

1/13 | Discrete 2 :

Math 30123

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Office Hours : T-F 2-5

D2L = TCU online

Weekly Planner :

quiz / hw / solutions / Exams

Text : 24books



learn.24books.com

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② TCUMATH 30123 Noll et Spring 2028

③ Subscribe

Calculator TI-89 or none

<u>Grades:</u>	5	AR	= Active reading
	10	HW	
	10	quiz	
	15	Exam 1	Feb 19
	15	2	March 19
	15	3	Apr 16
	30	Final	

Monday May 5, 5-730

Discrete I:

Direct

contrapositive

contradiction

counterexample

Induction

{ Proofs

Boolean logic / truth tables

Sets & functions

Counting

Number Theory

gcd

Euclidean algorithm

modular arithmetic

Demographics : 17 6
CS Month

When discrete I :

24 weeks

0

Discrete 2 :

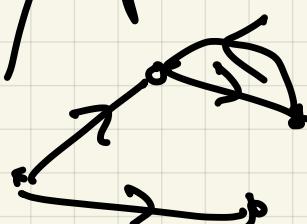
Counting

(more intense)

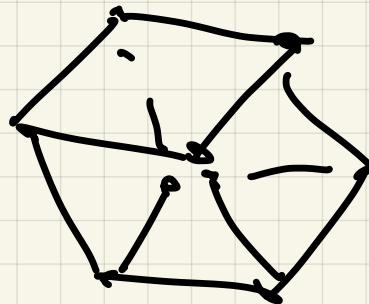
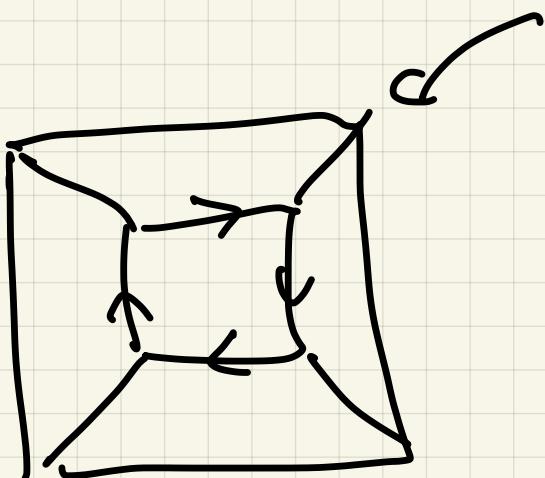
Relations

Equivalence relations / posets

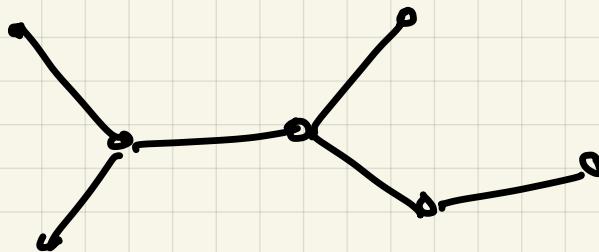
Directed graphs



Graphs



Trees



Recurrence relations

{ a_n } sequence

$$a_0 = 1, \quad a_{n+1} = 2a_n - 1$$

(a)

n	0	1	2	3	4	-
a_n	1	1	1	1	1	-

(b) $a_0 = 2, \quad a_{n+1} = 2a_n - 1$

n	0	1	2	3	4	-
a_n	2	3	5	9	17	-

$$a_n = 2^n + 1$$

Review of sets

Ex: common sets in Math
(a) $\mathbb{Z}, \mathbb{R}, \mathbb{Q},$

$$\mathbb{N} = \{0, 1, 2, 3, 4, 5, \dots\} \subset \mathbb{Z}$$

↑
Note

$$\begin{aligned}\mathbb{Z}^+ &= \{z \in \mathbb{Z} : z \text{ positive}\} \\ &= \{z \in \mathbb{Z} : z > 0\}\end{aligned}$$

