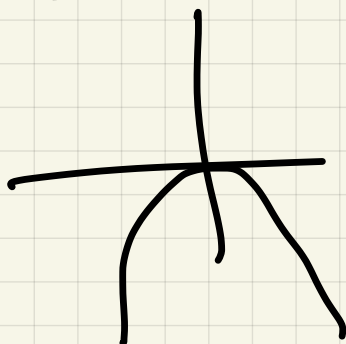


(c) $y = -(x-5)^2$

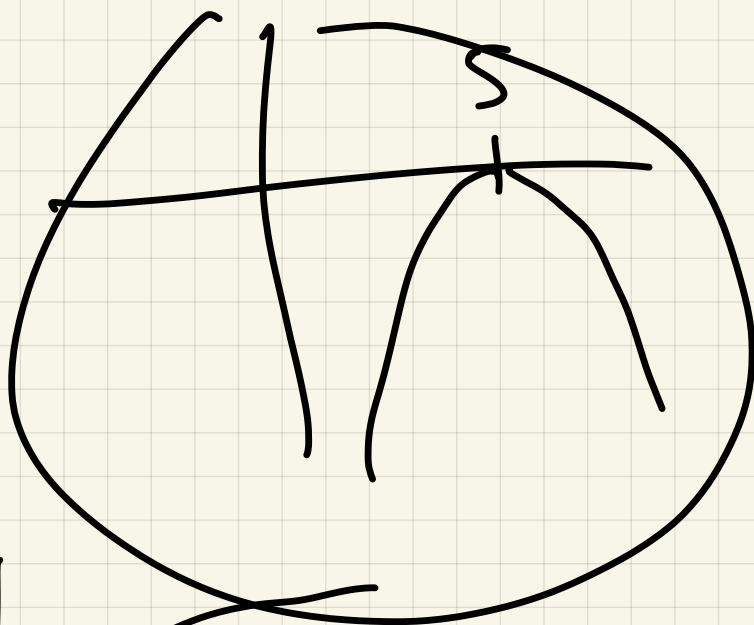
$y = x^2$



$y = -x^2$



$y = -(x-5)^2$

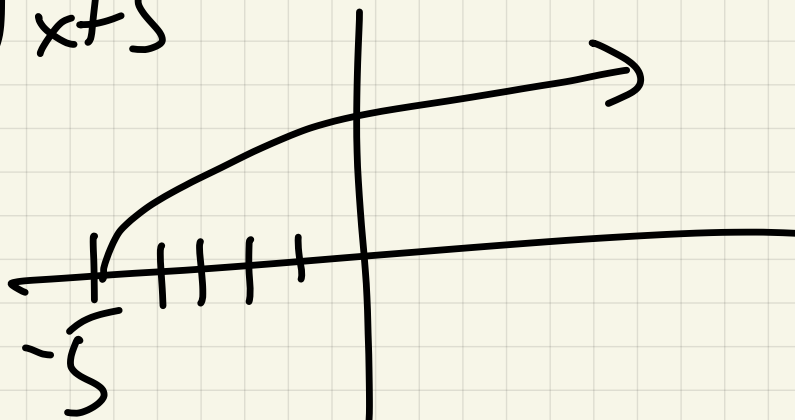


②

$y = \sqrt{x}$



$$(b) = y = \sqrt{x+5}$$



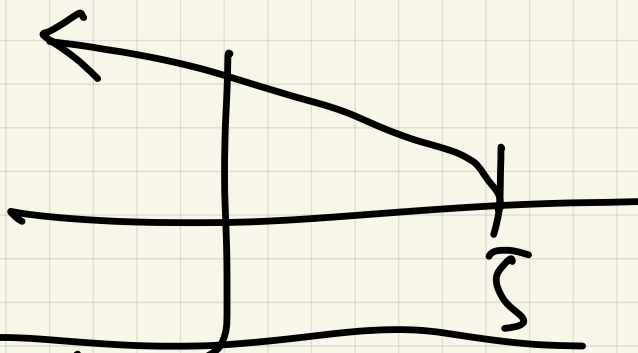
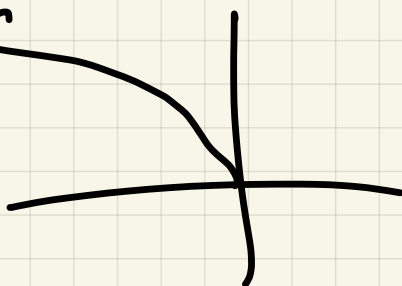
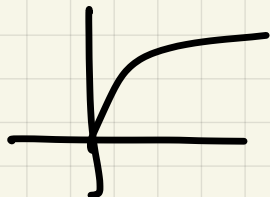
$$(c) \sqrt{5-x} = \sqrt{-(x-5)}$$

$$\sqrt{x}$$

$$\sqrt{-x}$$

→

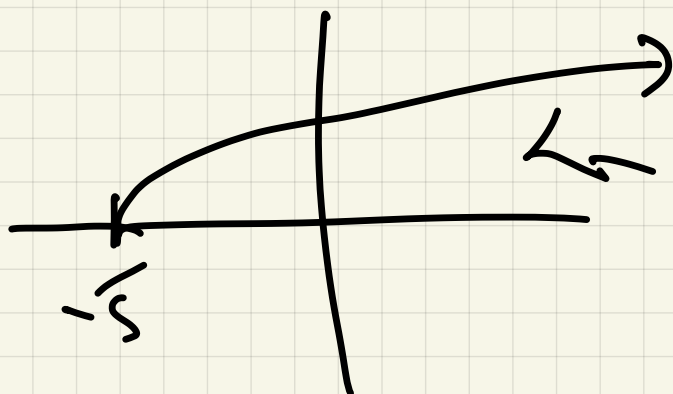
$$\sqrt{-(x-5)}$$



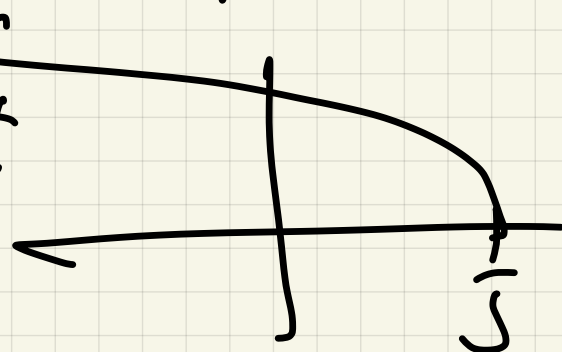
OK

$$y = \sqrt{5+x}$$

$$y = \sqrt{5-x}$$



← reflect



§2.6 Last time Combining functions

$$(f+g)(x) = f(x) + g(x)$$

$$(f-g)$$

$$(f \cdot g)$$

$$(f/g)$$

New $(f \circ g)(x) = f(g(x))$

Recognizing composition:

$$h(x) = \sqrt{3-7x} = (f \circ g)(x)$$

$$f(x) = \sqrt{x}$$

$$g(x) = 3-7x$$

§2.7 Inverses

Functions f, g are inverses

to each other i.f

$$\begin{cases} g(f(x)) = x & \text{all } x \text{ in dom } f \\ f(g(x)) = x & \text{all } x \text{ in dom } g \end{cases}$$

