1/15/ Disc 2

D2L 1/22

AR 3.1 - 3.4, 3.6 - 3.7

HW 1 on 3.7

Q 1 on

Last time

Syllabus

Sets + subsets

$A \subset B$ vs $A \subseteq B$

~~old~~ old

Operations $A \cup B$, $A \cap B$, $A - B$, $B - A$,

$A \oplus B$, $\bar{A}$

Venn diagrams

$A \times B$ $|A \times B| = |A| \times |B|$

New Partitions 3.7

Ex 1 $A = \mathbb{Z} = \text{Ints}$

$\left\{\begin{array}{l} A_e = \{\text{even ints}\} \\ A_o = \{\text{odd ints}\} \end{array}\right.$

partition of $\mathbb{Z}$

$\{k \in \mathbb{Z} : k \equiv 0 \bmod 2\}$

$\{k \in \mathbb{Z} : k \equiv 1 \bmod 2\}$

10.1 Two basic counting principles

Know: $|A \times B| = |A| \times |B|$

More generally: Product Rule

If $A_1 \ldots A_n$ are finite sets,

$$|A_1 \times A_2 \times \ldots A_n| = |A_1| \times |A_2| \times \ldots |A_n|$$

Notation: $\left|\prod_{i=1}^{n} A_i\right| = \prod_{i=1}^{n} |A_i|$

Ex1 How many 7-letter words are there?

(string of length 7 taken from $A = \{a, b, c, \ldots z\}$)

think: $|\underbrace{A \times A \times \ldots \times A}_{7 \text{ times}}| = |A|^7$

$= 26^7$

Leave it like this!

$(8.03 \times 10^{10} \neq 26^7)$

(b) How many with no vowels

a, e, i, o, u

$A - \{a, e, i, o, u\} = B \qquad |B| = 21$

Ans: 21^7

(c) How many start with a and end with k.

aardvark

$a\,\boxed{\_\ \_\ \_\ \_\ \_}\,k$

count these:

26^5

Ex2 How many 10 digit bit strings are there?

$A = \{0, 1\}, \quad |A^{10}| = |A|^{10}$

$= 2^{10} = 1024$

Ex8: How many license plate combinations have form

(a) $\underbrace{\_\ \_\ \_}_{\text{3 letters}}\ \underbrace{\_\ \_\ \_\ \_}_{\text{4 digits}}$

from $1, 2 \ldots 9$

$A, \ldots \ Z$

$A = \{A, \ldots Z\}$, $D = \{1, 2 \ldots 9\}$

$$|\underbrace{A \times A \times A}\times D \times D \times D \times D|$$

$$|A|^3 \times |D|^4$$

$$= 26^3 \cdot 9^4$$

(b) Realizing that they will run out of plate numbers, the state decides to also allow

$\underbrace{\_\ \_\ \_\ \_\ \_}_{5\text{ letters}}\ \underbrace{\_\ \_}_{2\text{ digits}}$

A_1 type II : give

$26^5 \cdot 9^2$ possibilities

A_2 Type I $26^4 \cdot 9^3$ (old ones)

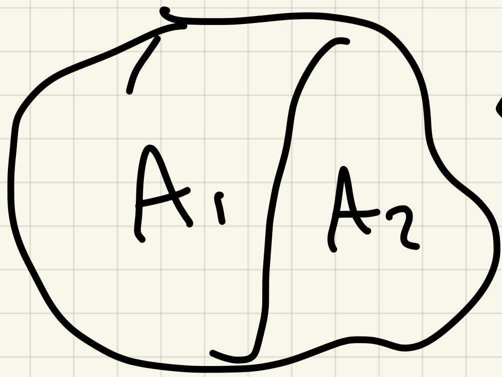
$$26^5 \cdot 9^2 + 26^4 \cdot 9^3$$

This illustrates a second counting principle:

If $A = A_1 \cup A_2$ and

$A_1 \cap A_2 = \emptyset$

Then $|A| = |A_1| + |A_2|$



A Idea

Same works with more sets:

Addition Rule

If $A_1 \cup A_2 \cup \dots \cup A_n = A$

and $A_i \cap A_j = \emptyset$, then

$$|A| = \sum_{i=1}^{n} |A_i|$$

(i.e. If A_i is a partition of A, then $|A| = \sum |A_i|$)

Ex 4 At Subway there are 20 sandwiches. A lunch combo consists of a sandwich ord (function)

(a) drink + chips

3 sizes

5 chips

(b) drink + cookie

4 kinds

(c) drink + apple slice

How many ways to make lunch combo

(a) $|S \times D \times Ch|$

sandwich

drink size

chip choice

$|S| \times |D| \times |Ch| =$

$20 \times 3 \times 5 = 300$

(b) $|S \times D \times C_o|$

$20 \times 3 \times 4 = 240$

(c) $|S \times D \times A|$

$20 \times 3 \times 1 = 60$

So total # is $300 + 240 + 60$
$= 600$

Ex5 A football fan creates a 4 digit ATM PIN for bank account

any number from $0, 1, 2 \ldots 9$

He wants his PIN to begin

34 Walter Payton

4 Brett Favre

84 Randy Moss

10 Fran Tarkenton

How many possible PINs?

34 _ _ (10 10)	100
4 _ _ _	1000
84 _ _	100
10 _ _	100
	1300

Rmk: If we add Alvin Kamara to list, #41

41 _ _ 100 + 1

1400

What's wrong?