Note: A, B sets, $A \subset B$
means

$$x \in A \Rightarrow x \in B, \text{ but } \underline{\underline{A \neq B}}$$

$$\text{Allow} = , \quad A \subseteq B$$

$$(b) S = \{1, 2, 3, 4, 5\} = \{1, 6, 3, 5, 4, 3\}$$

$$= \{x \in \mathbb{N} : 1 \leq x \leq 5\}$$

$$= \{x \in \mathbb{N} : |3-x| \leq 2\}$$

$$|S| = 5$$

$$(c) T = \{1, 2, -5, E, \{5, 6\}, R\}$$

$$|T| = 6$$

$$(d) A = \{a, b\}$$

$$P(A) = \{ \text{all subsets of } A \}$$

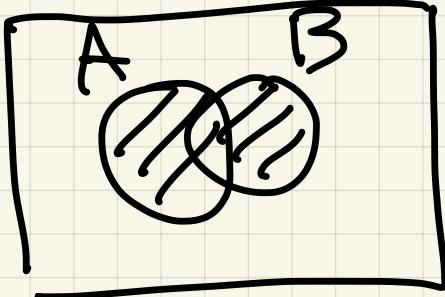
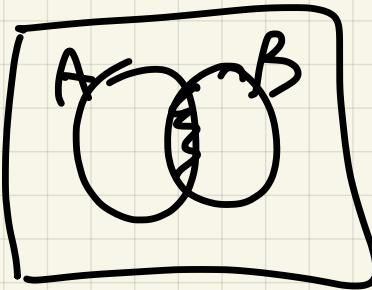
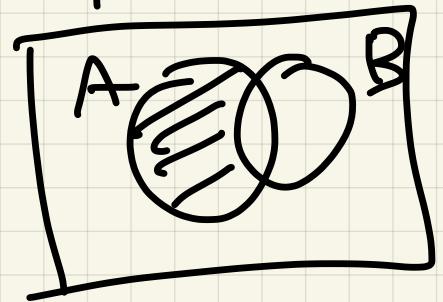
Powered set

$$= \{\{a\}, \{b\}, \{a, b\}, \emptyset\}$$

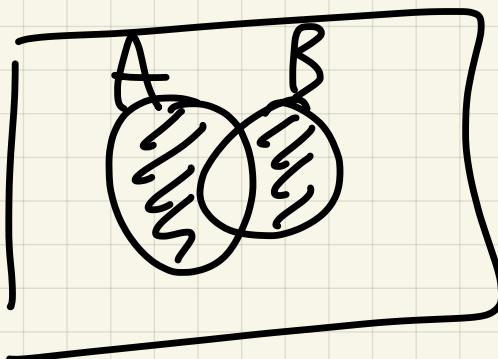
$$|P(A)| = 2^{|A|}$$

Set operations :

Given sets A, B can form new sets:

$A \cup B$  $A \cap B$  $A - B$ 

Symmetric Difference

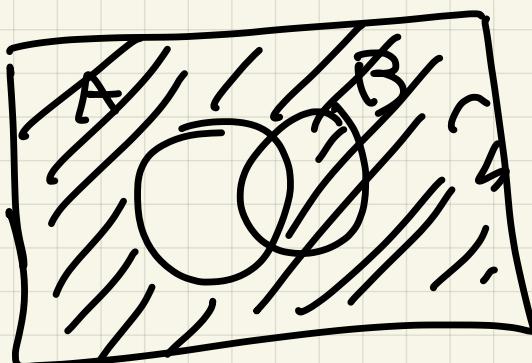


Schönemann

 $A \Delta B$

Für vs :

