

10/13 / Precalc { Vertical shifts
Horizontal shifts
Reflections

Vertical stretch/shrinks

$y = f(x) \Rightarrow y = cf(x)$ $c > 1$ stretches
in y -direction
 $c > 0$
 $0 < c < 1$ shrinks
in y -direction

Horizontal stretch/shrinks

$y = f(x) \Rightarrow y = f(cx)$

$c > 1$
shrinks graph
 x -direction

$0 < c < 1$
stretches
 x -direction

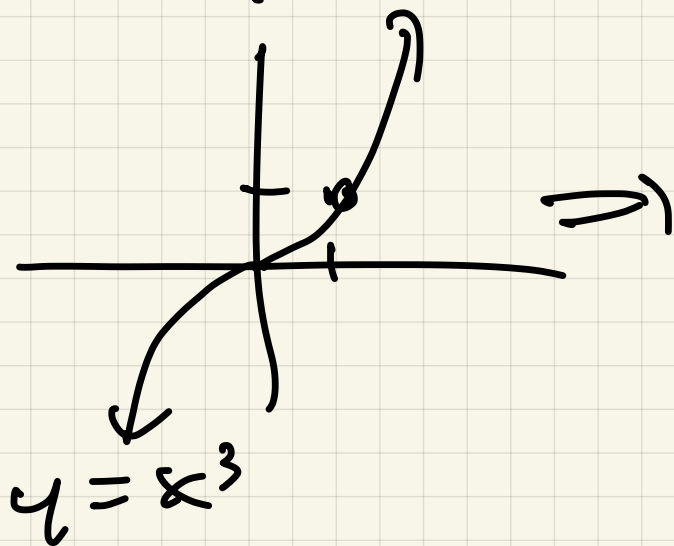
h_u 1/c

function 1/c

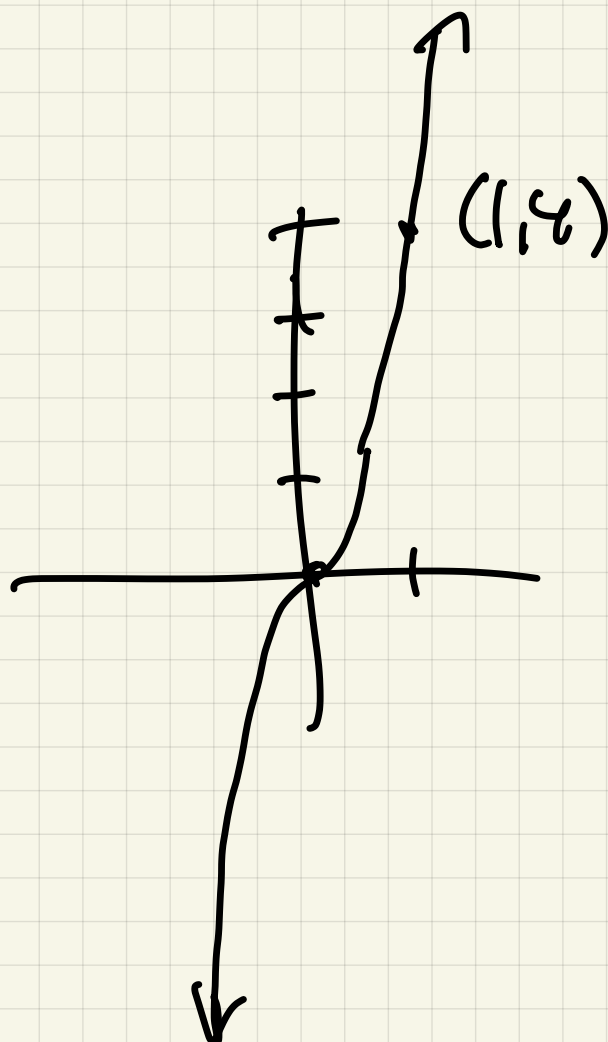
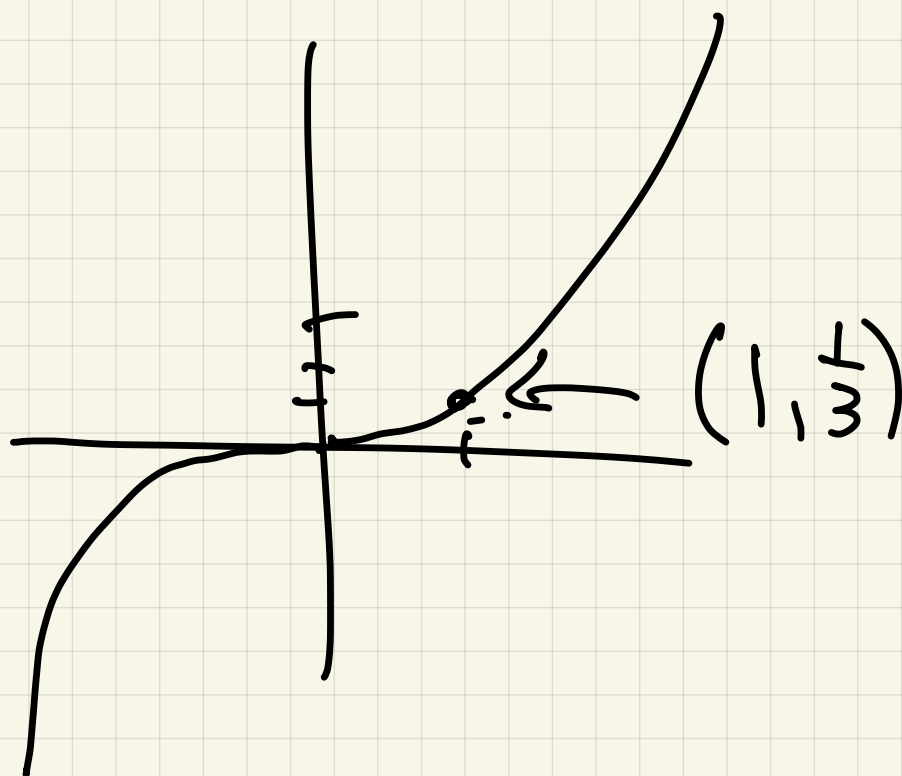
Ex)

