**Exam 2** **October 2023**
**Discrete Mathematics II** **Prof. Nollet**

The second exam covers sections 11.3 and 6.1-6.5, 6.7, 6.9 in zybook.

11.3 The pigeon-hole principle (PHP). Structure of a PHP argument and the generalized PHP (some pigeon-hole has at least $\lceil p/h \rceil$ pigeons for $p$ pigeons and $h$ pigeonholes), how many pigeons are needed to ensure some hole has at least $d$ pigeons.

6.1 Binary relation $R$ from $A$ to $B$, arrow diagrams, matrix representations, list representations. Digraph representation when $A = B$.

6.2 Relations $R$ on a domain $A$ that are reflexive ($\forall x \in A\ (xRx)$), anti-reflexive ($\forall x \in A\ (\neg xRx)$), symmetric ($\forall x, y \in A\ (xRy \Rightarrow yRx)$), anti-symmetric ($\forall x, y \in A\ (x \neq y \Rightarrow \neg(xRy \wedge yRx))$: or equivalently, $\forall x, y \in A\ (xRy \wedge yRx \Rightarrow x = y)$), and finally transitive property ($\forall x, y, z \in A\ (xRy \wedge yRz \Rightarrow xRz)$). Be able to work with these definitions.

6.3 Directed graph language: vertices and directed edges, head and tail of edge, self-loops, in-degree and out-degree, open and closed walks, length of walks, trails, circuits, paths and cycles.

6.4 Definition of composition $S \circ R$ of relations $R$ and $S$ on a set $A$.

6.5 Relation powers $R^k$ given by $(x, y)$ is an edge of $R^k \iff$ there is a walk from $x$ to $y$ of length $k$, transitive closure $R^+ = \bigcup_k R^k$.

6.7 Partial orders (reflexive, anti-symmetric and transitive) and posets, Hasse diagrams (remove redundant edges, such as self-loops).

6.9 Equivalence relations (reflexive, symmetric, transitive), associated equivalence classes $[a]$ form a partition of $A$.

Suggestions: Look over homework, quizzes, and examples from class. Checking book problems not assigned is a good idea, because I will look at them when I write the exam.