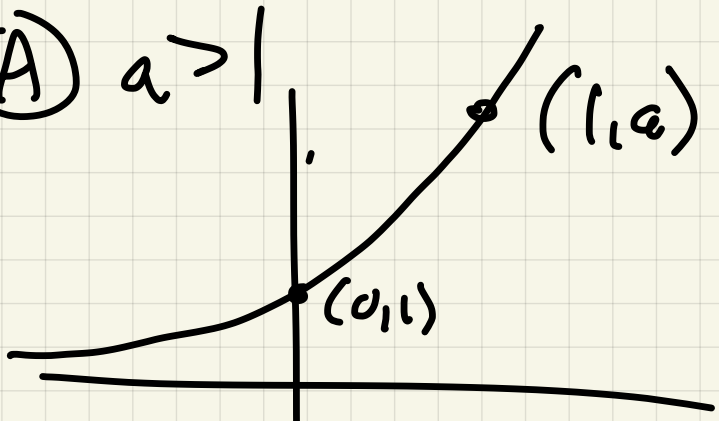
11/3/ Precalc

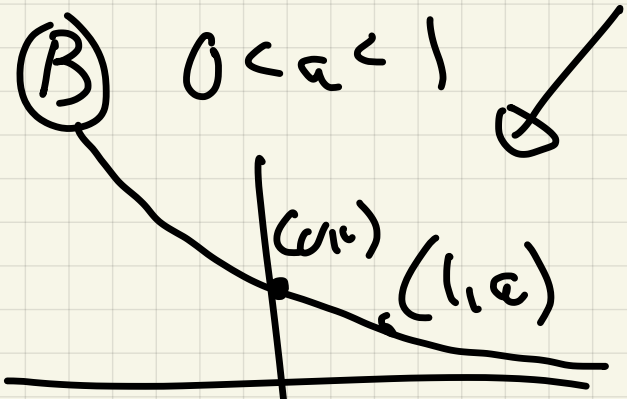
Last time $f(x) = a^x \quad 0 < a \neq 1$