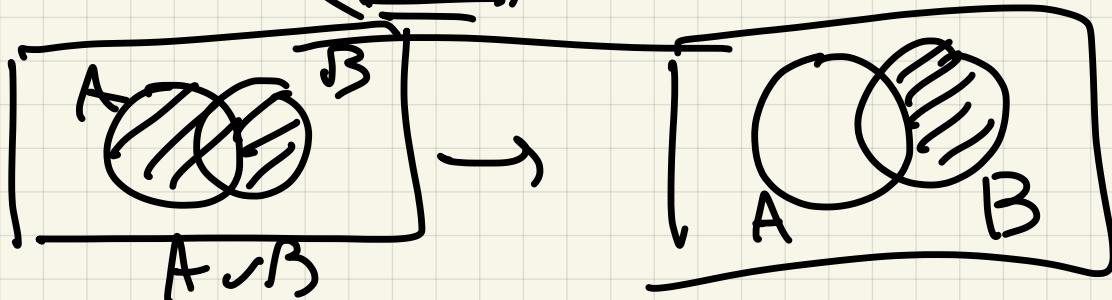
$$(A - B) \cup (B - A) = A \oplus B$$



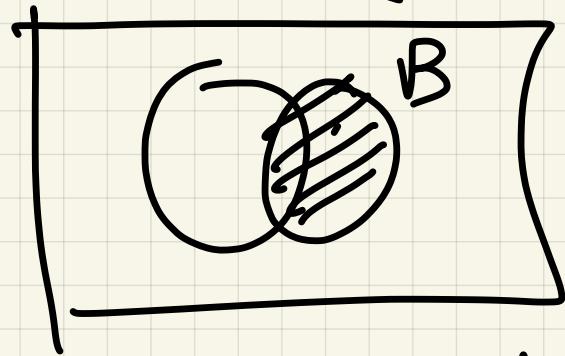
complement off A

 \bar{A} (or A^c)

Ex2 Is $(\underline{A \cup B}) - A = B$??



$$(A \cup B) - A$$



Answer: not in general :

Ex. $A = \{1\}$

$$B = \{1\}$$

$$(A \cap B \neq \emptyset)$$

but sometimes true :

$$A = \{1\} \quad B = \{2\}$$

Products :

