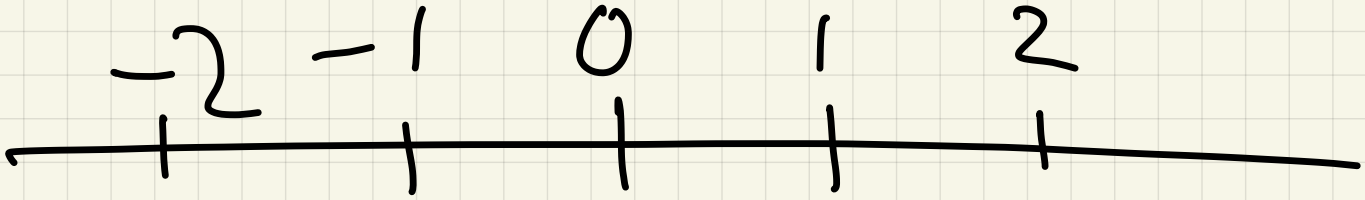
8/25 / Precal

email — weekly planner
webassign
HW 1 — HWPs1
HWP.2

Last time:



Integers $\mathbb{Z}$

rational $\mathbb{Q}$

irrationals $\pi, e, \sqrt{2}$

Comparing numbers

$0 < 2$ — less than

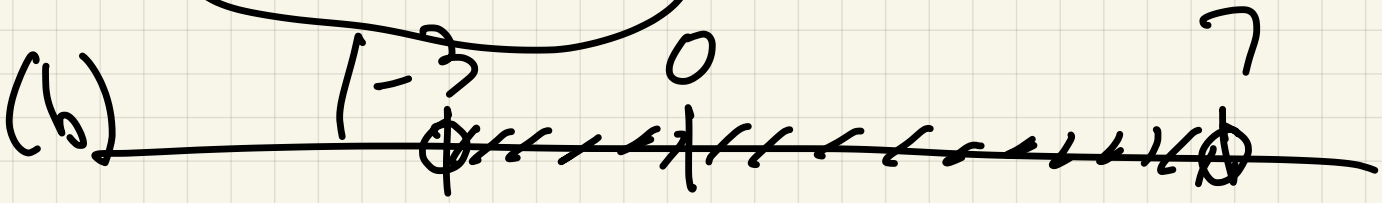
$0 > -1$ — greater than

$0 = 0$ — equal

Ex1 Set of all numbers that are greater than -3 by less than 7

(a)

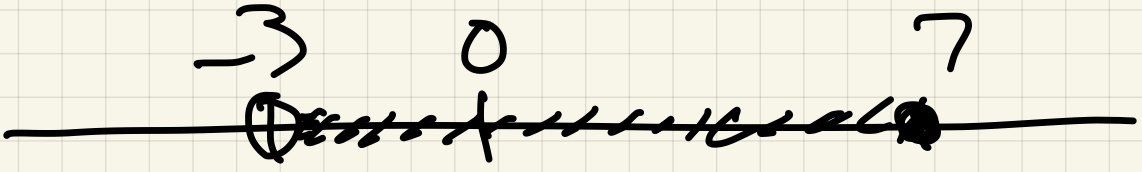
(b)



