8/27/ Precal

Last time

Real number

Intervals

dist and absolute value

Integer powers a^n

n integer

Properties

① $a^m a^n = a^{m+n}$

② $\frac{a^m}{a^n} = a^{m-n}$

③ $a^{-n} = \frac{1}{a^n}$

④ $\boxed{(ab)^n = a^n b^n}$

⑤ $(a^m)^n = a^{m \cdot n}$

Further:

$a^{m/n} = \sqrt[n]{a^m}$

Note: $a<0$, m odd, n even

$a^{m/n}$ not defined

Notation: $\sqrt{x} = \sqrt[2]{x}$

Ex 0 $\quad 5^{2/3} \quad 5^{3/2} \quad 5^{6/4}$

$(-5)^{2/3} \quad \boxed{(-5)^{3/2}} \quad (-5)^{6/4}$

Not defined!

Rest OK

Ex 1 $\quad f(x) = \sqrt{1-x}$

$\left(\text{defined for } 1-x \geq 0 \atop x \leq 1 \right)$

Ex 2 $\sqrt{a}\sqrt{b} = \sqrt{ab}$ prop 4

Ex 3 $(x^2)^{3/2} = x^3$ but false:

$(x^2)^{3/2} = |x|^3$

Ex 4 Simplify: $\sqrt[3]{54}$

(a)

$2 \cdot 27 = 2 \cdot 3^3$

$\sqrt[3]{3^3 \cdot 2}$

$\parallel$ prop 4

$\sqrt[3]{3^3} \cdot \sqrt[3]{2} = 3\sqrt[3]{2}$

$3^{3/3} = 3^1 = 3$

(b) $\sqrt[3]{54} + 7\sqrt[3]{16}$

$$7\cdot\sqrt[3]{2^4}$$

$$7\cdot\sqrt[3]{\underbrace{2^3}_{4}\cdot 2}$$

$$\underbrace{3\sqrt[3]{2} + 7\cdot 2\cdot\sqrt[3]{2}}$$

$$\underbrace{(3 + 7\cdot 2)}\sqrt[3]{2}$$

$$17\sqrt[3]{2}.$$

(c) $$\frac{7\sqrt{7}}{\sqrt[3]{7^8}} = \frac{7^1\cdot 7^{1/2}}{(7^8)^{1/3}} = \frac{7^{3/2}}{7^{8/3}} =$$

$$7^{3/2 - 8/3} = 7^{9/6 - 16/6} =$$

$$7^{-7/6} = \frac{1}{7^{7/6}} =$$

$$1 + 1/6$$

$$\frac{1}{7^1 \cdot 7^{1/6}} = \frac{1}{7\sqrt[6]{7}}$$

Rationalizing denominator

~~Ex 3(c)~~ Ex 4 revisit $\quad \dfrac{1}{7\sqrt[6]{7}}$

$$\frac{1}{7 \cdot \underbrace{\sqrt[6]{7} \cdot \sqrt[6]{7^5}}} \cdot \sqrt[6]{7^5} = \boxed{\frac{\sqrt[6]{7^5}}{7^2}}$$

$$7^{1/6} \cdot 7^{5/6} = 7^1$$

Ex 5 $\quad \dfrac{1}{(5+\sqrt{2})} \cdot \dfrac{(5-\sqrt{2})}{(5-\sqrt{2})}$

"multiplying by conjugate"

$$(5+\sqrt{2})(5-\sqrt{2}) =$$
$$5\cdot 5 - \cancel{5\sqrt{2}} + \cancel{\sqrt{2}\cdot 5} - \underline{\sqrt{2}\sqrt{2}}$$
$$25-2=23$$

$$\frac{5-\sqrt{2}}{23}$$

(b) $$\frac{5}{(4-\sqrt{3})} \frac{(4+\sqrt{3})}{(4+\sqrt{3})} = \frac{20+5\sqrt{3}}{13}$$