Notation: $g(x) = f^{-1}(x)$

Ex) Check that f, g inverses:

$$(a) \quad f, g: \mathbb{R} \rightarrow \mathbb{R}, \quad \begin{array}{l} f(x) = x+2 \\ g(x) = x-2 \end{array}$$

$$f(g(x)) = f(x-2) = (x-2) + 2 = x$$

$$g(f(x)) = g(x+2) = (x+2) - 2 = x$$

$$(b) \quad f(x) = 3x$$

$$g(x) = \frac{1}{3}x$$

$$f(g(x)) = f\left(\frac{1}{3}x\right) = 3\left(\frac{1}{3}x\right) = x$$

$$(c) \quad f, g: (0, \infty) \rightarrow (0, \infty)$$

$$f(x) = \frac{1}{x} \quad g(x) = \frac{1}{x}$$

$$f(g(x)) = f\left(\frac{1}{x}\right) = \frac{1}{\left(\frac{1}{x}\right)} = x \checkmark$$

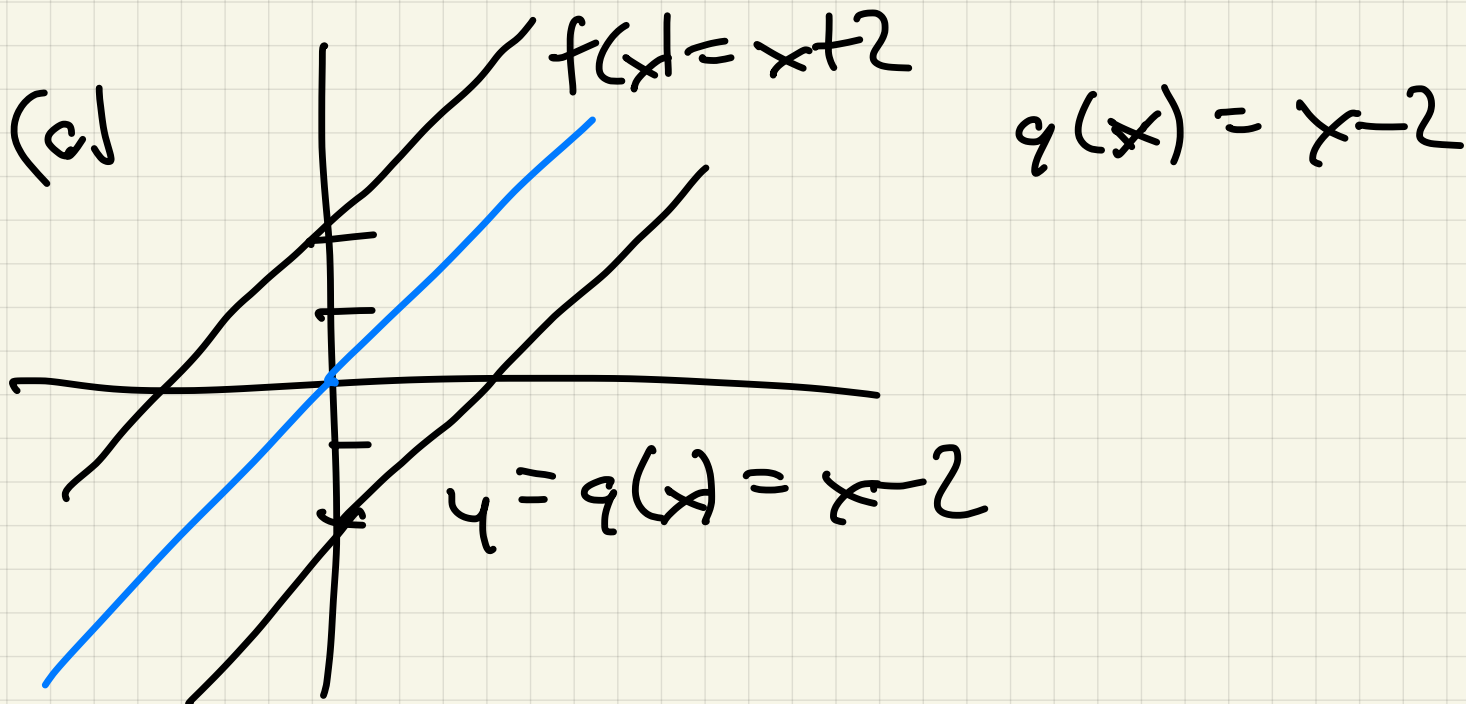
$$(d) = f(x) = \sqrt{x}, \quad x \geq 0$$