Sketch

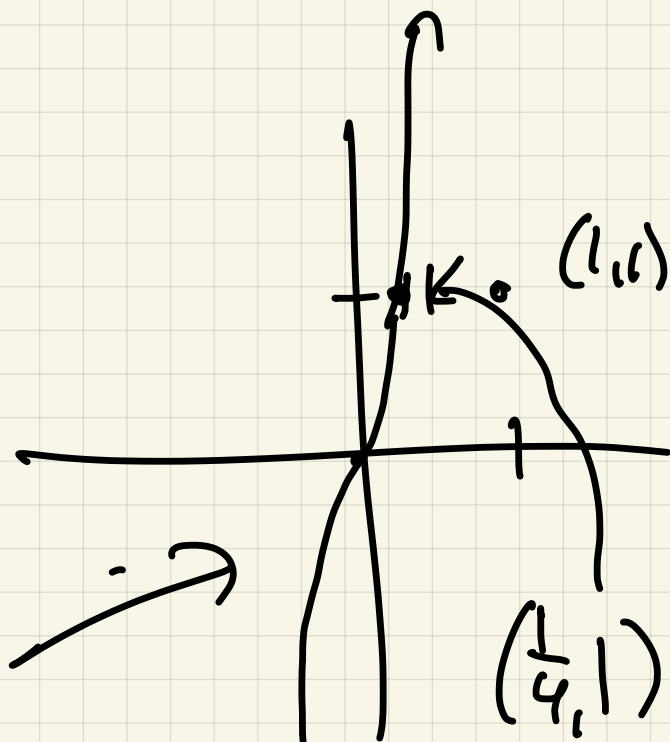
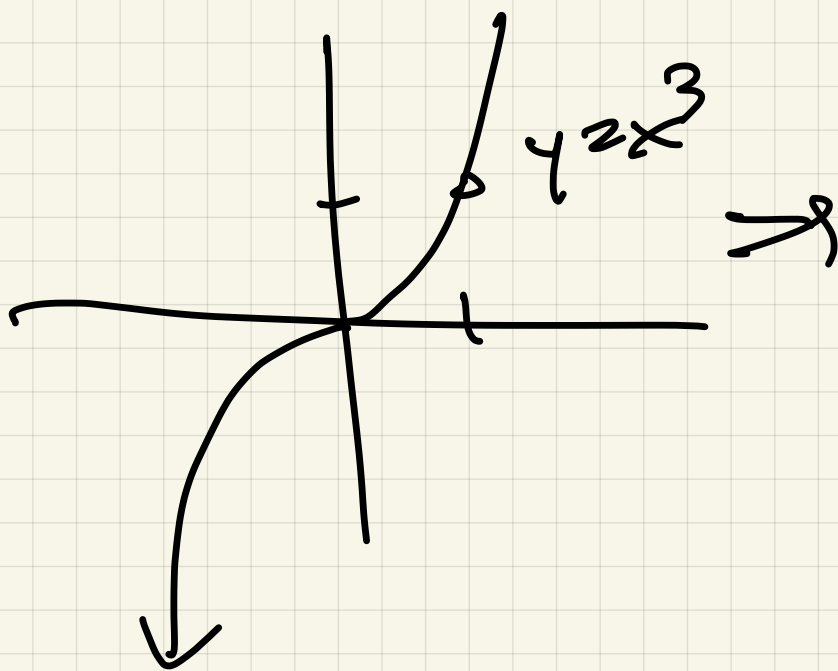
(a) $y = 4x^3$