$$\underbrace{(4-\sqrt{3})(4+\sqrt{3})}_{16-3}$$

<u>Calc I Applic</u>: $$\lim_{x\to 3} \frac{x-3}{2-\sqrt{x+1}} =$$

$$\lim_{x\to 3} \frac{(x-3)\ (2+\sqrt{x+1})}{(2-\sqrt{x+1})(2+\sqrt{x+1})} =$$

$$\lim \frac{(x-3)(2+\sqrt{x+1})}{\underbrace{4-(x+1)}_{3-x}} =$$

$\frac{x-3}{3-x} \rightarrow -1$

$$\lim (-1)(2+\sqrt{x+1}) = -4$$

P3 - P4

Polynomials

Defn A polynomial in x has form

$$p(x) = a_n x^{n} + a_{n-1}x^{n-1} + \dots + a_0 x^0$$

a_k are real number

a_n = leading coeff $\neq 0$

constant coefficient

~~Ex 2 $p(x)=$~~

$n =$ degree of $p(x)$

Ex 1 (a) $p(x) = 3x^2 + 10x + 7$

$\deg p(x) = 2$

leading coeff 3

const coeff 7

(b) $p(x) = 5$

$\deg p(x) = 0$

leading = const = 5

(c)

$p(x) = \underline{\underline{0}}x^2 + \pi x + 10^{18}$

deg = 1

leading $= \pi$

const $= 10^{18}$

Operations

Addition/subtraction - easy

Ex 2 $(3x^2+15x+10)+(x^3+7x-14)$

(a) $x^3+3x^2+22x-4$

(b) $(3x^2+15x+10)-(x^3+7x-14)$

$= -x^3+3x^2+8x+24$

(c) $(x^3+3x^2-10x+1) -$

$(x^3+3x^2+5x+7) =$

deg 3

$$0x^3 + 0x^2 - 15x - 6$$

$$= -15x - 6$$

deg 1

Multiplication

High school

FOIL

first out in last

$$(2x+7)(4x+5) =$$

$$\underset{F}{8x^2} + \underset{O}{10x} + \underset{I}{28x} + \underset{L}{35} =$$

$$8x^2 + 38x + 35$$

$$(2x-7)(4x-5)$$

$$8x^2 - 10x - 28x + 35$$

$$= 8x^2 - 38x + 35$$

(c) $(x^4 - \pi x)(x^3 + \pi x) =$

$$x^7 + \pi x^5 - \pi x^4 + \pi^2 x^2$$

Special Cases

1. squaring binomials

(a) $(3x+7)(3x+7) =$

$$9x^2 + 42x + 49$$

(b) $(2x-5)(2x-5) =$

$$4x^2 - 20x + 25$$

2. "difference of squares"

(a) $(3x+7)(3x-7)$

$$9x^2 - 49$$

(b) $(x+\pi)(x-\pi) = x^2 - \pi^2$

diff of squares

(c) $((2x+1)-3)((2x+1)+3)$

<u>one way</u>: diff of squares:

$$\underbrace{(2x+1)^2}_{\text{FOIL}} - 3^2$$

$$4x^2 + 2x + 2x + 1 - 9$$

$$4x^2 + 4x - 8 \quad \checkmark$$

<u>Also</u> simplify first

$$\underbrace{((2x+1)-3)}((2x+1)+3)$$

$$(2x-2)(2x+4)$$

FOIL

$$4x^2+8x-4x-8$$
$$\shortparallel$$
$$4x^2+4x-8 \checkmark$$

~~Ex~~ <u>In general??</u>

$$(2x^2+4x+8)(\underline{\underline{x-2}})$$

systemic!

$x-2$

$\times x$ $\times 2$

$$-4x^2-8x-16$$
$$2x^3+4x^2+8x$$

$$2x^3+0x^2+0x-16$$

$$=2x^3-16$$

$$(x^3+x+3)(x^2+2x-2)$$
$$\shortparallel$$

$$-2x^3 - 2x - 6$$

$$2x^4 + 2x^2 + 6x$$

$$x^5 + x^3 + 3x^2$$

$$x^5 + 2x^4 - x^3 + 5x^2 + 4x - 6$$