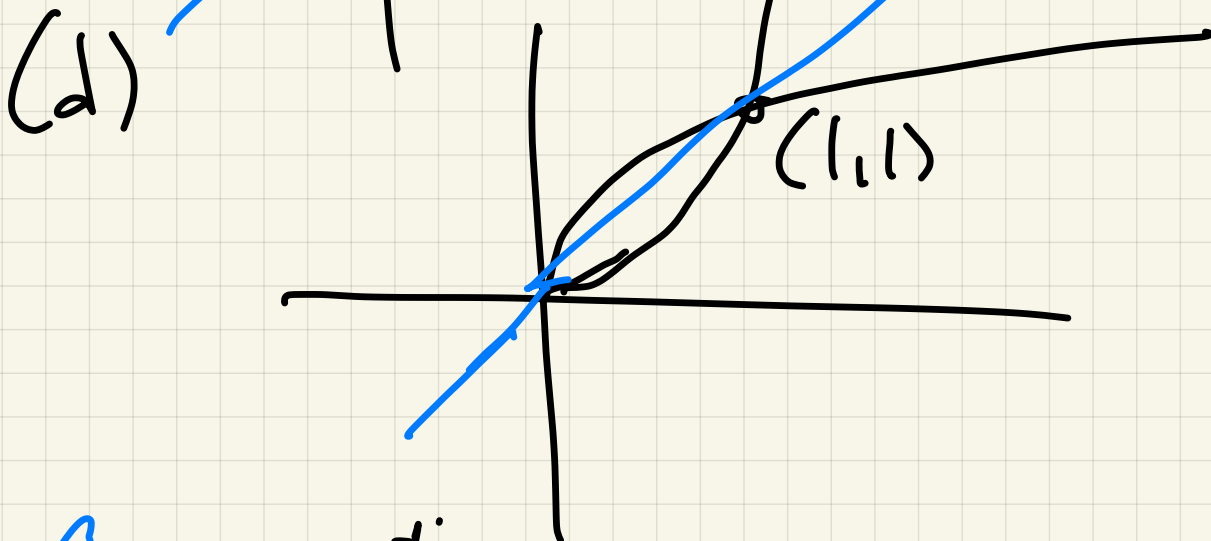
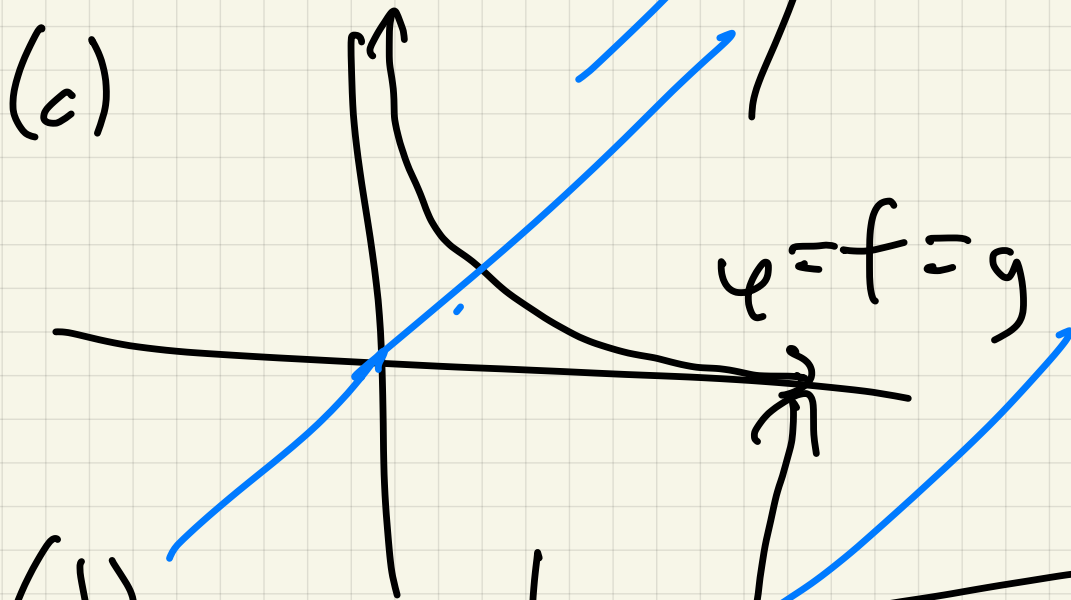
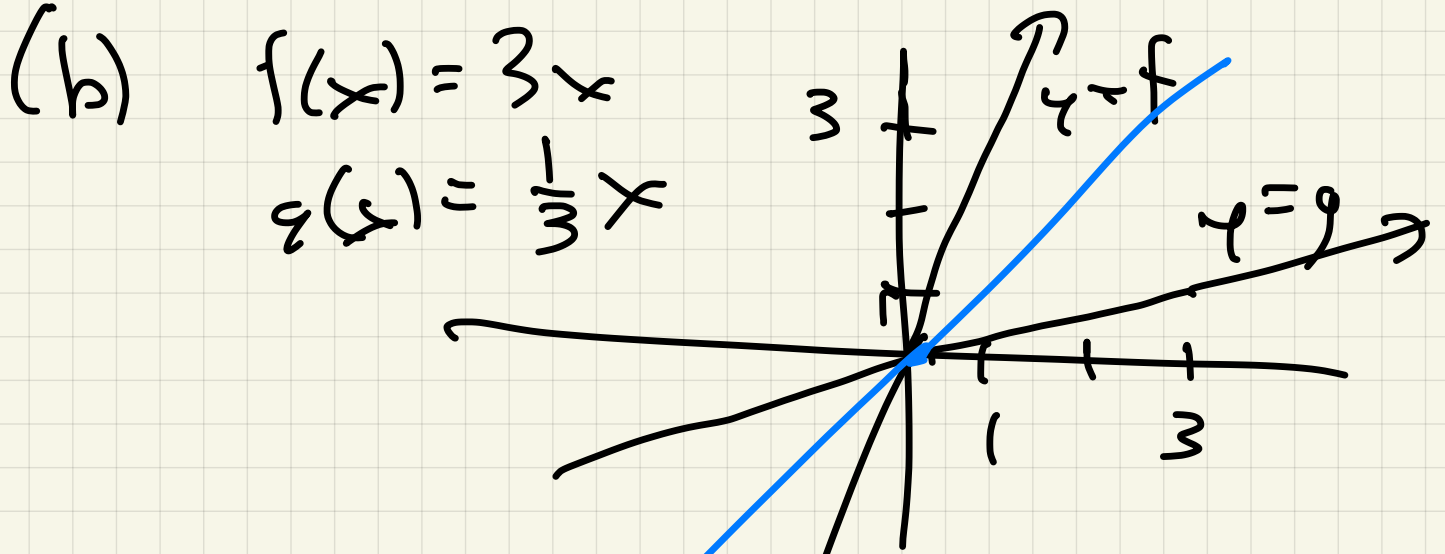
$$g(x) = x^2, \quad x \geq 0$$