(A) $a > 1$



exponential growth

(B) $0 < a < 1$

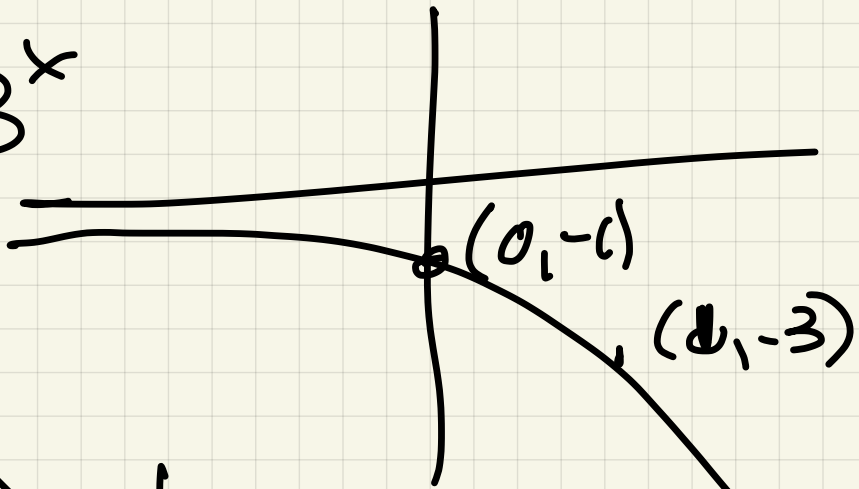


exponential decay

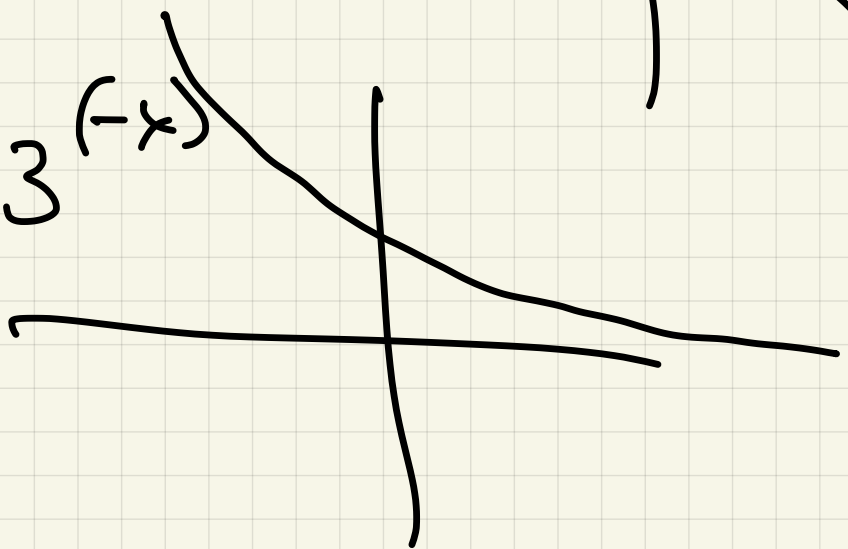
Note $y = 0$ is HA to graph

Ex1 $y = -3^x$

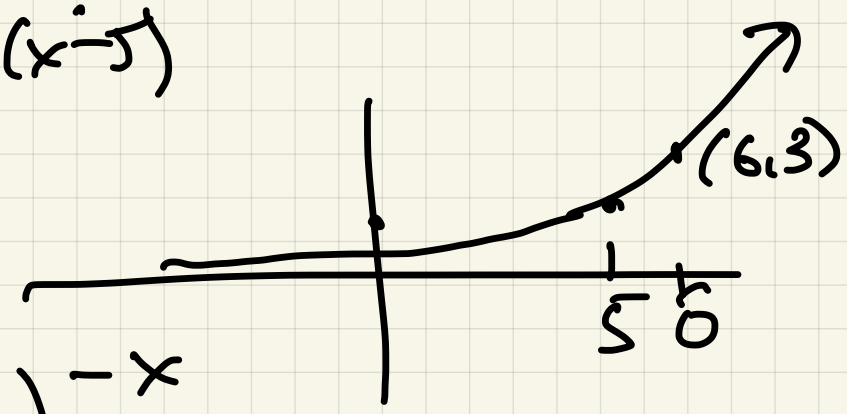
(a1



(b) $y = 3^{(-x)}$

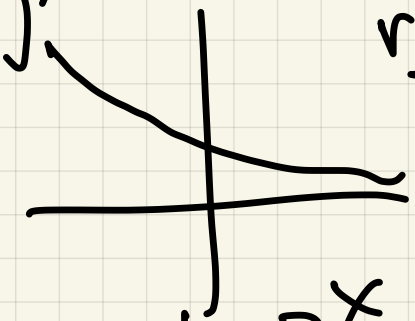


(a) $y = 3^{(x-5)}$

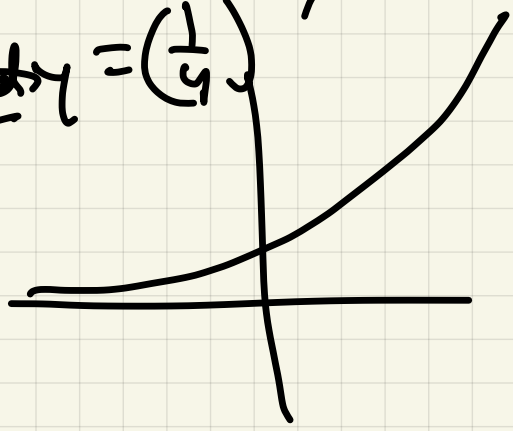


(d) $y = \left(\frac{1}{4}\right)^{-x}$

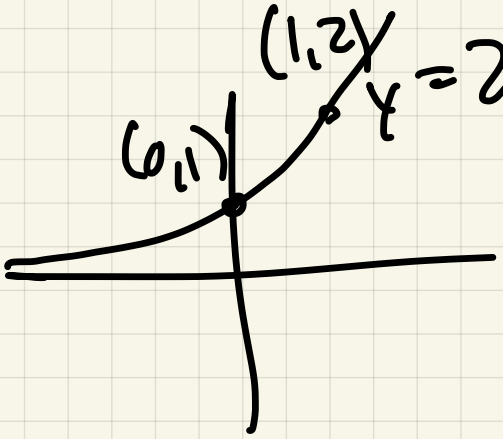
$y = \left(\frac{1}{4}\right)^{x}$



