11/5 Precalc Quiz 19

1. $y = \frac{3x^2+9x}{1x^2-9} = \frac{3x(x+3)}{(x-3)(x+3)}$

$= \frac{3x}{x-3}$, $x \neq -3$

HA: $y = \frac{3}{1} = 3$

hole $x = -3$

$(-3, 3/2)$

VA $x = 3$

2. $y = \frac{3x+6}{x^2+6x+8} = \frac{3(x+2)}{(x-2)(x-4)}$

HA $y = 0$

no holes

VA $x = 2, 4$

[3] $y = \dfrac{5x^3 - 5x}{x^2 - 4x} = \dfrac{5x(x^2-1)}{x(x-4)} =$

$\dfrac{5(x^2-1)}{x-4}, \quad x \neq 0$

HA none

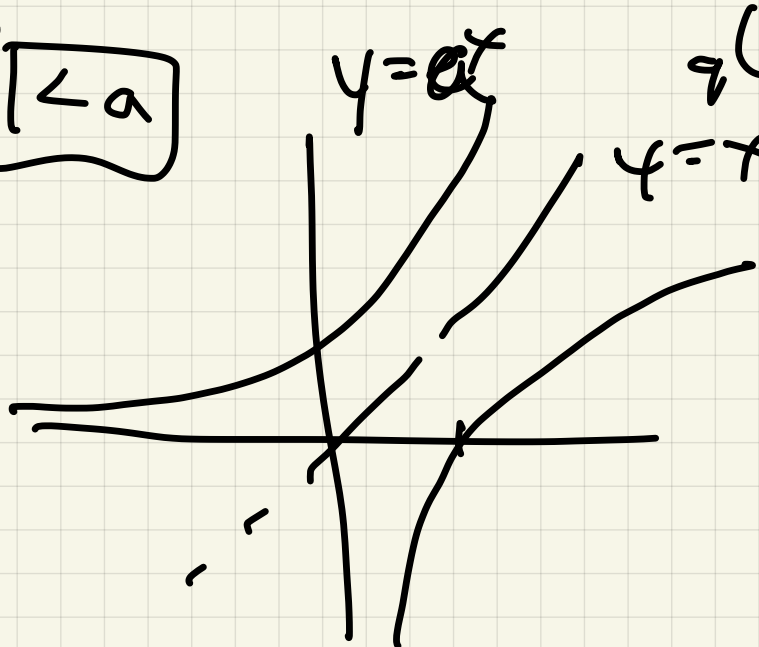
VA $x = 4$

hole $x = 0$ $\left(0, \frac{5}{4}\right)$

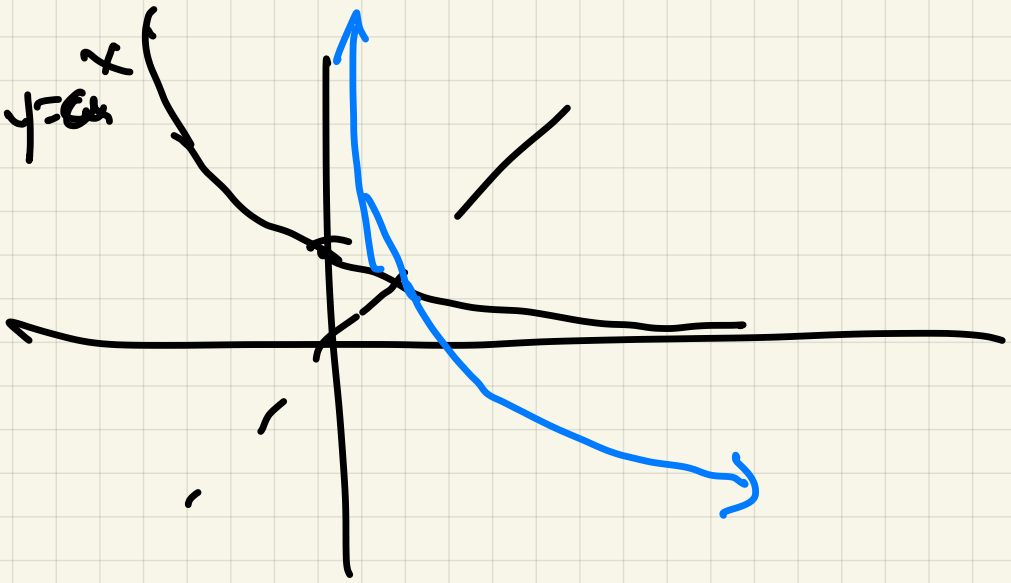
Last time: $f(x) = a^x$ expon

$g(x) = \log_a x$ log

[1<a]



$0 < a < 1$ $y = a^x$



Calculus $a = e = 2.718...$

Calculator: $\log x = \log_{10} x$

$\ln x = \log_e x$

Inverses properties:

$$\begin{cases} \log_a(a^x) = x \\ a^{\log_a x} = x \quad (x > 0) \end{cases}$$

Solve $5^x = 90$

$\Rightarrow x = \log_5(5^x) = \log_5 90$

Properties:

① $\log_a(uv) = \log_a u + \log_a v$

② $\log_a\left(\frac{u}{v}\right) = \log_a u - \log_a v$

③ $\log_a(u^p) = p \cdot \log_a(u)$

Idea: want $A \cdot B$

Table ⇒ $\log A$ $\log B$

or $\log AB = \log A + \log B$

$\log AB$: table → AB

$\log A + \log B$: easy

Ex1: If we know

$\ln 2 = .6931$ $\ln 3 = 1.0986$

can compute

(a) $\ln 12 = \ln 2 \cdot 2 \cdot 3 =$

$\ln 2^2 \cdot 3 = \ln 2^{②} + \ln 3$

$= 2\ln 2 + \ln 3$

$= 2(.6931) + 1.0986 =$ ...

(b) $= \ln \frac{2}{81} = \ln \frac{2}{3^4} =$

$\ln 2 - \ln 3^4 = \underline{\ln 2} - 4\underline{\ln 3}$

(c) $.6931 - 4(1.0986) =$ ...

$\ln 2^{10} \overset{③}{=} 10 \ln 2$

$= 6.931$

(d) $\ln \sqrt{2} = \frac{1}{2} \ln 2$

$= \frac{1}{2}(.6931) = .3465$

Ex2: Condense into a

single log

(a) $\frac{1}{2}\ln x + 3\ln(x+1)$

$= \ln x^{1/2} + \ln(x+1)^3$

$= \ln \sqrt{x}(x+1)^3$

(b) $7\log_3(x+2) - 5\log_3(x-2) - 2\log_3 x$

$= \log_3(x+2)^7 - \log_3(x-2)^5 - \log_3 x^2$

$= \log_3 \frac{(x+2)^7}{(x-2)^5 x^2}$

Ex3 Expand

(a) $\log \frac{(x+1)^2(x-1)}{x^5} =$

$\log (x+1)^2 \cdot (x-1) - \log x^5$

$\log (x+1)^2 + \log (x-1) - \log x^5$

$2\log (x+1) + \log (x-1) - 5\log x$

(b) $\log_5 \sqrt{\frac{(x+1)^7}{x^6}} =$

$\frac{1}{2} \log_5 \frac{(x+1)^7}{x^6}$

$= \frac{1}{2}\left[\log_5 (x+1)^7 - \log_5 x^6 \right]$

$= \frac{7}{2} \log_5 (x+1) - 3 \log_5 x$

Rmk: Many calculators only have $\log x$ and $\ln x$

How to compute $\log_a x$??

Ans: Change of base formulas

To compute $\log_a x = y$

$\Updownarrow$

$x = a^y$

$\log x = \log a^y$

$\Downarrow$

$= y \cdot \log a$

$y = \dfrac{\log x}{\log a}$ and $y = \log_b x$

Conclusion: $\log_a x = \dfrac{\log_b x}{\log_b a} =$

$$\frac{\log x}{\log a} = \frac{\ln x}{\ln a}$$

Ex 4

(a) $\log_2 3 = \frac{\ln 3}{\ln 2} = \frac{1.0986}{.6931}$

$\log_2 1 = 0 \quad 2^0 = 1$

$\log_2 2 = 1 \quad 2^1 = 2$

$\log_2 4 = 2 \quad 2^2 = 4$

$2^? = 3$???

(b) $\log_3 2 = \frac{\ln 2}{\ln 3} = \frac{.6931}{1.0986}$

5.4 Solving equations with log & exponentials

Keys:

$$a^{\log_a x} = x \qquad x > 0$$

$$\log_a a^x = x \qquad \text{all } x$$

Ex! Solve for x

(a) $\log_2 16 = x \iff 2^x = 16$

① OR $x = 4$

② $\log_2 16 = \dfrac{\ln 16}{\ln 2} = \dfrac{\ln 2^4}{\ln 2} =$

↑ change of base

$= \dfrac{4 \ln 2}{\ln 2} = 4$ ✓

③ $\log_2 16 = x$

$16 = 2^{\log_2 16} = 2^x$

↳ $x = 4$

(b) $\log_2 15 = \dfrac{\ln 15}{\ln 2} = \dfrac{2.708}{.6931} \approx 3.9$

COB

(c) $\log_3 x = -4$

$x = 3^{\log_3 x} = 3^{-4} = \frac{1}{81}$

(d) $\log_5 x = 2$

$x = 5^{\log_5 x} = 5^2 = 25$

(e) $e^{x^2+8} = e^{6x}$

$\ln e^{x^2+8} = \ln e^{6x}$

$x^2+8 = 6x$

$x^2 - 6x + 8 = 0$

$(x-2)(x-4) = 0 \quad x = 2, 4$

(f)