$$f(g(x)) = f(x^2) = \sqrt{x^2} = |x| = x \quad x \geq 0$$

$$g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 = x \quad x \geq 0$$

Ex2 Sketch graphs
of f, g from Ex1



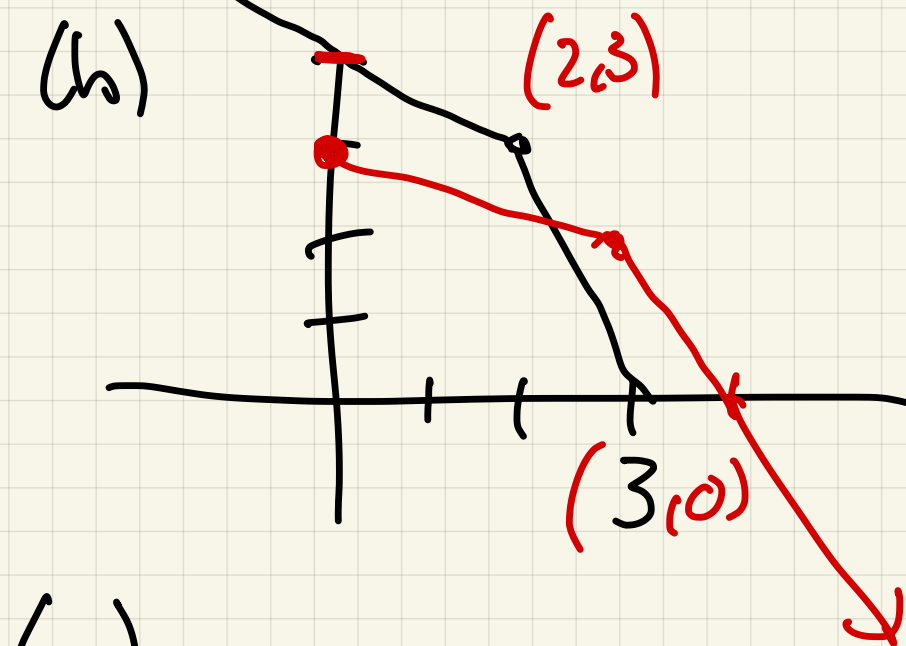
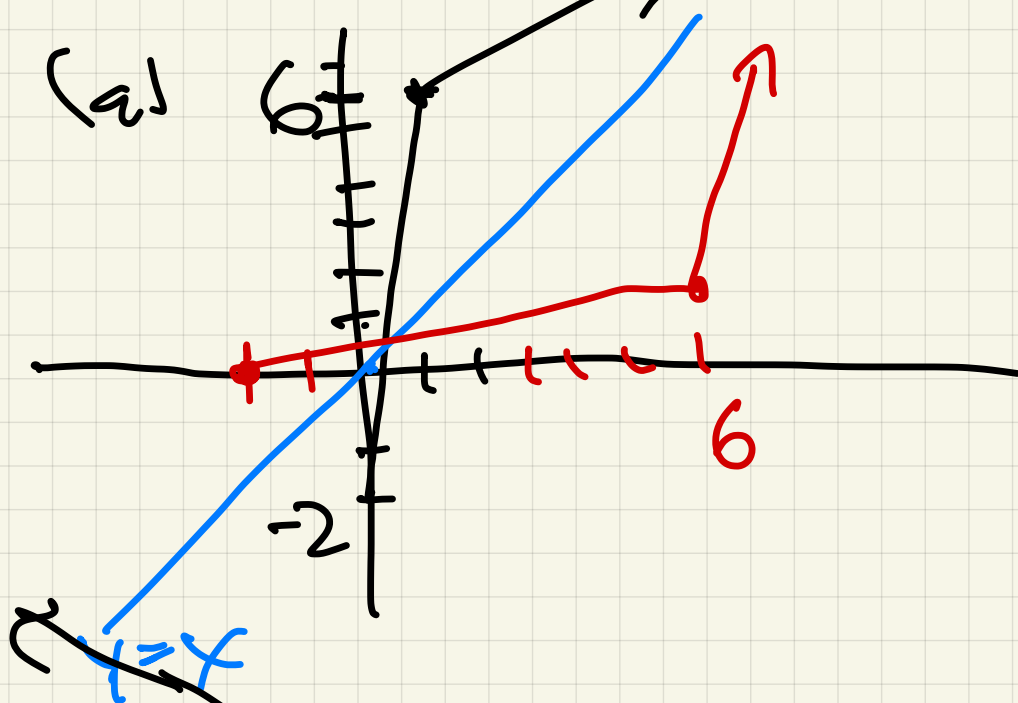


Connection:

Graph of $y = f^{-1} =$
 (Graph of $y = f$) reflected about

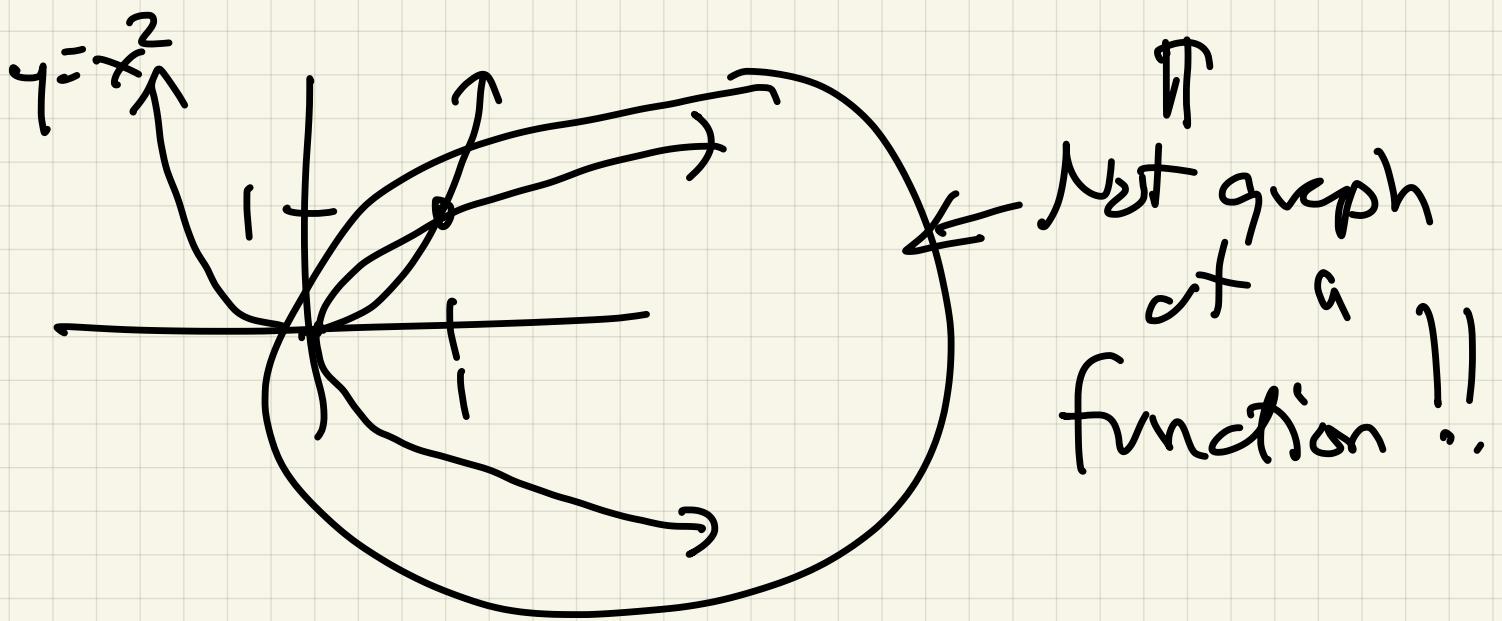
line $y=x$
 $y=f^{-1}$

Ex3 Sketch graph of
 given graph of f



(c)