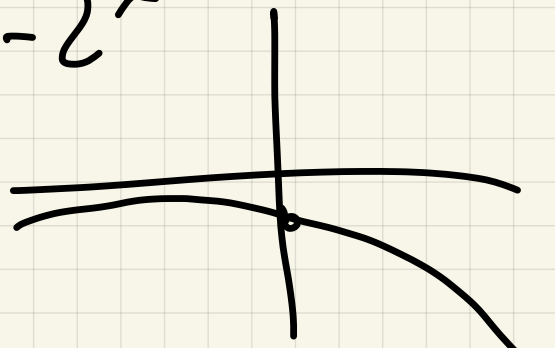
reflect $y = \left(\frac{1}{4}\right)^{-x}$

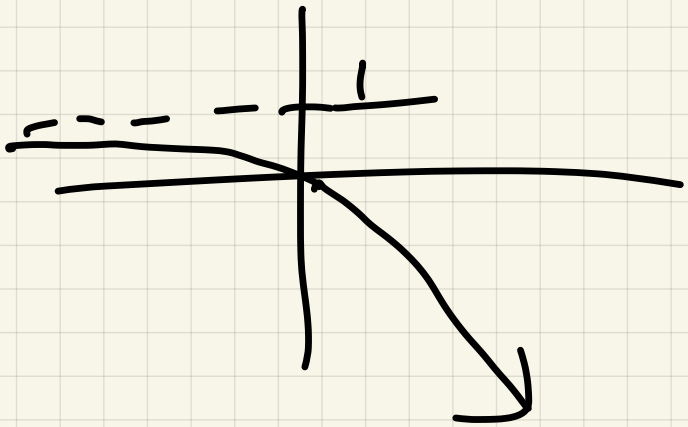


(e) $y = 1 - 2^{x}$



$y = -2^{x}$





Calculus: not $a = 2, 3, \frac{1}{4}$,

but $a = e = 2.71828818\ldots$

$= 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \ldots$

Application: Finance

P = principal

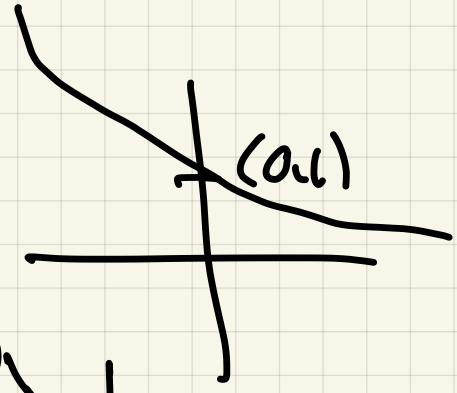
r = int rate

t = amt of time

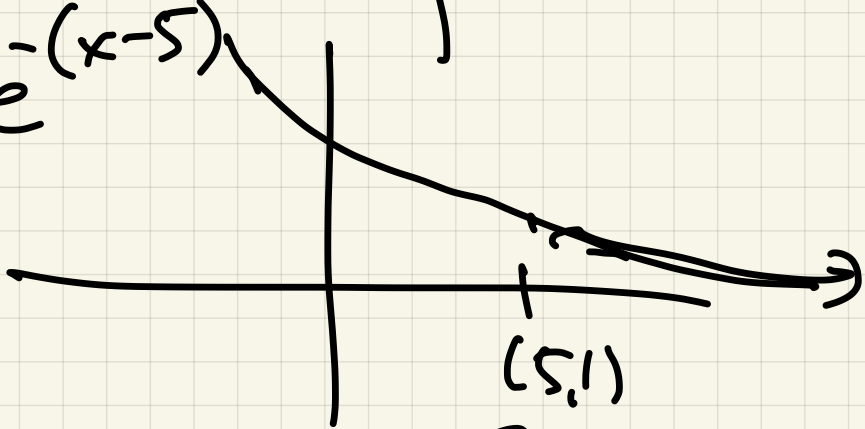
amt after t units of time $= A = Pe^{rt}$