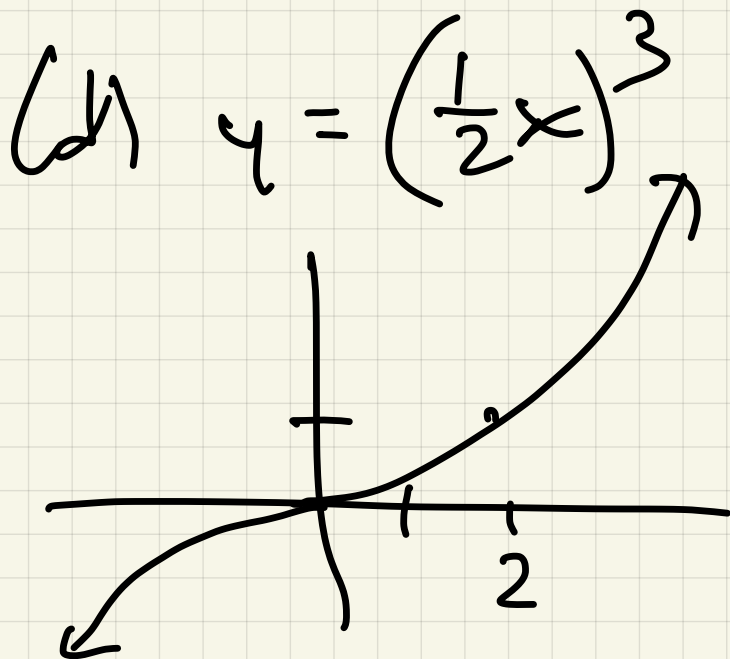
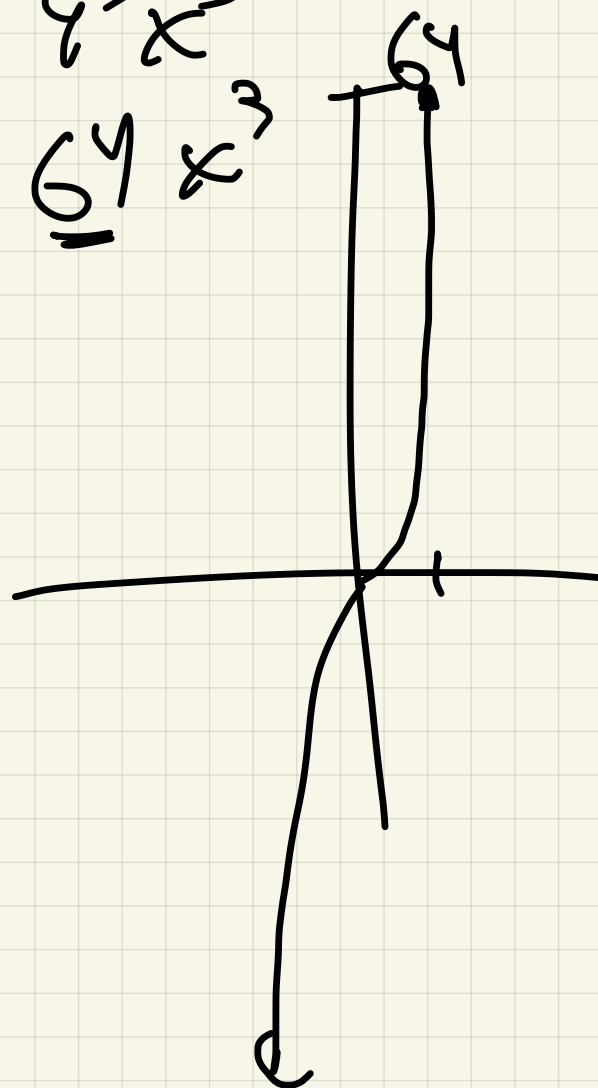
(b) $y = \frac{1}{3}x^3$



(c) $y = (4x)^3$



Also : $(4x)^3 = 4^3 x^3$
 $= 64 x^3$



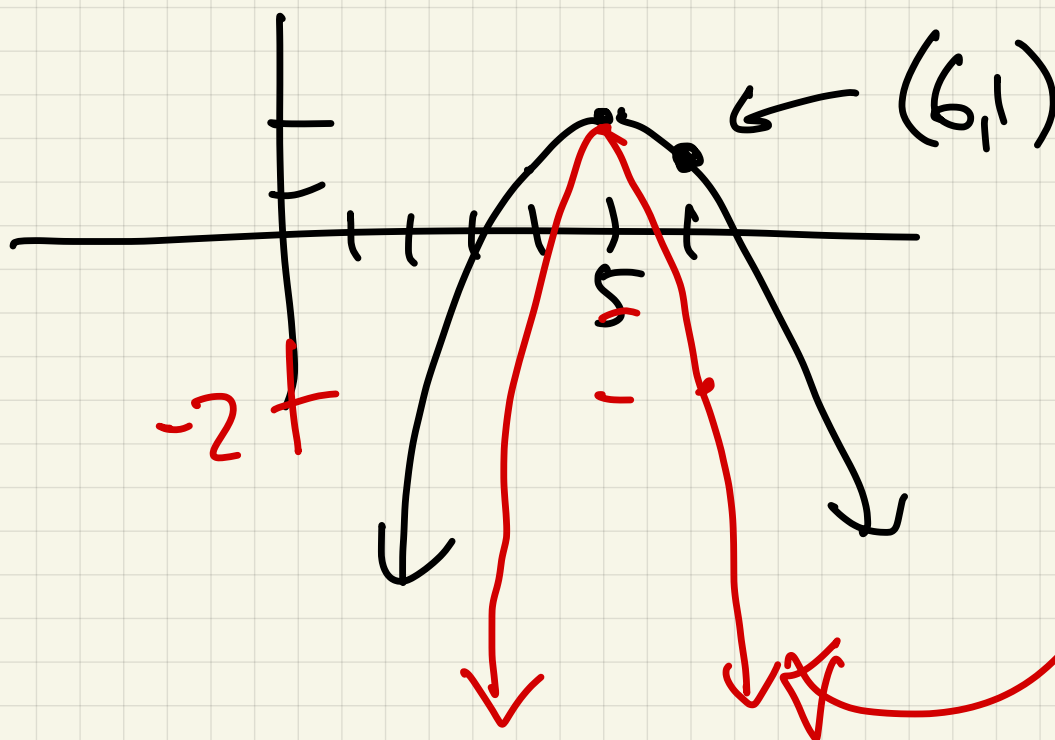
Ex 2

Combinations:

(a)

$$y = 2 - 4(x-5)^2$$

Ref: $y = 2 - (x-5)^2$



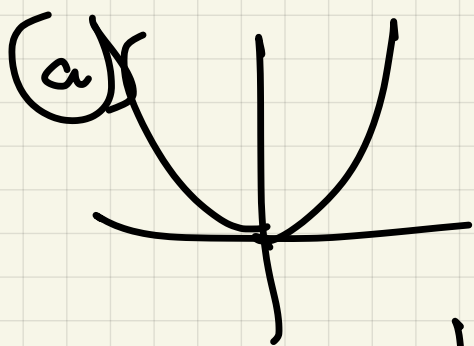
(a) $y = x^2$

(b) $y = (x-5)^2$

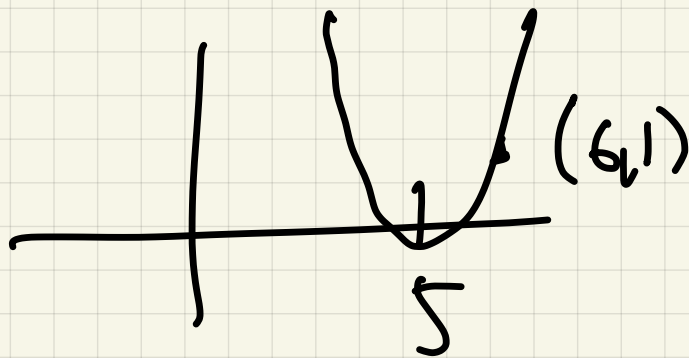
(c) $y = 4(x-5)^2$

(d) $y = 2 - 4(x-5)^2$

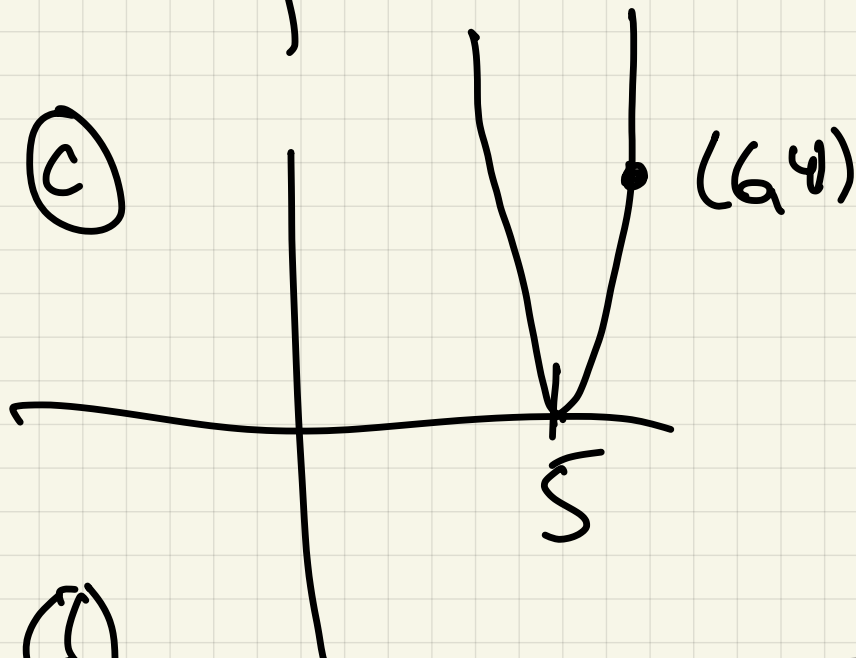
(e) $y = -4(x-5)^2$



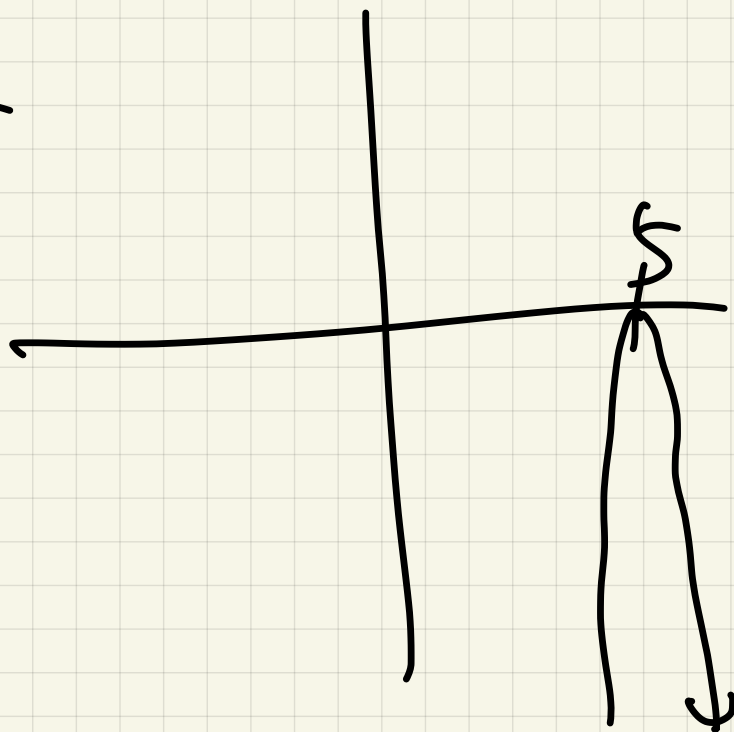
(b)