$y=f(x)=x^2$ does
NOT have inverse

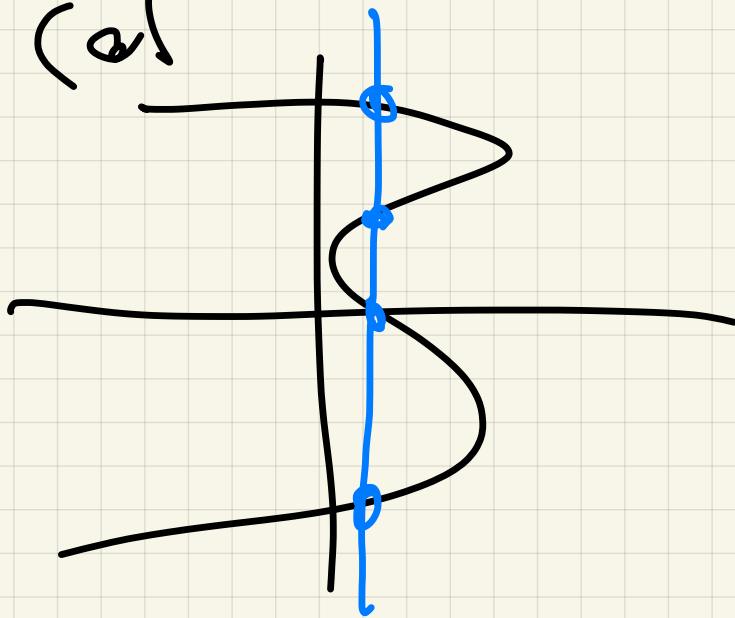


Take away:

- ① A set of points G in plane
is graph of a function $\Leftrightarrow$
every vertical line intersects
 G at most once.
- ② A function $y = f(x)$ has an
inverse $\Leftrightarrow$ Graph of $f(x)$
intersects each horizontal line
at most once

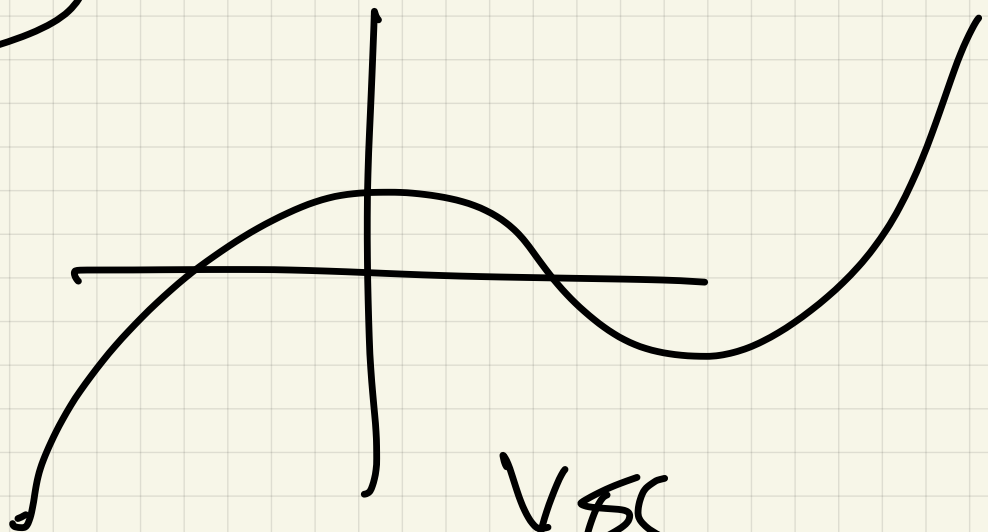
Ex 4 Are these graphs of functions?

(a)



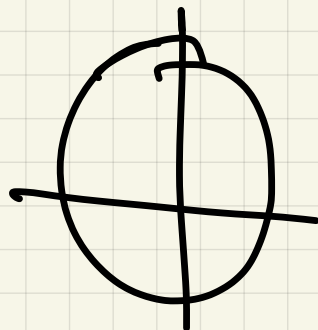
NO

(b)



YES

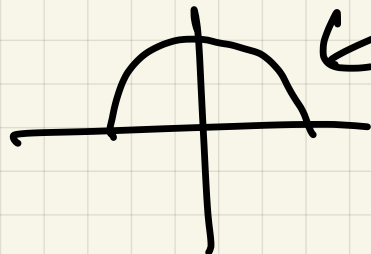
(c)