Ex 2

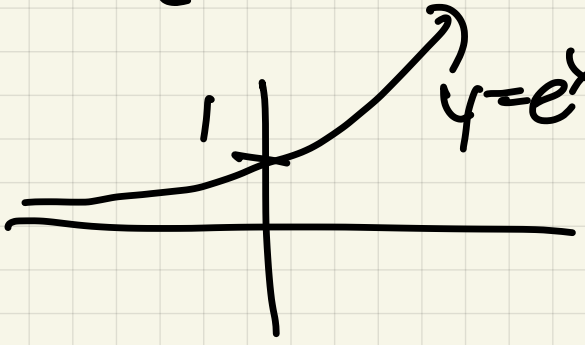
(a) $y = e^{-x}$

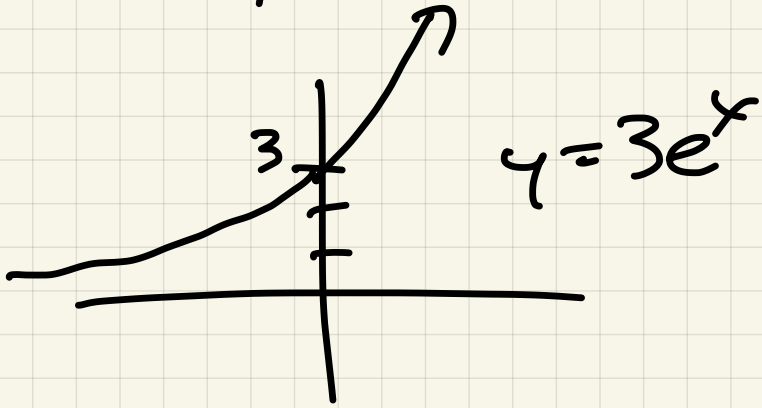


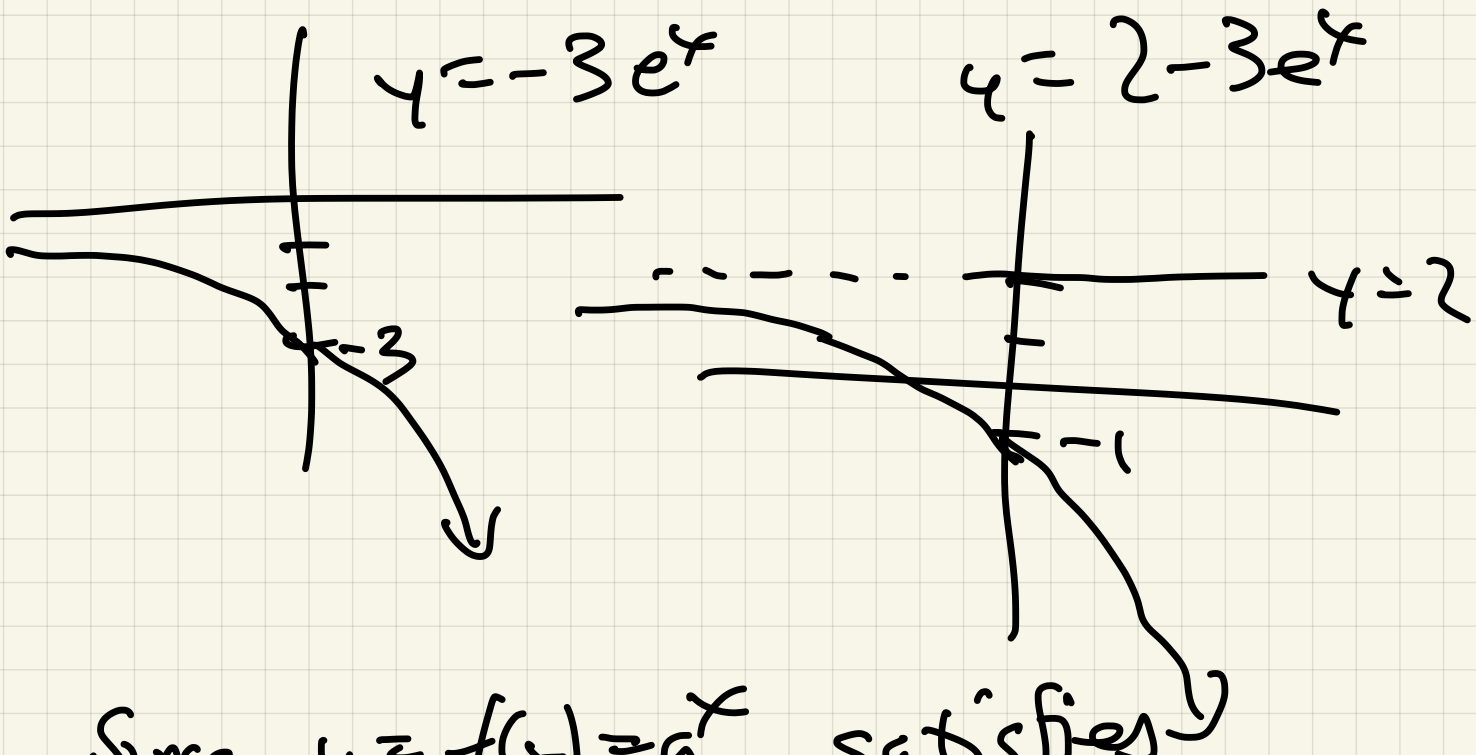
(b) $y = e^{5-x} = e^{-(x-5)}$



(c) $2 - 3e^{x}$







Since $y = f(x) = a^x$ satisfies horizontal line test, $f(x)$ is 1-1

Consequence 1: $\boxed{a^x = a^y \Rightarrow x = y}$

Ex 3 Solve!

(a) $3^{(2x+1)} = 9 = 3^{(2)}$