(c) $\{x: -3 < x < 7\}$

(d) $(-3, 7)$ interval notation

Ex2 less than or equal to 7

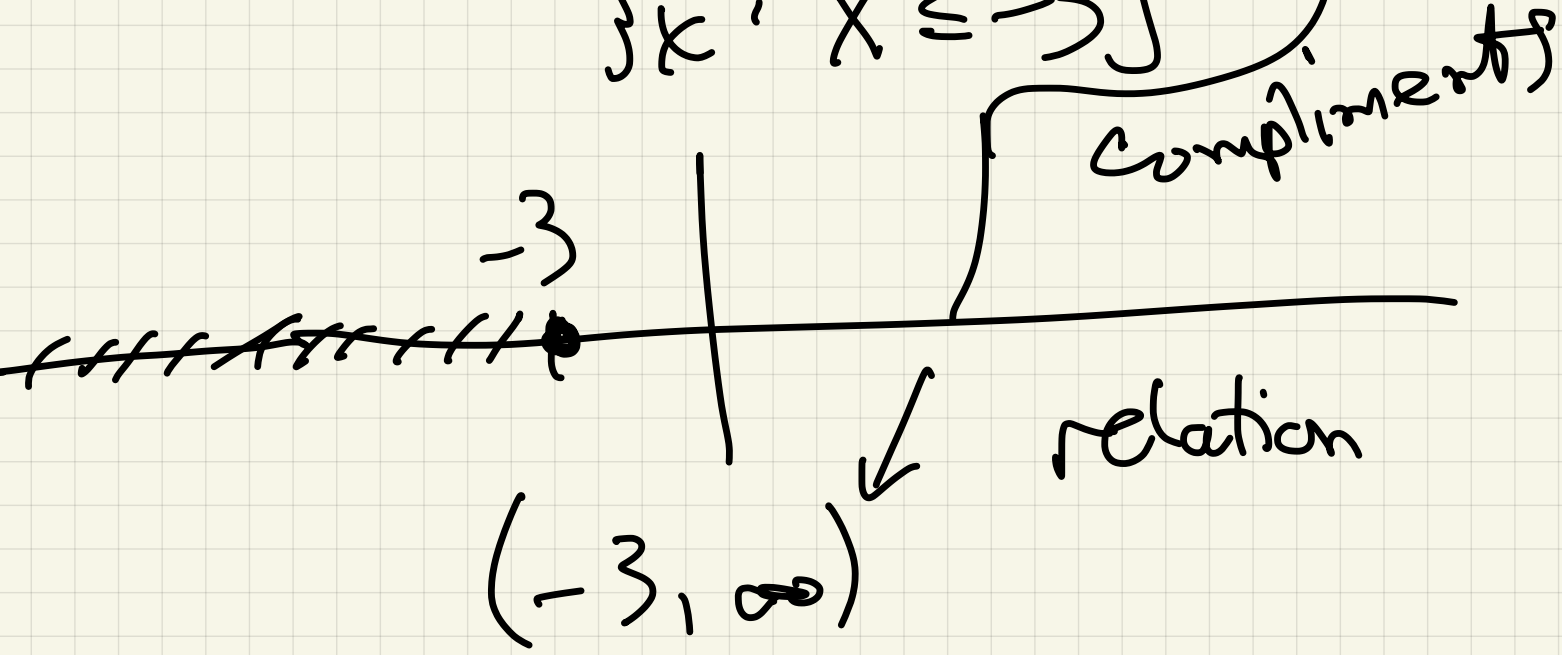


$\{x: -3 < x \leq 7\}$

$(-3, 7]$

Ex3 $(-\infty, -3]$

$\{x : x \leq -3\}$

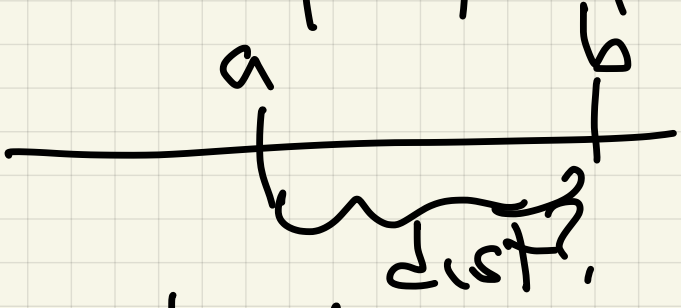


$(-3, \infty)$

Absolute value:

Defn: $|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$

$|25| = 25$ $\quad |-7| = 7$



Application

The distance ~~between~~

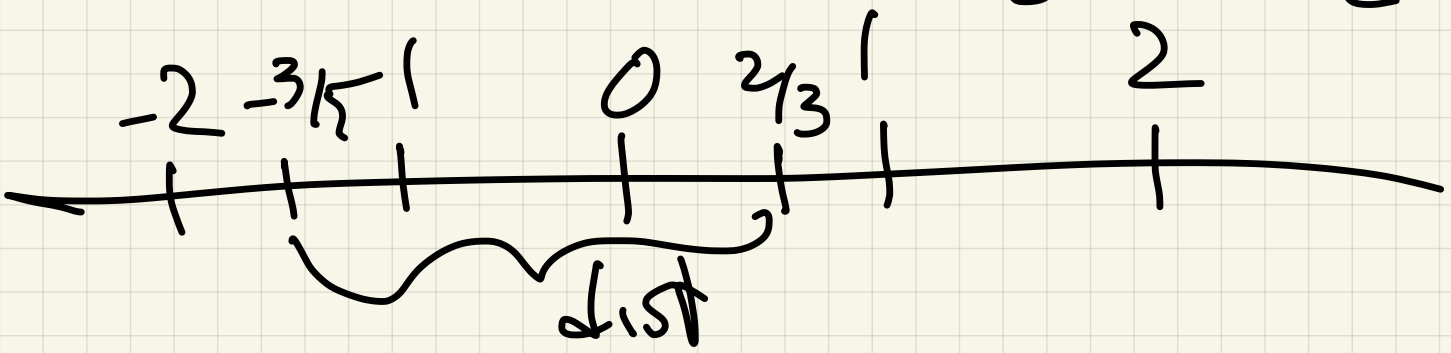
from a to b is $|a-b|$

Eg. $a=-3$ $b=7$ (Ex)

(a) $|-3-7| = |-10| =$

10

(b) distance from $2/3$ to $-3/2$



$\left|\frac{2}{3}-\left(-\frac{3}{2}\right)\right| = \left|\frac{2}{3}+\frac{3}{2}\right| =$

$\left|\frac{4}{6}+\frac{9}{6}\right| = \left|\frac{13}{6}\right| = 13/6$

(c) dist from π to $10 =$

$|\pi - 10| = -(\pi - 10) =$

$10 - \pi$

(d) dist from 10 to π is

$|10 - \pi| = 10 - \pi$ same

Expressions combinations of variables & numbers with operations

a) $x^2 + 5$ } Calc 1-2

b) $2x + 7y$

c) $x^2 + y^2 + z^2$ } Calc 3

Can **evaluate** by choosing values for variables

a) x^2+5

$x=0 \Rightarrow 5$

$x=5 \Rightarrow 30$

$x=-5 \Rightarrow 30$

c) $x^2+y^2+z^2$

$x=3$

$y=4$

$z=5$

$3^2+4^2+5^2$

$9+16+25=50$

(d) $\dfrac{|x+17|}{x+17}$

$x=80 \Rightarrow \dfrac{27}{27}=1$

$x=1 \Rightarrow 1$

$x=-5 \Rightarrow 1$

$x=-20 \Rightarrow -1$

$x = -67 \Rightarrow \frac{|-50|}{-50} = \frac{50}{-50} = -1$

$x = -17$

$\frac{0}{0}$ DNE

$$\frac{|x+17|}{x+17} = \begin{cases} 1 & x > -17 \\ \text{DNE} & \\ -1 & x < -17 \end{cases}$$

Does not exist

Arithmetic with fractions

(a) $\frac{7}{6} - \frac{9}{8} = \frac{56}{48} - \frac{54}{48} =$

(×6 and ×8 to common denom 24, 48)

subtraction

$\frac{2}{48}$

$\frac{1}{24}$

$$\frac{56}{48} - \frac{54}{48} = \frac{2}{48} = \frac{1}{24}$$

same

$$\frac{7}{6} - \frac{9}{8} = \frac{28}{24} - \frac{27}{24} = \frac{1}{24}$$

$\times 4$

multiplication / division

Easy: (b) $\frac{2}{3} \cdot \frac{6}{7} = \frac{12}{21} = \frac{4}{7}$

$$\frac{2}{3} \div 3/2 = \frac{2}{3} \times \left(\frac{1}{3/2}\right)$$

multiply by reciprocal

(c) $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

(d) $\frac{4}{7} \div \frac{8}{5} = \frac{1}{4} \times \frac{5}{8_2} = \frac{20}{56} =$

P2 Exponentials & Radicals

Powers $a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n}$

n pos integer n

Ex $2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$

$(-2)^5 = -2 \cdot -2 \cdot -2 \cdot -2 \cdot -2 = -32$

$3^5 = 243$

look it.