$$6e^{3x} + 2 = 20 \quad -2$$

$$6e^{3x} = 20 - 2 = 18 \quad \div 6$$

$$e^{3x} = \frac{18}{6} = 3$$

$$3x = \ln e^{3x} = \ln 3$$

$$x = \frac{\ln 3}{3} = \ln \sqrt[3]{3}$$

(a) $e^{2x} - 5e^x + 6 = 0$

$$(e^x)^2 - 5(e^x) + 6 = 0$$

$y = e^x$ $\qquad y^2 - 5y + 6 = 0$

$$(y-2)(y-3) = 0$$

$e^x = y = 2, 3$

$e^x = 2$, $e^x = 3$

$\Downarrow$ $\Downarrow$

$x = \ln(e^x) = \ln 2 \neq .6931$

$x = \ln(e^x) = \ln 3 = 1.0986$

(h) $5 - 2\ln x = 4 \quad -5$

$-2\ln x = 4 - 5 = -1 \quad \div -2$

$\ln x = \frac{-1}{-2} = \frac{1}{2}$ so

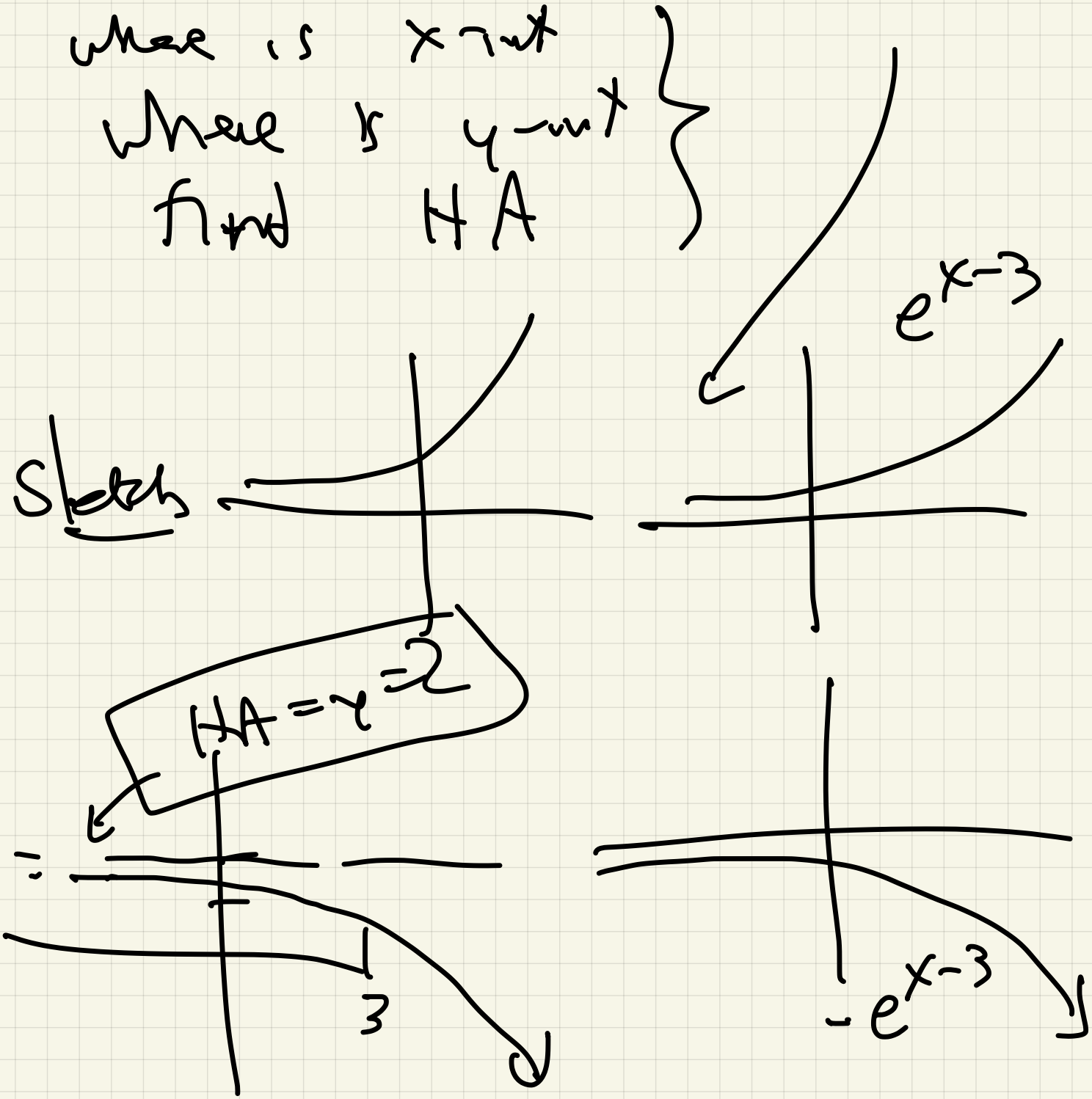
$x = e^{\ln x} = e^{1/2} = \sqrt{e}$

Ex) Sketch $y = 2 - e^{(x-3)}$

where is $x \to \infty$

where is $y \to \infty$

find HA



y int: $y = 2 - e^{-3} = 2 - \frac{1}{e^3}$

$(0, 2 - \frac{1}{e^3})$