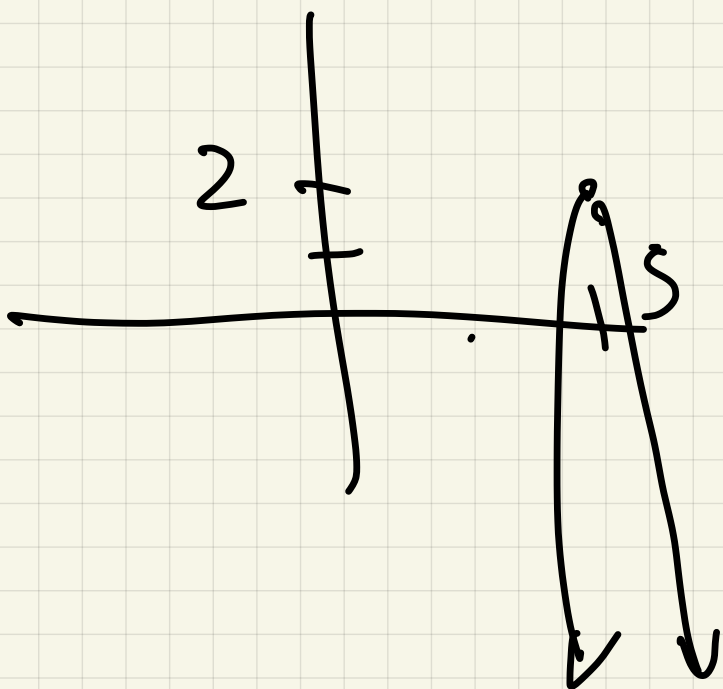
(c)



(d)

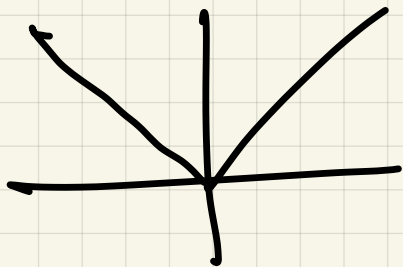


(e)