Ex $A = \{a, b, c\} \quad B = \{1, 2\}$

$$A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)\}$$

$$|A \times B| = |A| \times |B|$$

$$B \times A$$

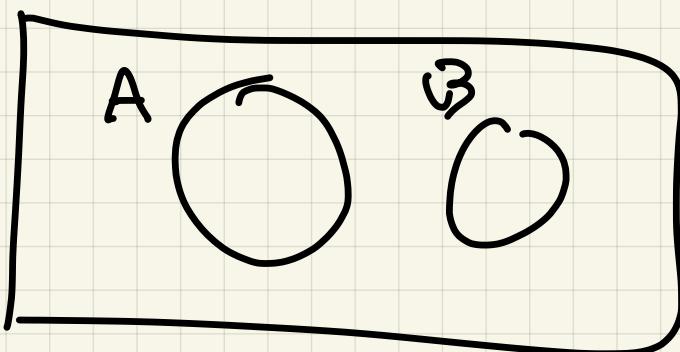
$$(3.1-3.4, 3.6)$$

3.7

Partitions

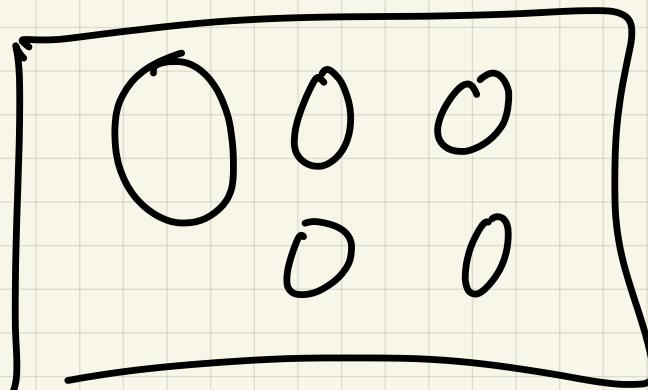
Two sets A & B are disjoint

If $A \cap B = \emptyset$



A_1, \dots, A_n are pairwise disjoint

If $A_i \cap A_j = \emptyset$ for all $i \neq j$



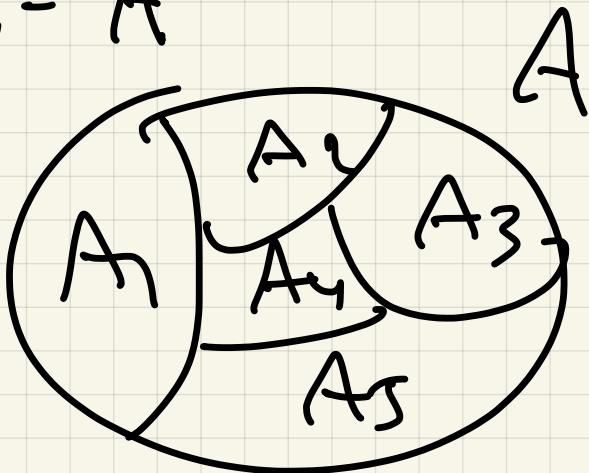
set
↓

Definition: A partition of A is a collection of subsets A_i so that \wedge_{nonempty}

① A_i are pairwise disjoint

② $\cup A_i = A$

Idea:



Ex 3 $A = \{1, 2, 3\}$

Find partitions.

$$\textcircled{1} \quad A_1 = \{1, 2, 3\}$$

$$\textcircled{2} \quad A_1 = \{1, 2\}, \quad A_3 = \{3\}$$

$$\textcircled{3} \quad A_1 = \{1, 3\}, \quad A_3 = \{2\}$$

$$\textcircled{4} \quad A_1 = \{2, 3\} \quad A_3 = \{1\}$$

$$\textcircled{5} \quad A_1 = \{1\}, \quad A_2 = \{2\}, \quad A_3 = \{3\},$$

$$\underline{\text{Ex 4}} \quad A = \mathbb{Z}$$

$$A_k = \{z \in A : z \equiv k \pmod{5}\}$$

$$A_0 = \{0, \pm 5, \pm 10, \dots\}$$

$$A_1 = \{1, 6, 11, 16, 21, 26, \dots\}$$

-9, -9, -14, -19, -24, -29, -34, -39, -44, -49, -54, -59, -64, -69, -74, -79, -84, -89, -94, -99, -104, -109, -114, -119, -124, -129, -134, -139, -144, -149, -154, -159, -164, -169, -174, -179, -184, -189, -194, -199, -204, -209, -214, -219, -224, -229, -234, -239, -244, -249, -254, -259, -264, -269, -274, -279, -284, -289, -294, -299, -304, -309, -314, -319, -324, -329, -334, -339, -344, -349, -354, -359, -364, -369, -374, -379, -384, -389, -394, -399, -404, -409, -414, -419, -424, -429, -434, -439, -444, -449, -454, -459, -464, -469, -474, -479, -484, -489, -494, -499, -504, -509, -514, -519, -524, -529, -534, -539, -544, -549, -554, -559, -564, -569, -574, -579, -584, -589, -594, -599, -604, -609, -614, -619, -624, -629, -634, -639, -644, -649, -654, -659, -664, -669, -674, -679, -684, -689, -694, -699, -704, -709, -714, -719, -724, -729, -734, -739, -744, -749, -754, -759, -764, -769, -774, -779, -784, -789, -794, -799, -804, -809, -814, -819, -824, -829, -834, -839, -844, -849, -854, -859, -864, -869, -874, -879, -884, -889, -894, -899, -904, -909, -914, -919, -924, -929, -934, -939, -944, -949, -954, -959, -964, -969, -974, -979, -984, -989, -994, -999\}

$$A_2 = \{2, 7, \dots\}$$

$$A_3 = \{3, 8, -2, -7, \dots\}$$

$$A_4 = \{4, 9, 14, -1, -6, \dots\}$$

Ex 5 $A = \text{deck of cards}$

$$= \{\spadesuit, \clubsuit, \heartsuit, \diamondsuit\} \times \{A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K\}$$

