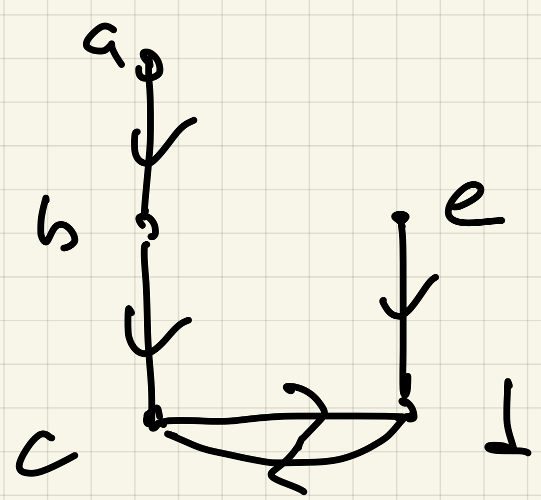
3/31/ Disc 2

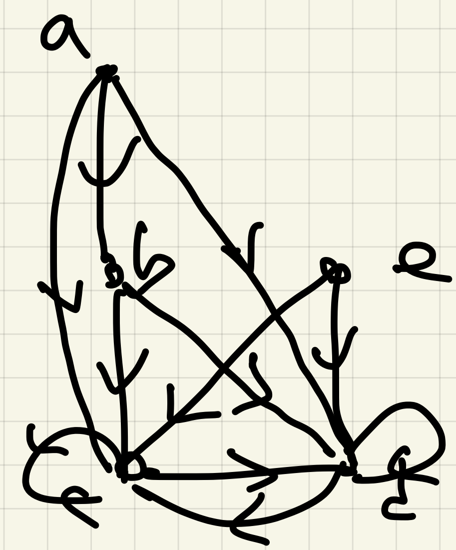
Exam 2 Wednesday

11.3, 6.1 – 6.5, 6.7, 6.9
Pitf

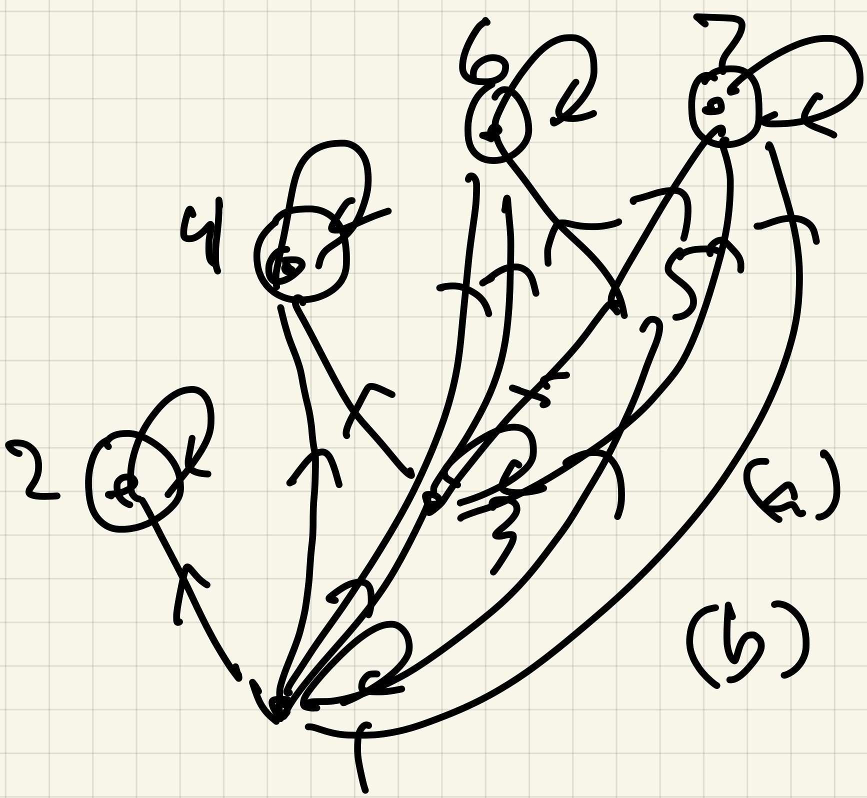
Quiz 8

1.





2.