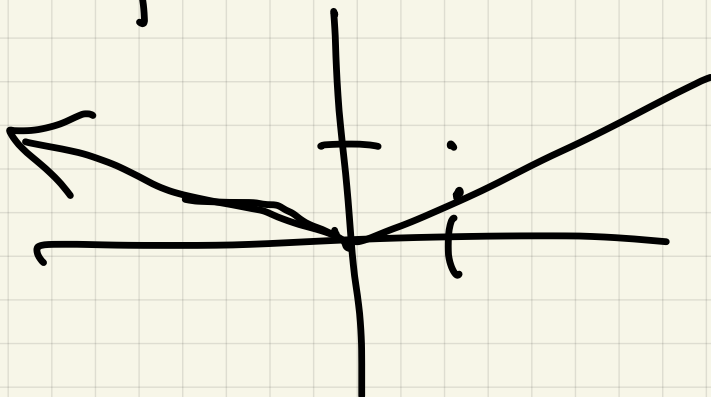


(b) $y = -1 + \frac{1}{2}|x-4|$

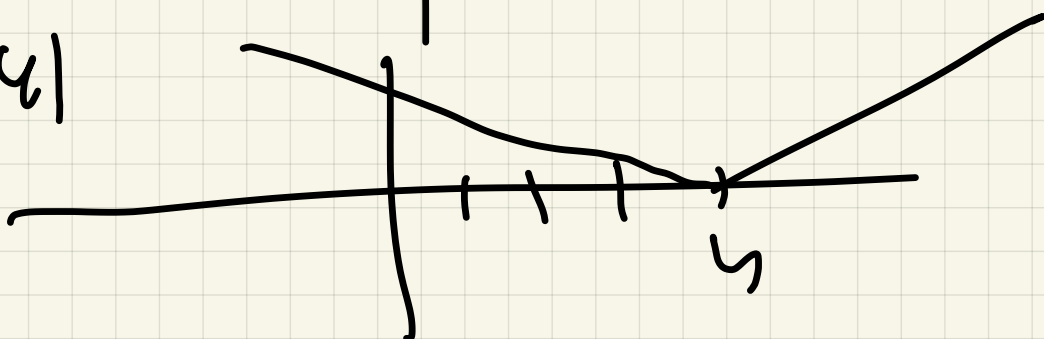
$$y = |x|$$



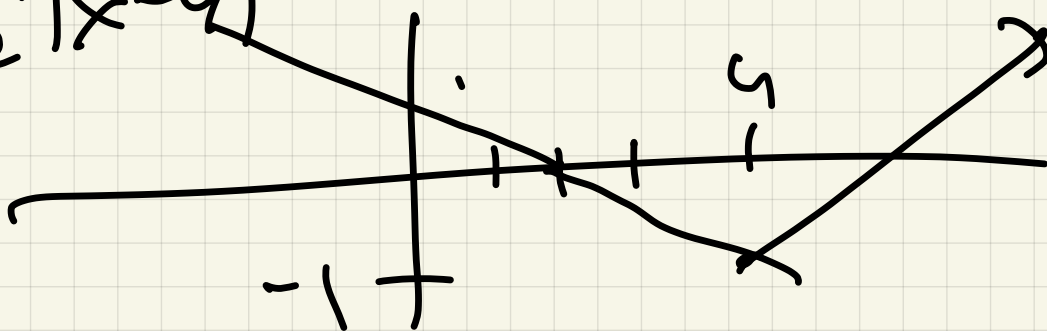
$$y = \frac{1}{2}|x|$$



$$y = \frac{1}{2}|x-4|$$

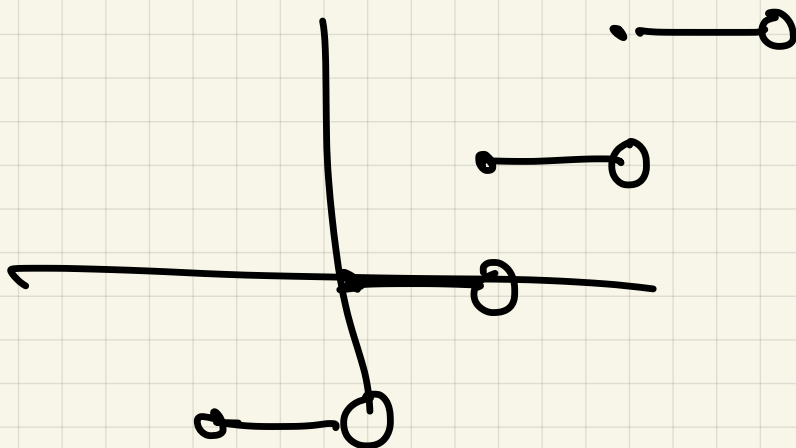


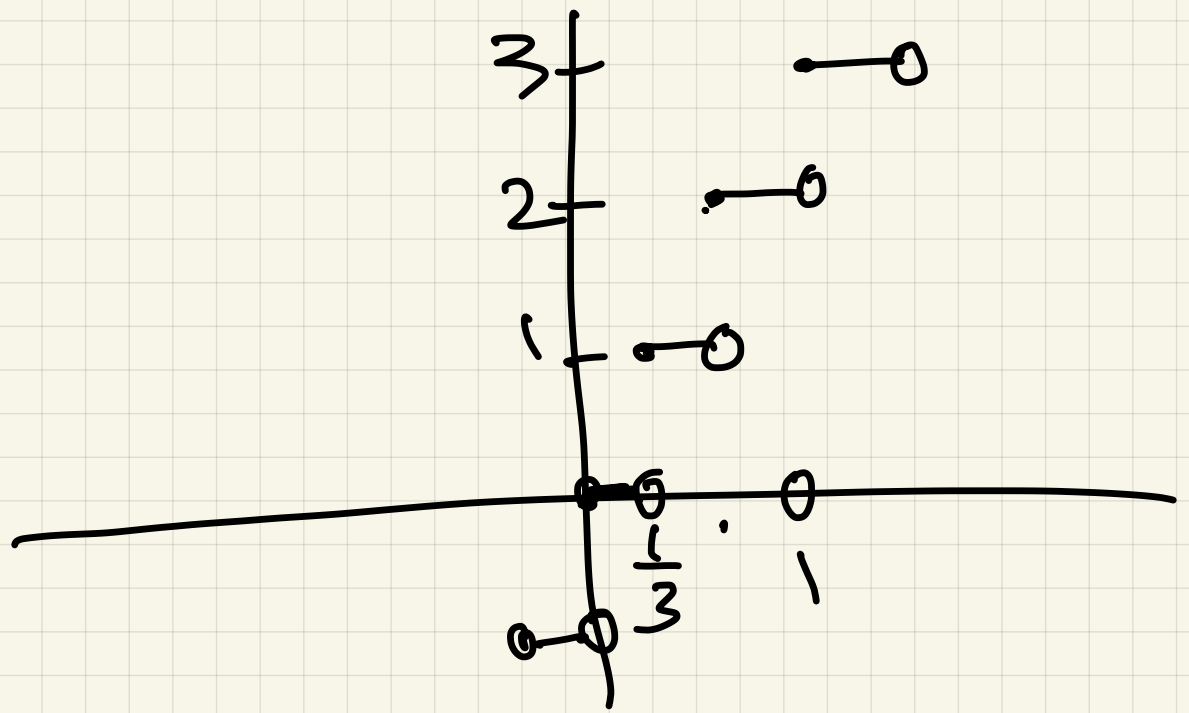
$$y = -1 + \frac{1}{2}|x-4|$$



(c) $y = \lfloor 3x \rfloor = \llbracket 3x \rrbracket$

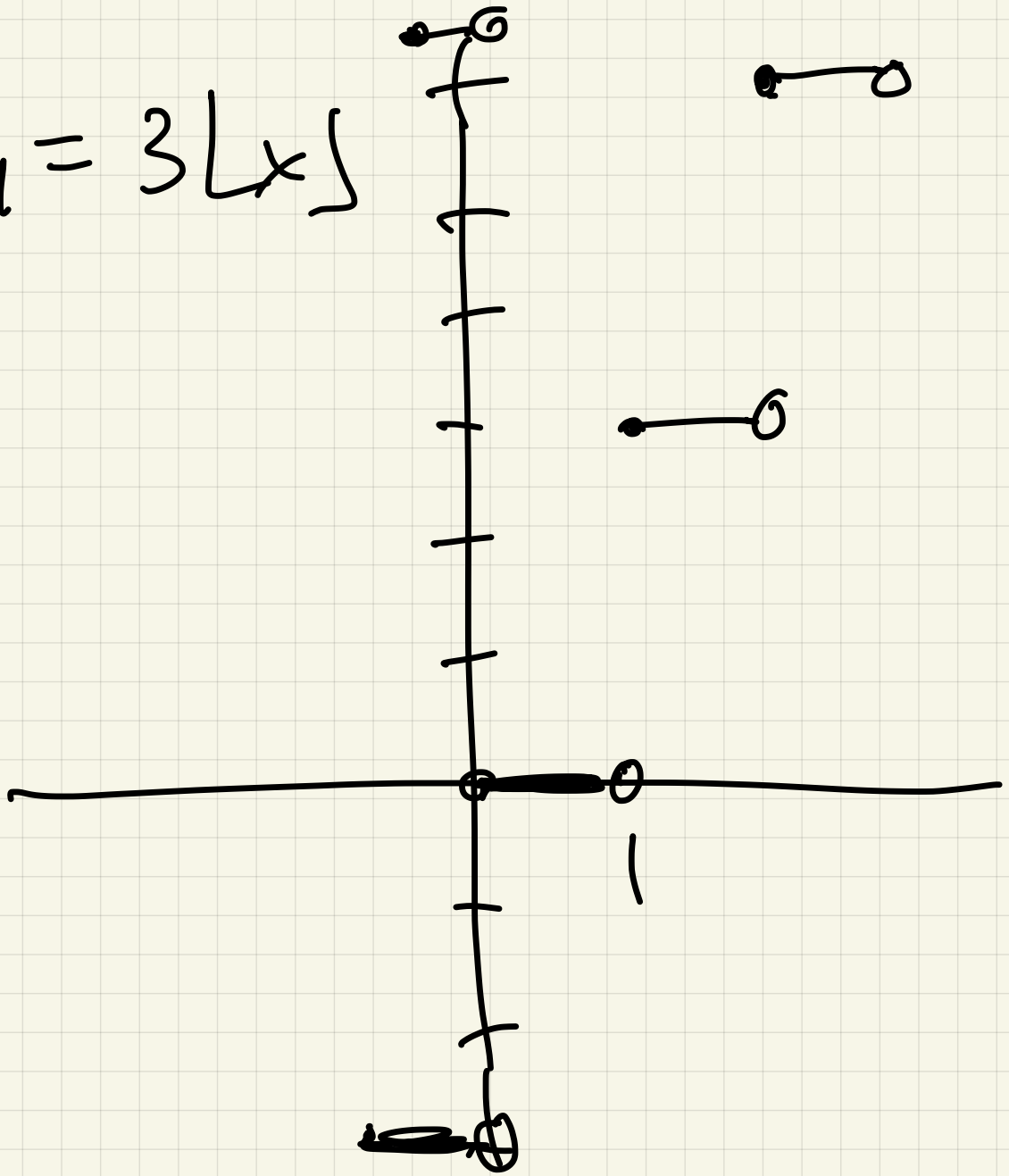
$$y = \lfloor x \rfloor$$





(d)

$$y = 3 \lfloor x \rfloor$$



2.6 Combining functions

In Chp, we combined rational expressions

Def: Can do same with functions:

Def: If $y = f(x)$ and $y = g(x)$

$$\textcircled{1} (f+g)(x) = f(x) + g(x) \quad \underline{\text{Sum}}$$

$$\textcircled{2} (f-g)(x) = f(x) - g(x) \quad \underline{\text{Difference}}$$

$$\textcircled{3} (fg)(x) = f(x)g(x) \quad \underline{\text{Product}}$$

$$\textcircled{4} \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad \underline{\text{Quotient}}$$