$\Rightarrow 2x+1 = 2 \Rightarrow x = \frac{1}{2}$

(b) $2^{(-5x+2)} = \frac{1}{4} = 2^{(-2)}$

$\therefore -5x + 2 = -2$

$-5x = -4$

$x = \frac{-4}{-5} = 4/5$

(c) $10^{3x} = 1000000 = 10^6$

$6 = 3x \Rightarrow x = 2$

(d) $e^{x^2+8} = e^{6x}$

$x^2 + 8 = 6x \Rightarrow x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$

$x = 2, 4$

5.2

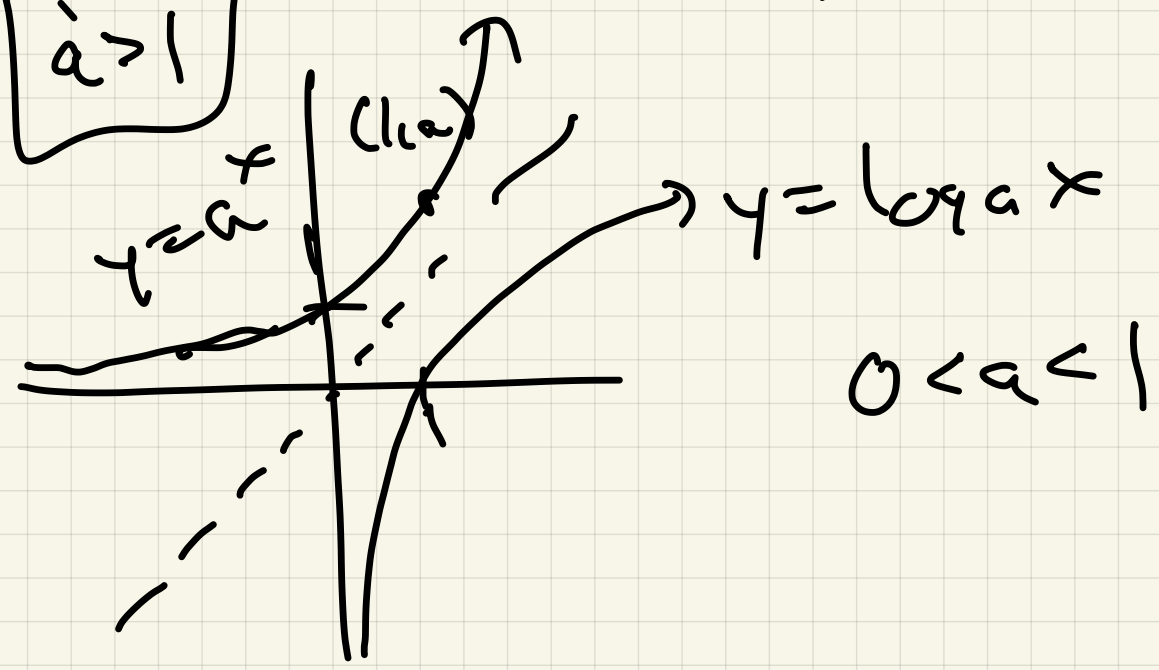
Consequence 2.