Negative powers: $a^{-1} = \frac{1}{a}$

$a^{-n} = \frac{1}{\underbrace{a \cdot a \cdot \ldots \cdot a}_{n}} = \frac{1}{a^n}$

n pos int

Ex $2^{-10} = \frac{1}{2^{10}} = \frac{1}{1024}$

$\left(\frac{1}{3}\right)^{-5} = \frac{1}{\left(\frac{1}{3}\right)^5} = \frac{1}{\left(\frac{1}{3^5}\right)} =$

$3^5 = 243$

Easy properties

① $a^m \cdot a^n = a^{m+n}$

(Why? m, n pos, $a^m = \underbrace{a \cdot \ldots \cdot a}_{m}$, $a^n = \underbrace{a \cdot \ldots \cdot a}_{n}$ =

$\underbrace{a \cdot a \cdot \ldots \cdot a}_{m+n} = a^{m+n}$)

② $\frac{a^m}{a^n} = a^{m-n}$ ←

③ $a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$

④ $(ab)^n = a^n b^n$

⑤ $(a^m)^n = a^{m \cdot n}$

Ex

$$\left(\frac{1}{3}\right)^{-5} = \left(\frac{1}{\left(\frac{1}{3}\right)}\right)^5 = 3^5 = 243$$

$$6^3 = (2 \cdot 3)^3 = 2^3 \cdot 3^3$$

$$8 \cdot 27 = 216$$

$$(2^5)^2 = 32^2 = ?$$

$$= 2^{5 \cdot 2} = 2^{10} = 1024$$

$$\frac{(-2)^4}{(-2)^7} = (-2)^{4-7} = (-2)^{-3} =$$

$$\frac{1}{(-2)^3} = \frac{1}{-8}$$

Ex Simplify:

(a) $(2\omega)^3 = 2^3 \cdot \omega^3 = 8\omega^3$

(b) $(-x)^5(3x^2) =$

$$(-1x^5)(3x^2) =$$

$$-3 \cdot x^5 x^2 = -3x^7$$

(c) $\left(\frac{x^2}{4^{-2}}\right)\left(\frac{x}{4^2}\right)^3 =$

$4^{-2} = \frac{1}{4^2}$ x^{-2}

$$\left(\frac{x}{y}\right)^{-2}\left(\frac{x}{y^2}\right)^3 = \frac{y^2}{x^2}\cdot\frac{x^3}{(y^2)^3} = \frac{x}{y^4}$$

$$\frac{y^2}{y^6} = y^{2-6} = y^{-4} = \frac{1}{y^4}$$

More complicated rational exponents

n positive integer

$a^{1/n} = \sqrt[n]{a}$ = largest number b so that $b^n = a$

In general, m,n positive,

then $a^{m/n} = \sqrt[n]{a^m} = (a^m)^{1/n}$

Ex $2^{5/2} = (2^5)^{1/2} =$

$\sqrt{2^5} = \sqrt{32} = \sqrt{4 \cdot 4 \cdot 2}$

$\sqrt{4} = 2$ $\qquad 2\sqrt{4 \cdot 2}$

$2 \cdot 2\sqrt{2} = 4\sqrt{2}$

$(-2)^{5/2}$ DNE

$(-2)^{2/5} = (4)^{1/5} = \sqrt[5]{4}$

Book: $\sqrt{a}\sqrt{b} = \sqrt{ab}$