(a) 2, 4, 6, 7

(b) 1

(c) total order?

NO: 2, 3 not comparable
2, 4, 2, 6, 6, 7
4, 5

(d)
$$\underbrace{6 + 6}\overbrace{4)} = 19$$

Last time linear homogeneous recurrence relations

$\{a_n\}$

$$(*) \begin{cases} a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k} \\ c_i \text{ constant,} \\ \text{Initial conditions} \end{cases}$$

Method to solve:

① Find characteristic equation:

subst $\underline{a_n = x^n}$

(+) $x^n = c_1 x^{n-1} + \dots + c_k x^{n-k}$

③ $x_1 \dots x_k$ are nonzero solutions to (+),

then

④ General solution is

$$a_n = b_1 x_1^n + b_2 x_2^n + \dots + b_k x_k^n$$

b_i constants

⑤ Use initial conditions to find $b_1 \dots b_k$

$k=4$

Ex1 $\begin{cases} a_n = 5a_{n-2} - 4a_{n-4} \\ a_0 = a_2 = 1,\ a_1 = a_3 = 0 \end{cases}$

$$\begin{cases} a_4 = 5a_2 - 4a_0 = 5 - 4 = 1 \\ a_5 = 5\underline{\underline{a_3}} - 4\underline{\underline{a_1}} = 0 \end{cases}$$

(*) $a_n = 5a_{n-2} - 4a_{n-4}$

(*) $x^n = 5x^{n-2} - 4x^{n-4}$

$\div x^{n-4}$

$$x^4 = 5x^2 - 4$$

$$x^4 - 5x^2 + 4 = 0$$

$y = x^2$

$y^2 - 5y + 4 = 0$

$(y-1)(y-4) = 0$

$y = 1, 4$

$x^2 = 1, 4$

$x^2 = 1$

$x^2 = 4,$

$x = \pm 1$

$x = \pm 2$

General Solution:

$$\boxed{a_n = A2^n + B(-2)^n + C(1)^n + D(-1)^n}$$

Hard part: Solve for A, B, C, D:

$n=0$ $\quad 1 = A + B + C + D \quad$ (1)

$n=1$ $\quad 0 = 2A - 2B + C - D \quad$ (2)

$n=2$ $\quad 1 = 4A + 4B + C + D \quad$ (3)

$n=3$ $\quad 0 = 8A - 8B + C - D \quad$ (4)

$A=0$ $\quad 6A = 3A + 3A = 0$

(3) − (1) $\Rightarrow$ $3A + 3B = 0$

(4) − (2) $\quad 6A - 6B = 0$

$A = B$

$$A = B = 0$$

$1 = C + D$

$0 = C - D$

$1 = 2C \rightarrow C = 1/2$

$D = 1/2$

So

$$a_n = \frac{1}{2} + \frac{1}{2}(-1)^n$$

a_n: 1 0 1 0 1 0 1 0 1 0 ...

Ex2

$$\begin{cases} \boxed{a_n = -10a_{n-1} - 25a_{n-2}} \\ a_0 = 10, \quad a_1 = 20 \end{cases}$$

(+) $x^n = -10x^{n-1} - 25x^{n-2}$

$x^2 = -10x - 25$

$\underbrace{x^2 + 10x + 25} = 0$

$(x+5)^2 = 0$

So $x = -5$

$$\boxed{a_n = A\cdot(-5)^n} \quad \underline{\text{Fails}}$$

$$\left.\begin{array}{l} a_0 = 10 \\ a_1 = 20 \end{array}\right\} \to \begin{array}{l} A = 10 \\ \# \\ A = -4 \end{array}$$

Idea: $n(-5)^n$ is also a solution!

