

1/27/Discrete 2:

Quiz 1

A

1. $\frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3}$ 3^8

2. A, CA
 $11 \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3}$ 3^6

3. $A_2: 12 \quad 22 \quad 3$
 $3^6 + 3^6 + 3^7$

4. $11 \quad 13$
 $12 \quad \text{missing} \quad 23$
 $22 \quad 21$
 3
strings start $13, 23, 21.$

Last time $\left\{ \begin{array}{l} \text{bijection rule} \\ k-1 \text{ rule} \end{array} \right.$
 $\left[\text{Generalized product rule} \right]$

m step process,

n_i outcomes at stage i

\Rightarrow Total # outcomes $\prod_{i=1}^m n_i$

Ex 1 20 students

10 boys / 10 girls

(a) $20!$ ways to line them up

$\overline{20} \overline{19} \overline{18} \overline{17} \dots \overline{1} \rightarrow 20!$

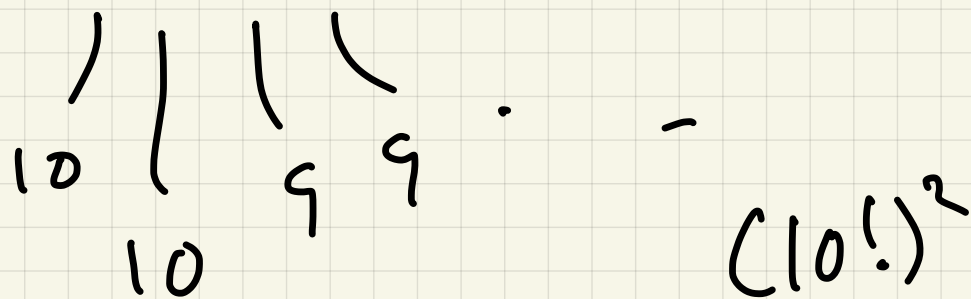
(b)

$\overline{10} \overline{9} \overline{1} \overline{10} \overline{9} \overline{8} \dots \overline{1}$

10 girls followed by 10 boys

$(10!)^2$

(c) BG BG BG . . .



(d) How many ways with
Bill₁ / Bob not next to each
other



Idea: How many ways with
B₁ / B₂ together?

→ 19 ways to place pair of them

→ 2 decide if B₁/B₂ or B₂/B₁

18! ways to place rest

$$2 \cdot \underline{19} \cdot 18! = 2 \cdot 19!$$

Find answer

$$20! - 2 \cdot 19!$$

$$= 20 \cdot \underline{19!} - 2 \cdot \underline{19!}$$

$$(20 - 2) \cdot 19! = 18 \cdot 19!$$

§ 10.4 Permutations

The most common application of generalized power rule is counting permutations

X set, $|X| = n < \infty$

Defn An r -permutation from X

is a sequence of r items taken from X in order without repetitions

Ex) Drawing 5 cards from

Deck X in order, without replacement

$\overline{52} \overline{51} \overline{50} \overline{49} \overline{48}$

way to do it is

$$\frac{52!}{47!}$$

In general, the number of r -permutations from a set X of size n is

$$n(n-1)(n-2) \dots (n-r+1)$$

$$\underbrace{\hspace{15em}} \Bigg|$$

$$\frac{n!}{(n-r)!}$$

Notation

$$\underline{\underline{P(n, r)}} = \frac{n!}{(n-r)!}$$

Ex 2 Drama club of 10 students has to select president VP, treasurer and secretary, no person holds 2 offices.
How many ways?

$$\overline{P} \overline{V} \overline{T} \overline{S} \Rightarrow P(10, 4) = \frac{10!}{6!} = 5040$$

Ex 3 How many ways to make a password from $X = \{a, b, c, \dots, z\}$ of length 10?

(a) 26^{10}

(b) If all characters are

distinct?

$$P(26, 6) = \frac{26!}{16!}$$

(c) No repetition, end xyz

-----xyz

$$P(23, 7) = \frac{23!}{16!}$$

(d) No repetitions, must contain the letter k.

10 · P(25, 9)
/ \
position k remaining letters

(e) No repetitions,
must contain letters
x, y and z.

Place x, y, z , rest

$$10 \cdot 9 \cdot 8 = P(10, 3)$$

$$P(23, 7)$$

(F) Must contain

a b c d e f g h i j

$$10! = P(10, 10)$$

Defn A permutation of a set X

is a sequence from X

where each item appears

exactly once

$$\# = |X|!$$

Ex 4 How many functions

(a) $f: \{0, 1, \dots, 9\} \rightarrow \{a, b, \dots, z\}$

$$f(0) \leftarrow 26$$

$$f(1) \leftarrow 26$$

⋮

⋮

$$f(9) \quad 26$$

$$26^{10}$$

(b) How many are 1-1?

$$P(26, 10) = \frac{26!}{16!}$$

(c) How many are onto?

NONE!

Ex 5 $X = \{a, b, c, d\}$

How many permutations of X

abcd

abdc

start with a

acbd

acdb

adbc

adcb

$$P(4, 4) = 4! = 24.$$

Ex 6: 20 school children

10 boys / 10 girls

(a) # ways to order in line

permutations of
20 children

$\therefore \# 20!$

(b) $B \quad \underbrace{BG \dots BG}$

$$10! \times 10! = 10!$$

(c) $B \underbrace{G B G \dots B G}_{10!}$

(d) How many with
Billy / Bob apart?