41 __ already accounted for!

Ans 1300

10.2 The bijection rule

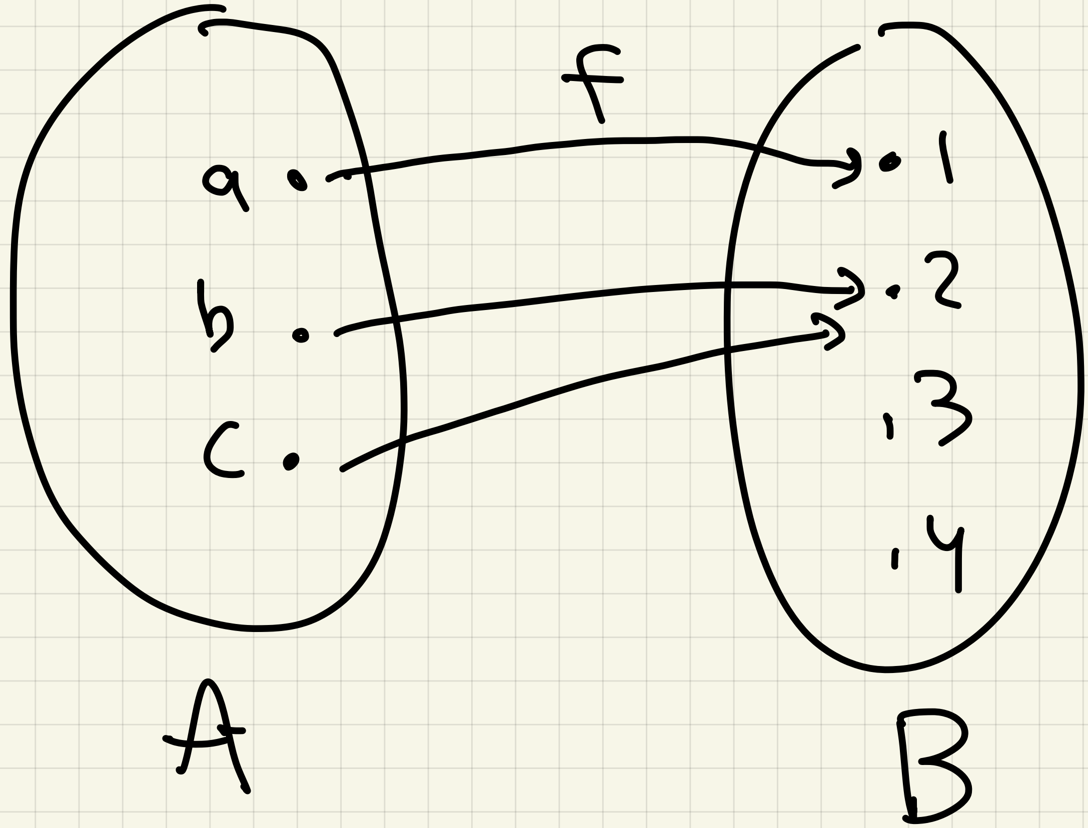
Recall (Discrete 1)

Let A, B be sets.

A function $f: A \to B$ is a rule that assigns each $a \in A$ to exactly one $b \in B$.

Notation $f(a) = b$

Ex)



$f(a)=1$
$f(b)=2$
$f(c)=2$

Graph $= \{(a,1),(b,2),(c,2)\}$

Defns ① f is 1-1 if

$$f(x)=f(y) \Rightarrow x=y$$

(equiv: $x\neq y \Rightarrow f(x)\neq f(y)$)

② f is onto if $\mathrm{Im} f = B$

i.e. $\forall b\in B \ \exists a\in A : f(a)=b$

③ f is a <u>bijection</u> if f is <u>1-1</u> and <u>onto</u>

<u>In Ex)</u>, f not <u>1-1</u>

$$f(b) = f(c) = 2$$
$$b \neq c$$

f is not <u>onto</u>:

$$3, 4 \notin \operatorname{Im} f$$

<u>Thm</u> (Scheinerman's book)

(a) If f is <u>1-1</u>, then $|A| \leq |B|$

(b) If f is <u>onto</u>, then $|A| \geq |B|$

<u>Consequence</u>: If f is a <u>bijection</u>, then ... $|A| = |B|$.

Ex 1 : Recall

7-letter strings from A ... Z

$B = \{ a \underbrace{\boxed{\_\_\_\_\_\_}} k \}$

$A = \{$ 5-letter words $\}$

$f : A \to B$

$f(x_1 x_2 x_3 x_4 x_5) =$

$a x_1 x_2 x_3 x_4 x_5 k$

words that start with a end with k

so f is bijection $\therefore$

$|B| = |A| = 26^5$ ✓

(b) Counts words of form

$B = \{ x_1 x_1 x_2 x_2 x_3 x_3 x_4 \}$

same same same

xxaabby

keys

$A = \{4 \text{ letter words}\}$

$f : A \longrightarrow B$

$f(xaby) = xxaabby$

f is bijection :

$|B| = |A| = 26^4$

(c) <u>Count</u> palindromes

$\underbrace{x_1 x_2 x_3 x_4}\, x_3 x_2 x_1$

$f: A \longrightarrow B$

$f(abcd) = abcdcba$

so $|B| = 26^4$

(d) Count palindromes with middle 3 letters same

$\boxed{a\ b\ c}\ c\ c\ b\ a$

So 26^3