A_{g}
 A_{d}
 A_{c}
 A_{h}
 Spaces decorated clubs hearts

Ex6 $A = \mathbb{R}$

$$A_1 = \mathbb{R}^+ = \{r \in \mathbb{R}, r > 0\}$$

$$A_2 = \mathbb{R}^- = \{r \in \mathbb{R}, r < 0\}$$

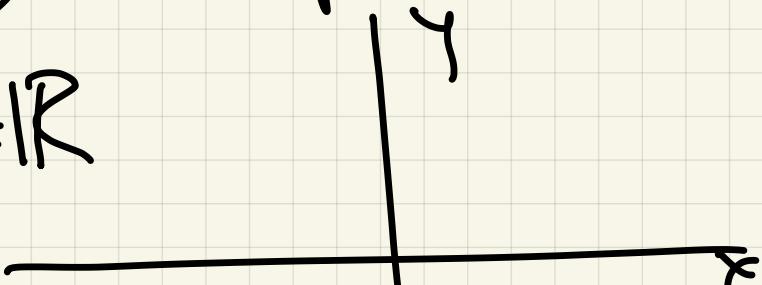
Not a partition

$$A_1 \cup A_2 \neq \mathbb{R}$$

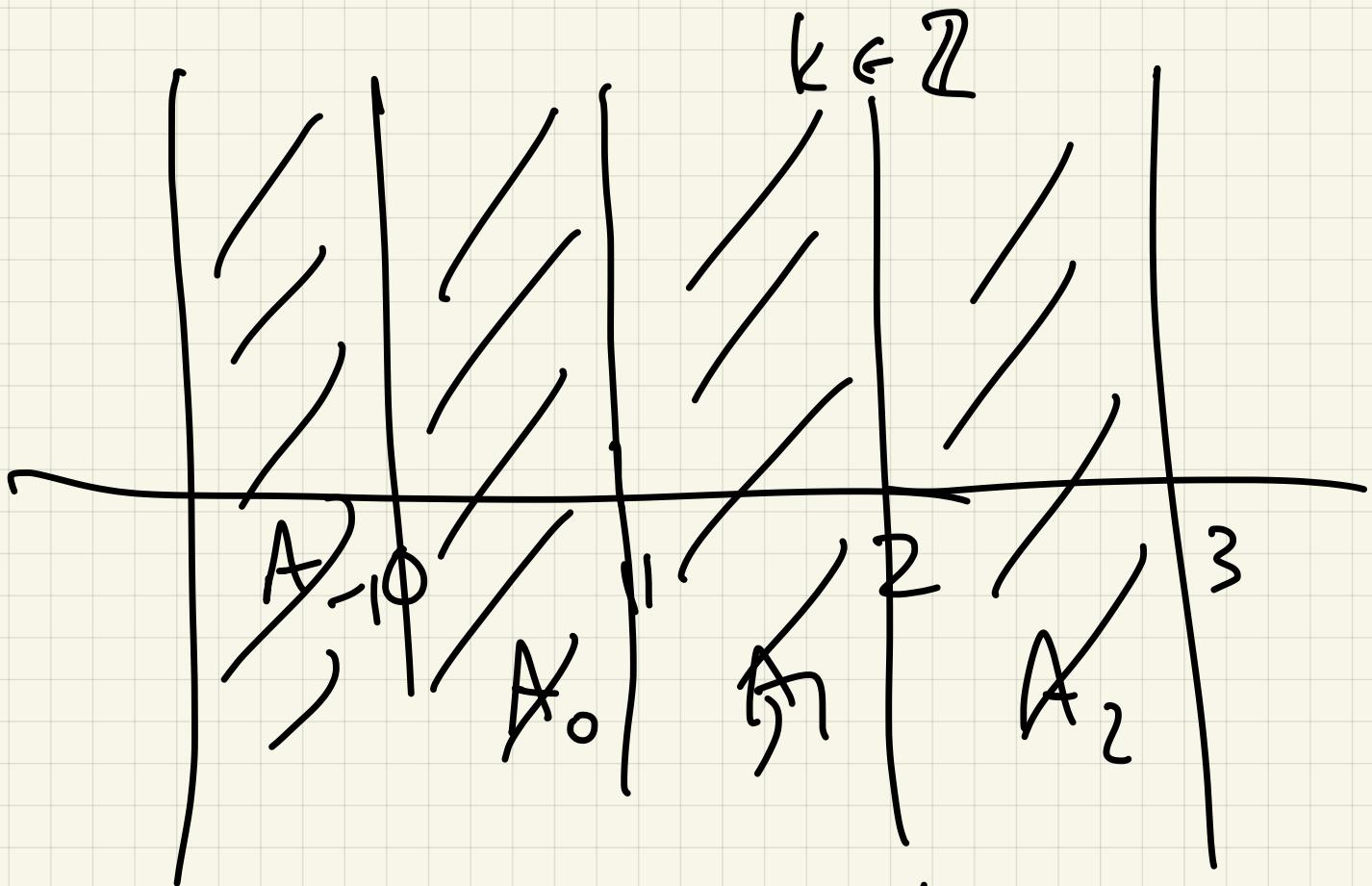
but

add $A_3 = \{0\}$
 A_1, A_2 is partition