NO

$$x^2 + y^2 = 1$$

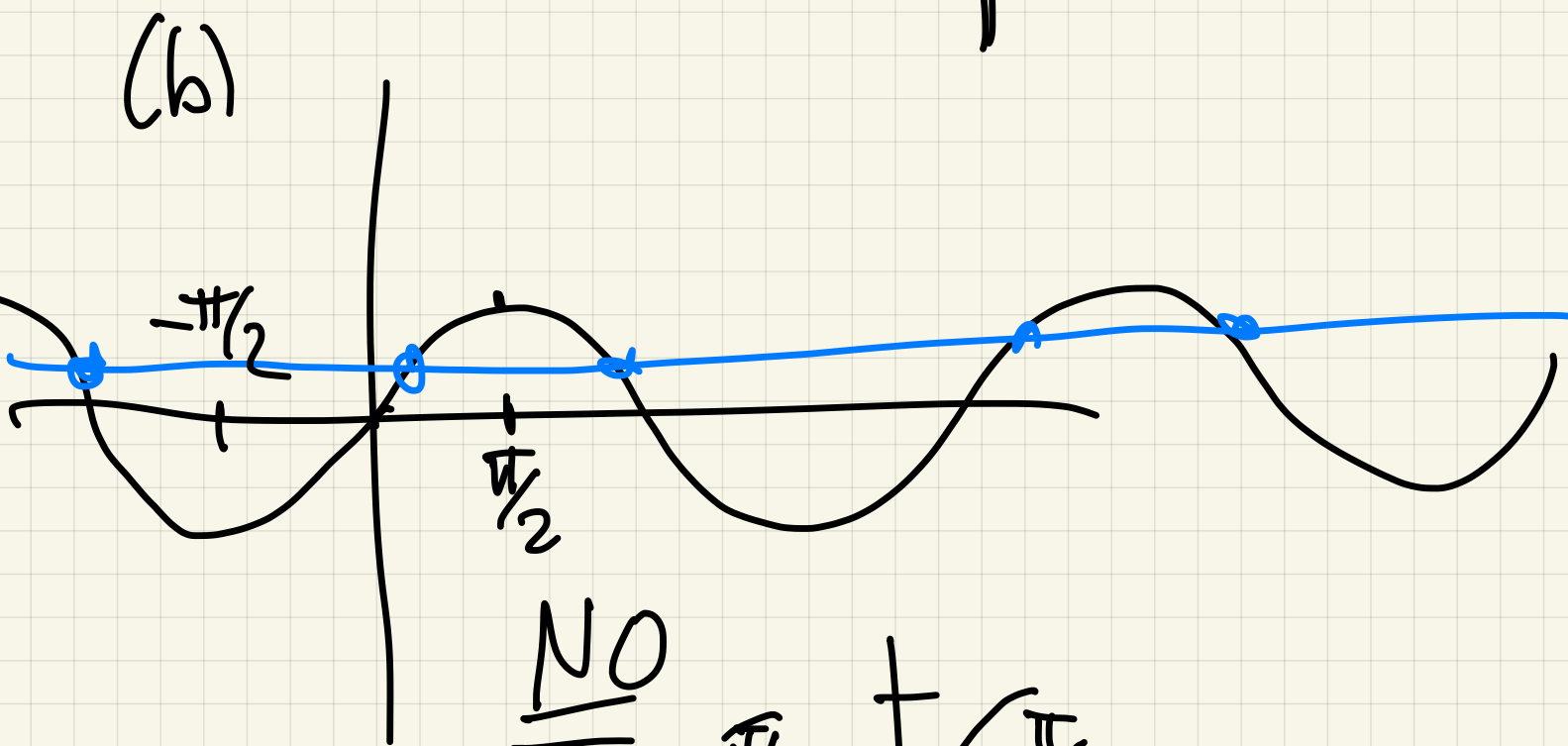
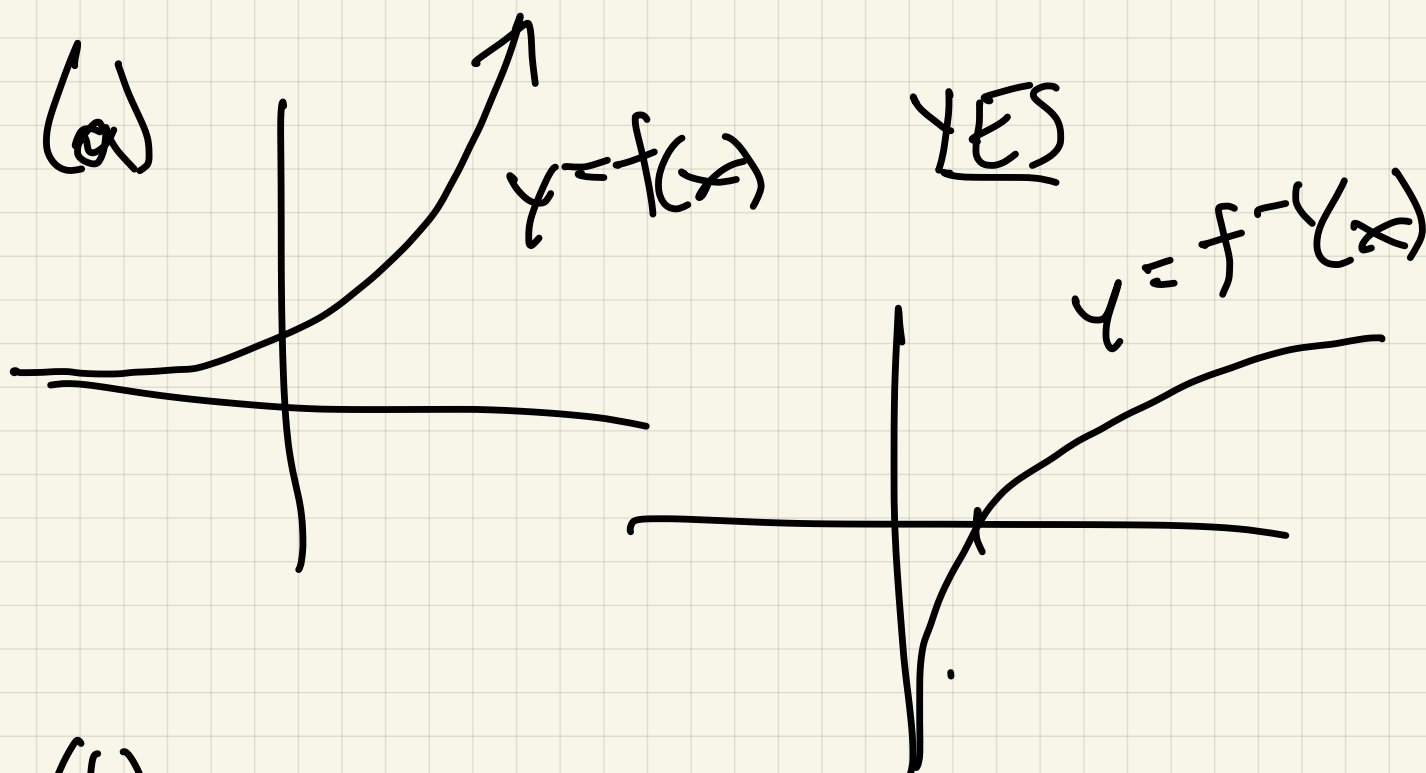
(d)



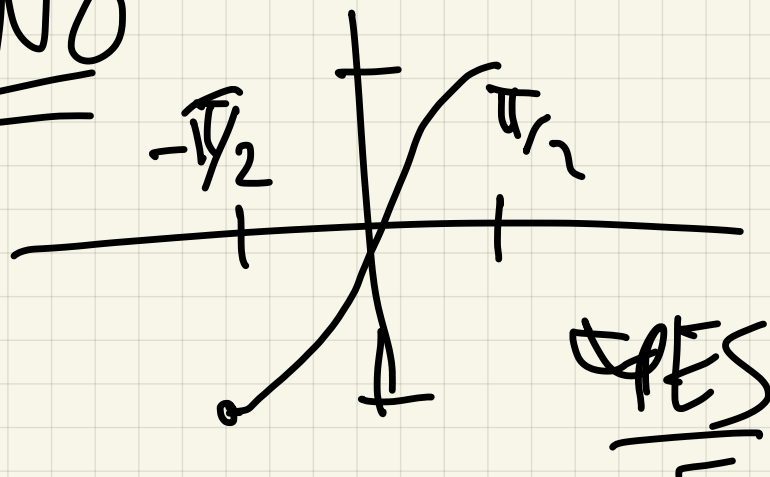
YES

$$y = \sqrt{1-x^2}$$

Ex 5 Do following functions have inverses?