$y = f(x) = a^x$ has an inverse function $g(x) = f^{-1}(x)$

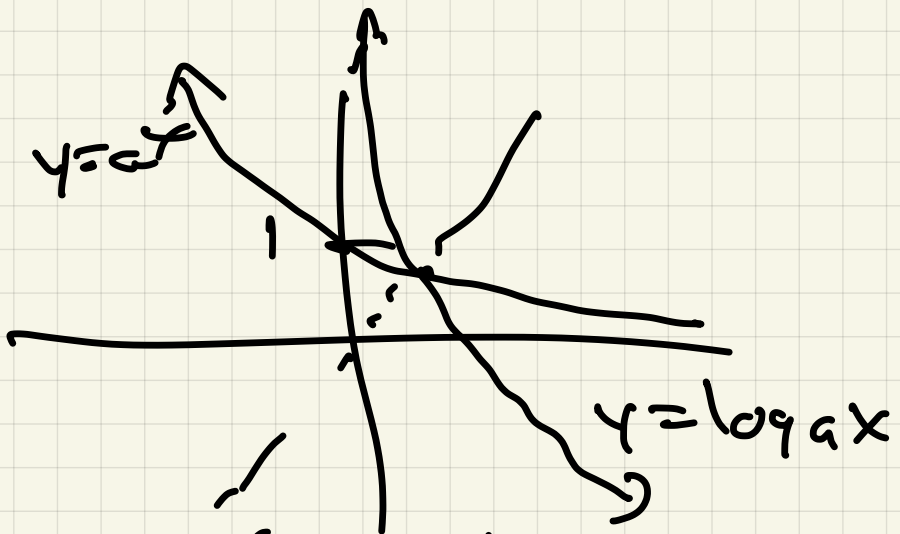
Notation: logarithmic function base a

$y = g(x) = \log_a x$

$a > 1$



$0 < a < 1$



Notice: Domain of $y=\log_a x$

is $(0,\infty)$

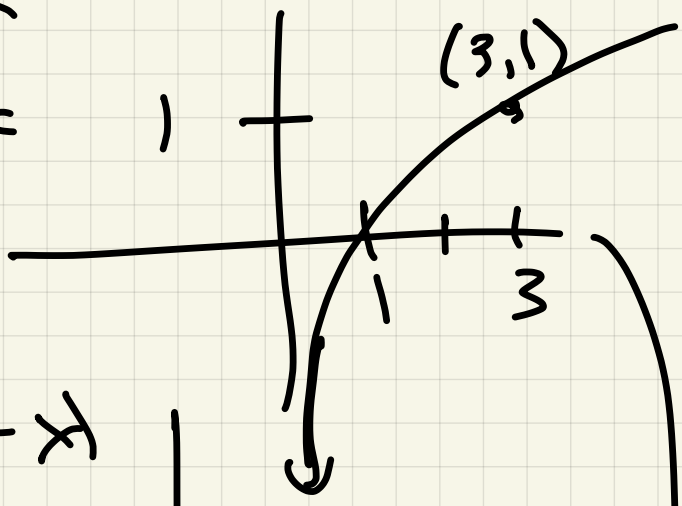
Def: For $x>0$, $0<a\neq 1$,

$$y=\log_a x \iff x=a^y$$

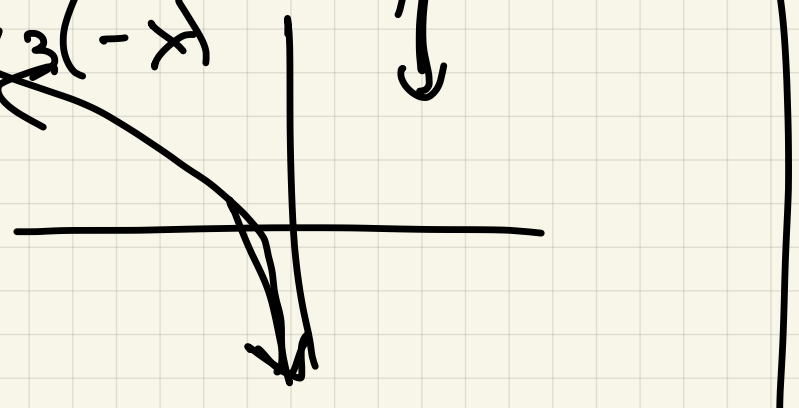
For $x\leq 0$, $\log_a x$ DNE

Ex 1 Sketch

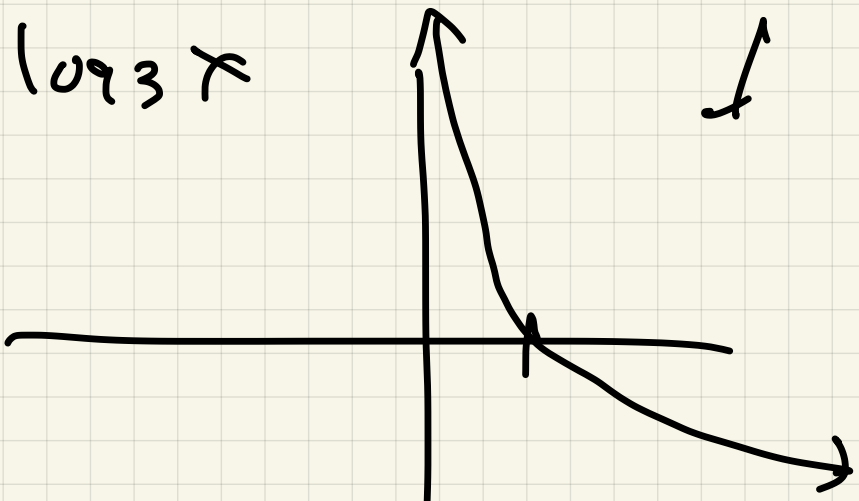
(a) $y=\log_3 x$



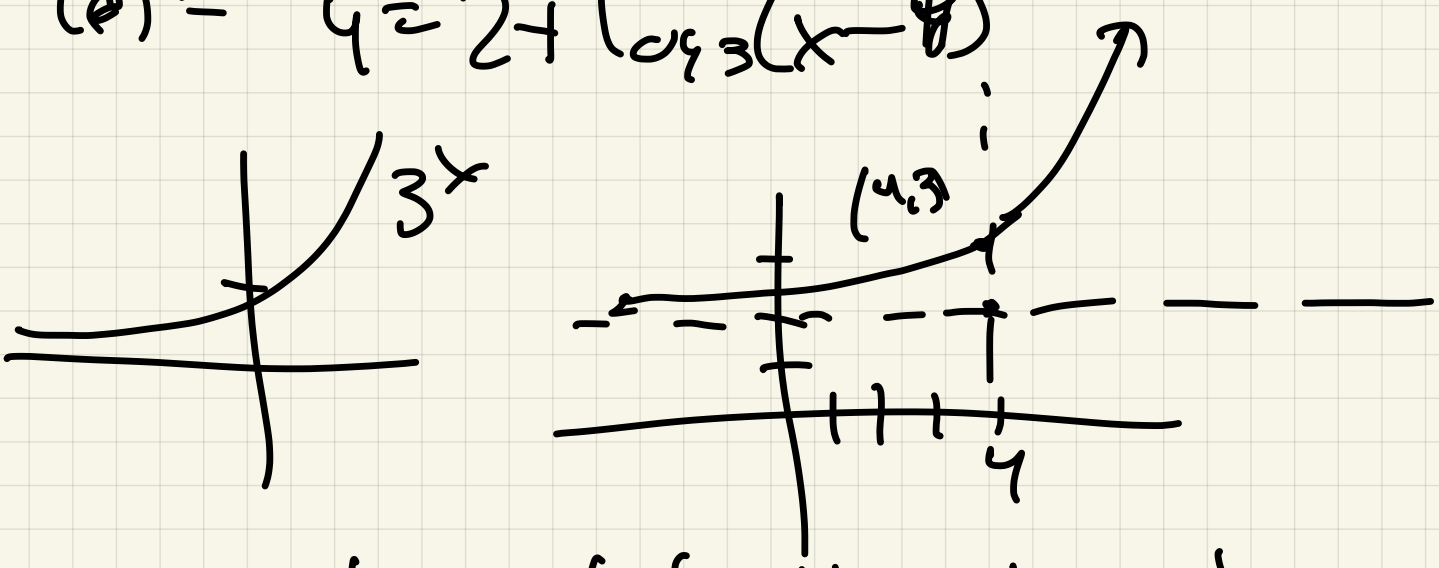
(b) $y=\log_3(-x)$



(c) $y = -\log_3 x$



(d) $y = 2 + \log_3(x-4)$



Ex 2 Use definition to solve equations:

(a) $\log_2 32 = y \Leftrightarrow 32 = 2^y$

$\Downarrow$

$y = 5$

(b) $\log_3 27 = y \Leftrightarrow 27 = 3^y$

$y = 3$

(c) $\log_{10} 1 = y \Leftrightarrow 10^y = 1$

$y = 0$

($\log_{10} 2 = y \Leftrightarrow 10^y = 2$)
HARD

(d) $\log_5 x = -3$ $\quad x = 5^{-3}$

$= \frac{1}{125}$

(e) $\log_2 x = -4$ $\quad x = 2^{-4} = \frac{1}{2^4}$

$= \frac{1}{16}$

Properties: ① $\log_a 1 = 0$ $\quad a^0 = a$

② $\log_a a = 1$ $\quad a^1 = a$

③ $\log_a a^x = x$

④ $\log_a x = \log_a y \Rightarrow x = y$

Special Cases: $\log x \equiv \log_{10} x$

$\ln x = \ln x \equiv \log_e x$

log base 10

"Natural Logarithm" = log base e

$\ln(e^5) = 5$

Properties $\begin{cases} \ln 1 = 0 \\ \ln e = 1 \\ \ln e^x = x \end{cases}$

$(\ln x = \ln y \Rightarrow x = y$

Ex Simplify:

$\ln \frac{1}{e} = \ln e^{-1} = -1$

$e^{\ln 10} = 10$

$q = f^{-1}$

$q(f(x)) = x$

$f(q(x)) = x$

$e^{2\ln 10} = (e^{\ln 10})^2 = 10^2 = 100$

Ex4 Find domains:

(a) $f(x) = \log_3 x$

$(0, \infty)$

(b) $g(x) = \ln(x+10)$

$x + 10 > 0$

$x > -10$

$(-10, \infty)$

(c) $y = \log(3-x)$

$3 - x > 0$

$3 > x$

$(-\infty, 3)$

(d) $y = 2 - \log(x+5) + \log(4-x)$

$x+5 > 0$

$x > -5$

$4-x > 0$

$4 > x$

$-5 < x < 4$

$(-5, 4)$

5.3 Properties:

① $\log_a(uv) = \log_a u + \log_a v$