Ex7 $A = \mathbb{R} \times \mathbb{R}$



$$A_k = \{(x, y) : k < x < k+1\}$$



Not a partition, but

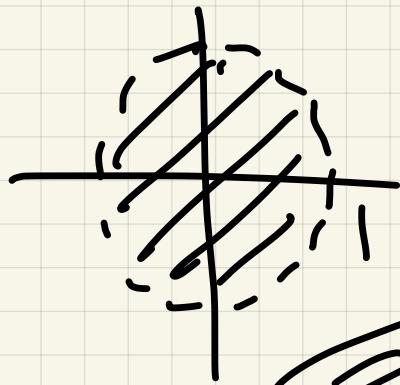
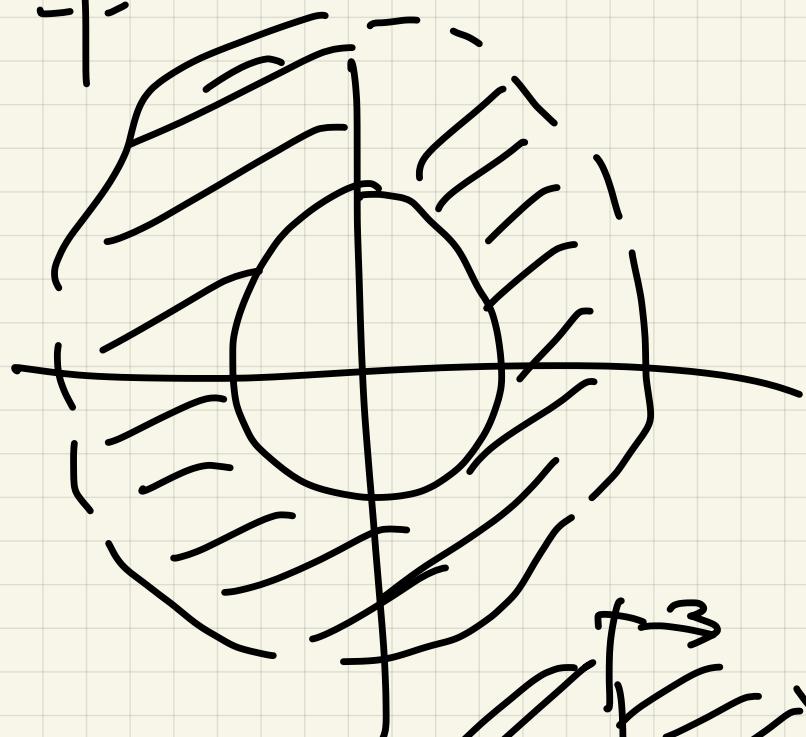
$$B_k = \{(x,y) : k \leq x < k+1\}$$

is a partition

Ex 8 $A = \mathbb{R} \times \mathbb{R}$

$$A_k = \{(x,y) : k \leq \sqrt{x^2+y^2} < k+1\}$$

$$k \in \mathbb{N}$$

A_0  $A_1 =$  $A_3 =$ 