Idea

$B_1 \quad B_2$
 $B_1 B_2$ one entity

$$19! \times 2 =$$

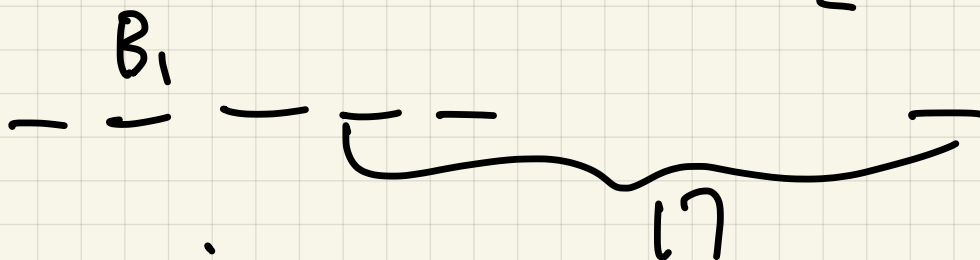
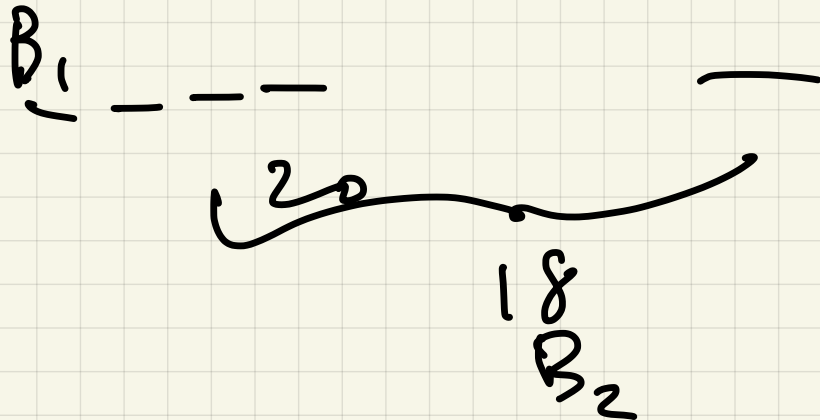
$$B_1/B_2 + B_2/B_1$$

$$\text{Total} = 20! - 19! \cdot 2$$

Idea 2

Count directly.

① Place $B_1 = \text{Billy}$



2 cases:

②

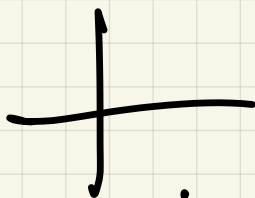
B_1 on end of line |

①8

B_2 |

(18!)

rest



(18)

B_1 is rest on end

(17)

B_2

(18!)

Rest

$$(2) \underline{18} \cdot \underline{18!} + \underline{18} \cdot (17) \underline{18!}$$

$$18 \cdot 18! (2 + 17)$$

$$\underbrace{\hspace{10em}}_{19}$$

$$18 \cdot 19! \checkmark$$

(e) How many ways with

Billy / Rob a part

and Gina / Gloria a part

Use first strategy!

ways Billy/Bob together

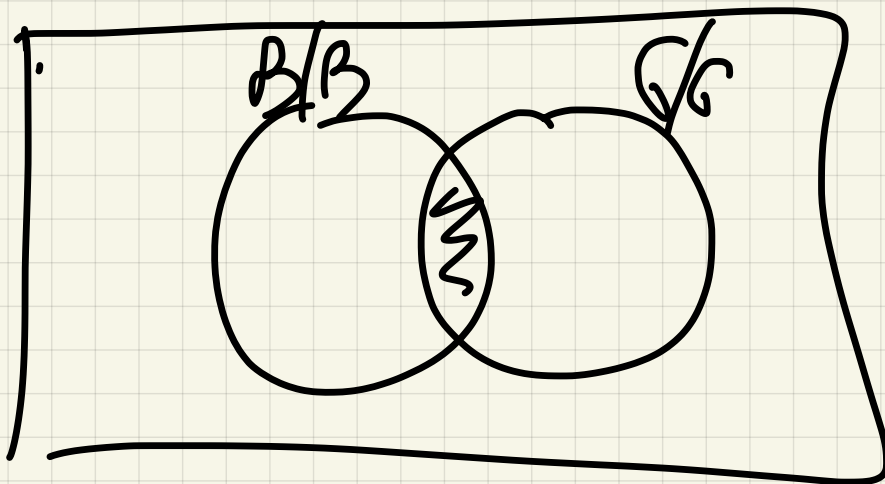
$$2 \times 19!$$

ways Gina/Glenda together

$$2 \times 19!$$

$$20! - 2 \times 19! - 2 \times 19! = \text{ans.}$$

B/B G/G



to correct this, need
add # ways with
B/B together AND
G/G together

B/B

19×2

B_1/B_2

G/G

18×2

G/G_2

$16!$

$19 \cdot 18 \cdot 2 \cdot 2 \cdot 16!$