(c)



Book: If $f(x)$ has inverse,

then Notation:

f is 1-1

Algebraically:

Ex 6 $y = f(x) = \frac{x-2}{x-5}$

(a) sketch graph

(b) Find equation for $y = f^{-1}(x)$.

(b) $y = \frac{x-2}{x-5}$ $x = \frac{y-2}{y-5}$

equation for $f(x)$ equation for $f^{-1}(x)$

$$x = \frac{y-2}{y-5} \Rightarrow x(y-5) = y-2$$

$$\Downarrow xy - 5x = y - 2$$

$$4y - 4 = 5x - 2$$

$$y(x-1) = 5x-2$$

$$y = \frac{5x-2}{x-1}$$

Check: $f(f^{-1}(x)) = x$

$$f(f^{-1}(x)) = f\left(\frac{5x-2}{x-1}\right) = \frac{\left(\frac{5x-2}{x-1} - 2\right)}{\left(\frac{5x-2}{x-1} - 5\right)} = \frac{x-1}{x-1} = 1$$

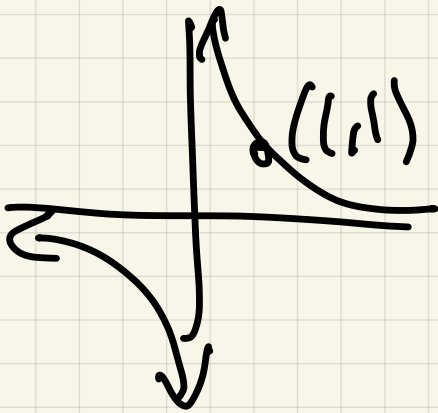
$$\frac{\cancel{5x-2} - 2(x-1)}{\cancel{5x-2} - 5(x-1)} = \frac{3x}{5-7} = \frac{3x}{-2} = -\frac{3x}{2}$$

~~$-5x+5$~~

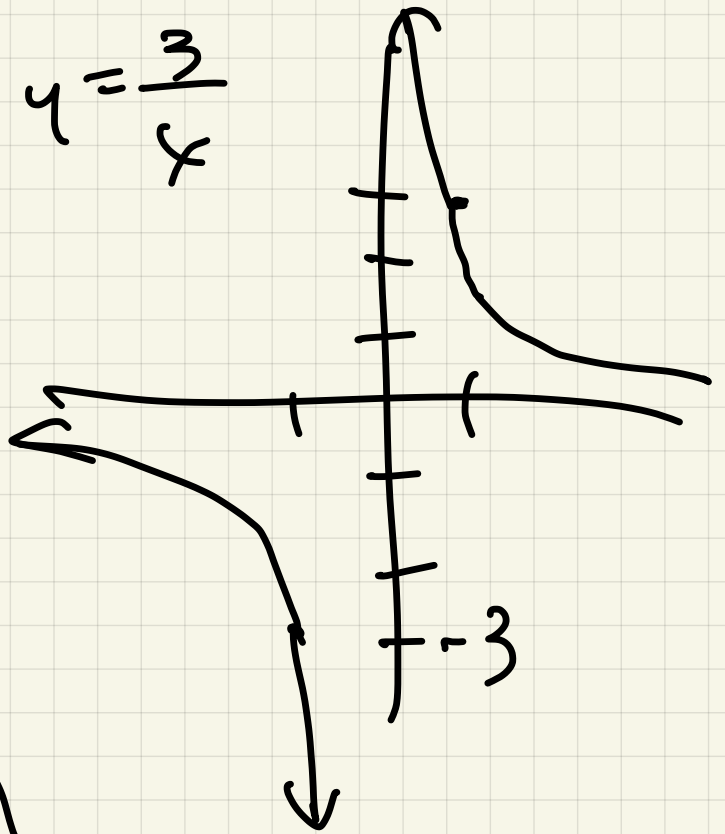
(a) sketch $y = \frac{x-2}{x-5} = \frac{x-5+3}{x-5}$