Note: Each of these is defined for x in Domain

und x in Domain g

Ex $f(x) = \sqrt{x}$ $g(x) = 3\sqrt{x}$

(a) $(f+g)(x) = \sqrt{x} + 3\sqrt{x} =$

$4\sqrt{x}, \quad x \geq 0$

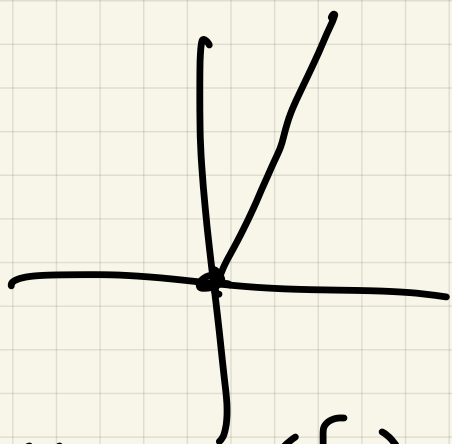
(b) $(f-g)(x) = \sqrt{x} - 3\sqrt{x} = -2\sqrt{x}$

$x \geq 0$



(c) $(f \cdot g)(x) = \sqrt{x} \cdot 3\sqrt{x} = 3x$

$x \geq 0$



(d) $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{3\sqrt{x}} = \frac{1}{3},$

$x > 0$

Ex 2 Same for $f(x) = x-3$
 $g(x) = x^3 - 9x$

$$(a) (f+g)(x) = x^3 - 8x - 3$$

$$(b) (f-g)(x) = -x^3 + 10x - 3$$

$$(c) (fg)(x) = (x-3)(x^3 - 9x)$$

$$= x^4 - 9x^2 - 3x^3 + 27x$$

$$(d) \left(\frac{f}{g}\right)(x) = \frac{x-3}{x^3 - 9x} =$$

$$\frac{\cancel{x-3}}{x(\cancel{x-3})(x+3)} = \frac{1}{x(x+3)}$$

$x^2 < 9$

Defn: The composition of f and g is

$x \neq 0, \pm 3$

$$(f \circ g)(x) = f(g(x))$$

Ex 3 Find $f \circ g$ and $g \circ f$ of
in Ex 1 and Ex 2

Ex 1 $f(x) = \sqrt{x}$ $g(x) = 3\sqrt{x}$

$$f \circ g(x) = f(g(x)) =$$

$$f(3\sqrt{x}) = \sqrt{3\sqrt{x}} =$$

$$\sqrt{3} \sqrt{\sqrt{x}} = \sqrt{3} (x^{1/2})^{1/2} =$$

$$\sqrt{3} x^{1/4} = \sqrt{3} \sqrt[4]{x}$$

$$(g \circ f)(x) = g(f(x)) =$$

$$g(\sqrt{x}) = 3\sqrt{\sqrt{x}} = 3x^{1/4}$$

Ex 2 $f(x) = x - 3$

$$g(x) = \underline{\underline{x^3 - 9x}}$$

$$(f \circ g)(x) = f(g(x)) =$$

$$f(x^3 - 9x) = x^3 - 9x - 3$$

$$(g \circ f)(x) = g(f(x)) =$$

$$g(x-3) = \underline{(x-3)^3} - \underline{9(x-3)}$$

$$x^3 - 9x^2 + \underline{27x} - \cancel{27} - \underline{9x} + \cancel{27}$$

$$x^3 - 9x^2 + 18x$$

Ex 4 $f(x) = x^3$, $g(x) = x - 4$

$$(f \circ g)(x) = f(g(x)) = f(x - 4) =$$

$$(x - 4)^3$$

Ex 5 For given $h(x)$, find functions $f(x)$ and $g(x)$, so that $h(x) = (f \circ g)(x)$

$$(a) \quad h(x) = (x-4)^3$$

$$f(x) = x^3,$$

$$g(x) = x-4$$

(see Ex 4)

$$(b) \quad h(x) = \sqrt{2x+5}$$

" ① $2x+5$ ② $\sqrt{\text{result}}$ "

$$g(x) = 2x+5$$

$$f(x) = \sqrt{x}$$

$$\left[\begin{array}{l} \underline{\text{At } 5} \\ g(x) = 2x \\ f(x) = \sqrt{x+5} \end{array} \quad \underline{\text{works}} \right]$$

$$(c) \quad h(x) = \frac{2}{\sqrt{2x+5}}$$

$$g(x) = 2x+5$$

$$f(x) = \frac{2}{\sqrt{x}}$$

$$f(x) = \frac{2}{x}$$

$$g(x) = \sqrt{2x+5}$$

$$(d) \quad h(x) = \frac{4}{(2x+9)^3}$$

$$g(x) = 2x+9$$

$$f(x) = \frac{4}{x^3}$$

$$g(x) = (2x+9)^3$$

$$f(x) = \frac{4}{x}$$

$$(e) \quad h(x) = \frac{x^3-1}{8-x^3}$$

$$g(x) = x^3$$

$$f(x) = \frac{x-1}{8-x}$$

$$(f) \quad h(x) = \lfloor 2x+1 \rfloor$$

$$f(x) = \lfloor x \rfloor, \quad g(x) = 2x+1$$

2.7 Inverse functions

Concept : for a function $f: A \rightarrow B$
then an inverse function

$g: B \rightarrow A$ is a function
such that

$$\begin{cases} f(g(x)) = x & \text{all } x \text{ in domain} \\ g(f(x)) = x & \text{all } x \text{ in domain of } f \end{cases}$$