$$a_n = n(-5)^n:$$

$$n(-5)^n = -10(n-1)(-5)^{n-1} - 25(n-2)(-5)^{n-2}$$

$$\div (-5)^{n-2} \quad \Downarrow$$

$$n(-5)^2 = -10(n-1)(-5) - 25(n-2)$$

$$25n = 50(n-1) - 25(n-2)$$

$$25n = 50n - \cancel{50} - 25n + \cancel{50}$$

Now $\quad n(-5)^n \quad (-5)^n$

$\therefore$ Theorem $\Rightarrow$

d

$$a_n = A(-5)^n + B\,n(-5)^n$$

is a solution,

can match initial conditions

$n=0 \qquad 10 = A + 0B$

$n=1 \qquad 20 = A(-5) + B\cdot 1\cdot(-5)$

$A = 10 \qquad -5A - 5B = 20$

$$-50 - 5B = 20$$

$$-5B = 70$$

$$B = \frac{70}{-5} = -14$$

so $a_n = 10(-5)^n - 14n(-5)^n$

In general: If root x to (f) appears k times, then

$x^n, nx^n, n^2x^n \dots n^{(k-1)}x^n$

linear combination of these as general solution

Ex1: Given the characteristic equation, find general solution.

(a) $x - 7 = 0$

$x = 7 \Rightarrow$ gen'l solution $\boxed{a_n = a \cdot 7^n}$

(b) $x^2 - 1 = 0$

$(x-1)(x+1) = 0$ $\quad x = \cancel{}$

$x = 1, -1$

So $a_n = a1^n + b(-1)^n$

(c) $(x+2)^2(x+5) = 0$

$x = \underline{\underline{-2, -2}}, -5$

d

$a_n = a(-2)^n + bn(-2)^n + c(-5)^n$

(d) $(x+3)^4(x-10) = 0$

$-3, -3, -3, -3 \qquad 10$

$a_n = \underline{a}(-3)^n + \underline{bn}(-3)^n + \underline{cn^2}(-3)^n$

$+ \underline{dn^3(-3)}^n + e10^n$

(e) $x^2 - 16 = 0$ $\quad x = \pm 4$

$a4^n + b(-4)^n$

(f) $x^2 - 8x - 16$

$(x-4)^2 \qquad x = 4, 4$

$a4^n + b\,n \cdot 4^n$

(g) $\underbrace{x^3 + x^2} - \underbrace{x - 1} = 0$

$x^2(x+1) - 1(x+1) = 0$

$\underline{(x^2 - 1)}(x+1) = 0$

$(x-1)(x+1)(x+1) = 0$

$1 \quad -1 \quad -1$

So $a_n = a1^n + b(-1)^n + c(n)(-1)^n$

§ 8.16 Linear non-homogeneous recurrence relations

Ex| (a)

(a) $\underline{f_n = 2f_{n-1} + 3f_{n-2}} + 7$

(b) $\underline{f_n = 2f_{n-1} + 3f_{n-2}} + n^2$

(c) $\underline{f_n = 2f_{n-1} + 3f_{n-2}} + 5^n$

extra term

Two part strategy:

① Find general solution to the homogeneous part of the equation (underlined part)

f_n^h h = homogeneous

② Find one solution to actual equation f_n^p = particular

③ Add results

(4) ~~Solve~~ Match initial conditions.

How to find f_n^p ???

Ex 2 (a) $\underbrace{f_n = 2f_{n-1} + 3f_{n-2}} + 7$

Aside: $f_n = 2f_{n-1} + 3f_{n-2} \to$

$x^n = 2x^{n-1} + 3x^{n-2} \to$

$x^2 = 2x + 3 \to x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3, \ x = -1$

$f_n^h = A(3)^n + B(-1)^n$

How to find f_n^p ??

Guess: $f_n = c$ = constant

(b/c 7 constant)

$f_n = 2f_{n-1} + 3f_{n-2} + 7$

$c = \underline{2c} + \underline{3c} + 7$

$-4c = 7 \Rightarrow c = -7/4$

so

$f_n = A\,3^n + B(-1)^n - 7/4$

(b) $f_n = 2f_{n-1} + 3f_{n-2} + 5^n$

$\underline{\text{Guess}}$: $f_n = c \cdot 5^n$

$\frac{5^{n-2}}{5^n}$ $\quad \underline{c \cdot 5^n} + 2 \cdot c\,\boxed{5^{n-1}} + \underline{3c\,5^{n-2}} + \underline{\underline{5^n}}$

$c + 2c\frac{1}{5} + 3c\frac{1}{25} + 1$

$\Downarrow$

$$2\underline{5}c = 1\underline{0}c + \underline{3}c + 25$$

$$\cancel{c(38) = 25 \Rightarrow c = 25/38}$$

$$12c = 25 \Rightarrow c = 25/12$$