$$= 1 + \frac{3}{x-5}$$

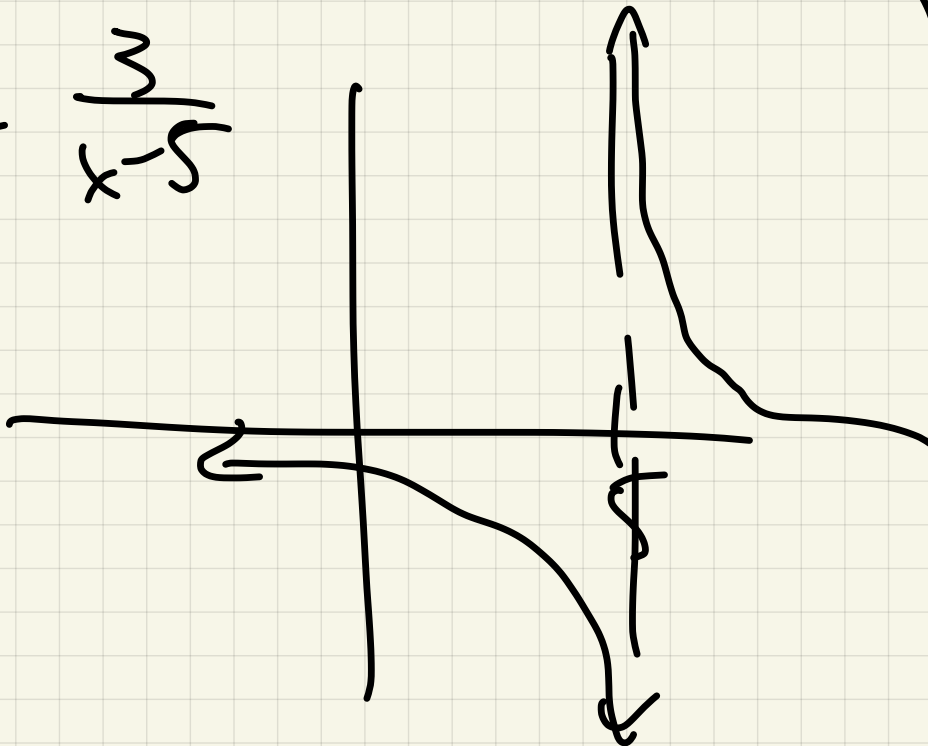
$$y = \frac{1}{x}$$



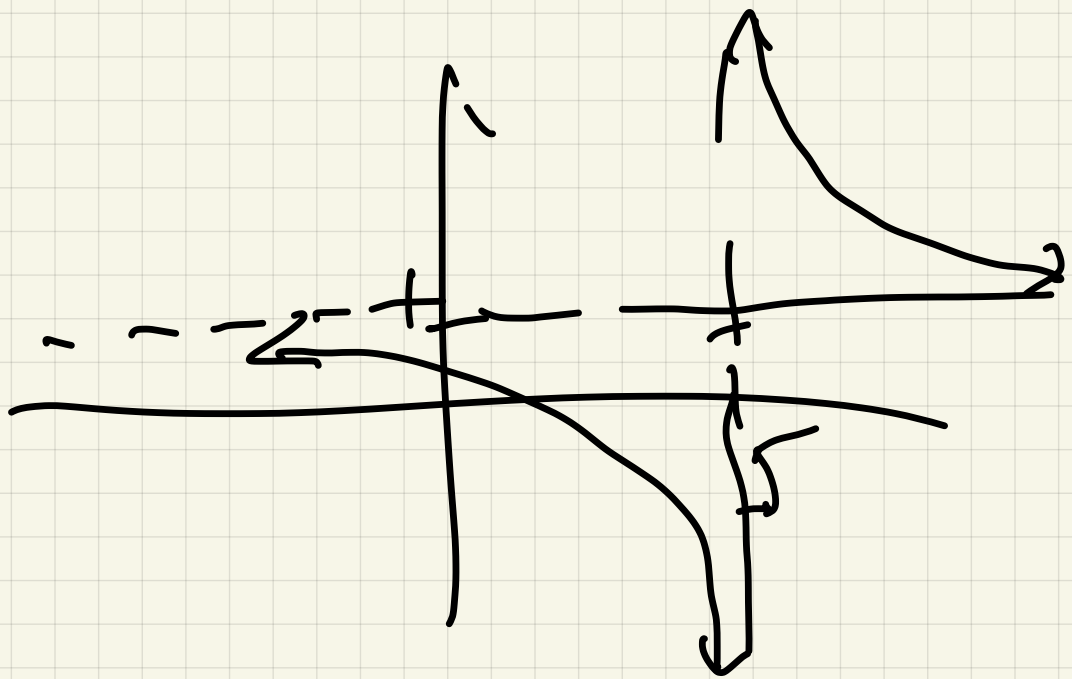
$$y = \frac{3}{x}$$



$$y = \frac{3}{x-5}$$



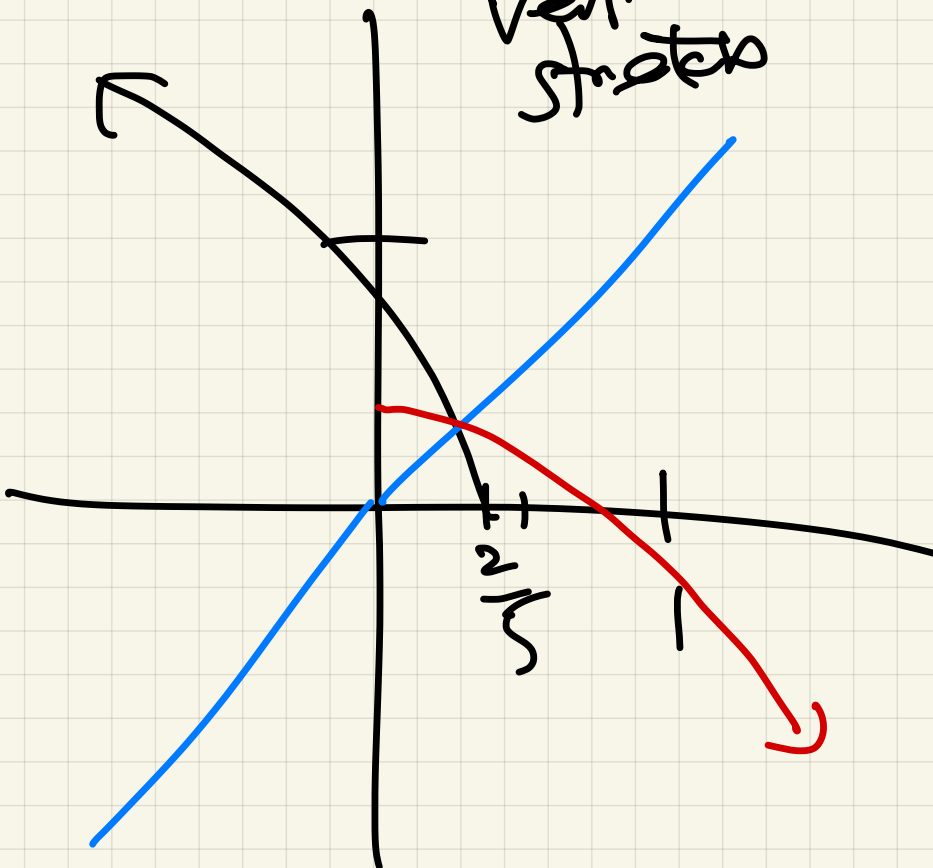
$$y = 1 + \frac{3}{x-5}$$



Ex 7 Same for $y = \sqrt{5x-2}$

Sketch: $y = \sqrt{5} \sqrt{x - \frac{2}{5}}$

$\nearrow$ vert stretch
 $\nwarrow$ right shift $\frac{2}{5}$



$$y = \sqrt{5x-2}$$

$$x = \sqrt{5y-2}$$

$$x^2 = 5y-2$$

$$x^2 + 2 = 5y$$

$$\frac{x^2 + 2}{5} = y$$