1/13/ Discrete2:

Math 30123

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off hrs: T-F 2-5

D2L = TCU Online

Weekly Planner:

quiz/HW/ solutions/ Exams

Text: Zybooks

① learn.zybooks.com
create account / sign in

(2) TCUMATH30123NollotSpring2025

(3) Subscribe

Calculator TI-89 or none

Grades:

5 AR = Active reading

10 HW

10 Quiz

15 Exam 1 Feb 19

15 2 March 19

15 3 Apr 16

30 Final

Monday May 5, 5-730

Discrete):

Proofs
- direct
- contrapositive
- contradiction
- counterexample
- Induction

- Boolean logic/truth tables
- Sets & functions
- Counting
- Number theory
  - gcd
  - Euclidean algorithm
  - modular arithmetic

<u>Demographics</u>: 14 CS, 6 Math

<u>When discrete I</u>:

<u>2 ly weeks</u> 0

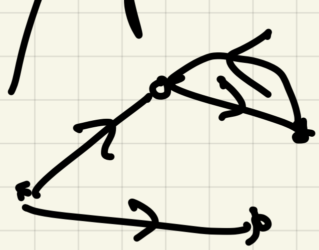
<u>Discrete 2</u>:

<u>Counting</u> (more intense)

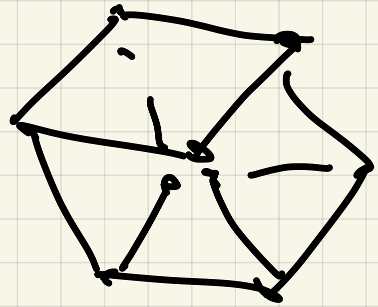
Relations

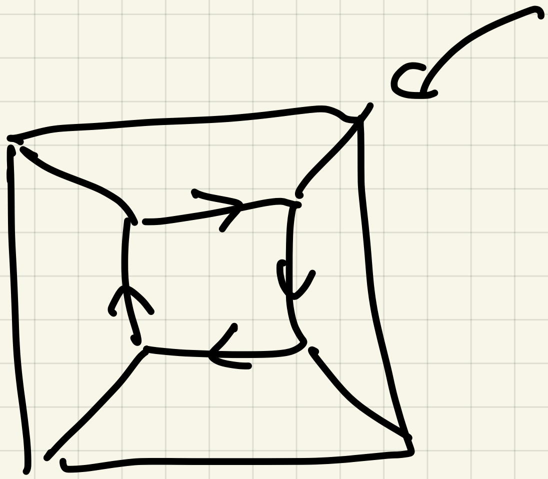
Equivalence relations / posets

Directed graphs

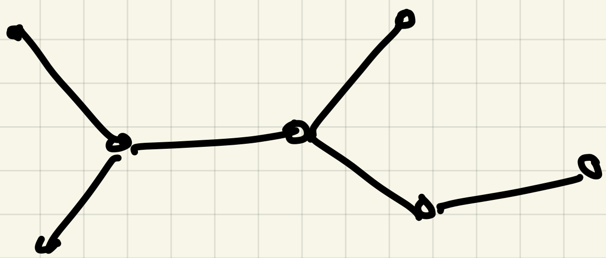


Graphs





Trees



Recurrence relations

$\{a_n\}$ sequence

$$a_0 = 1, \quad a_{n+1} = 2a_n - 1$$

(a)

n	0	1	2	3	4	
a_n	1	1	1	1	1	–

(b) $a_0 = 2$, $a_{n+1} = 2a_n - 1$

n	0	1	2	3	4
a_n	2	3	5	9	17

$a_n = 2^n + 1$

Review of sets

Ex) common sets in Math

(a) $\mathbb{Z}$, $\mathbb{R}$, $\mathbb{Q}$,

$\mathbb{N} = \{0, 1, 2, 3, 4, 5, \ldots\} \subset \mathbb{Z}$

Note

$\mathbb{Z}^+ = \{z \in \mathbb{Z} : z \text{ positive}\}$
$= \{z \in \mathbb{Z} : z > 0\}$

Note: A, B sets, $A \subset B$ means

$x \in A \Rightarrow x \in B$, but $A \neq B$

Allow =, $A \subseteq B$

(b) $S = \{1, 2, 3, 4, 5\} = \{1, 1, 2, 5, 4, 3\}$

$= \{x \in \mathbb{N} : 1 \le x \le 5\}$

$= \{x \in \mathbb{N} : |3 - x| \le 2\}$

$|S| = 5$

(c) $T = \{1, 2, -5, \mathbb{E}, \{5, 6\}, \mathbb{R}\}$

$|T| = 6$

(d) $A = \{a, b\}$

$P(A) = \{$ all subsets of $A\}$

power set

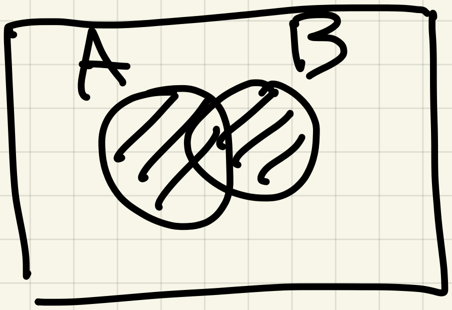
$= \{\{a\}, \{b\}, \{a, b\}, \phi\}$

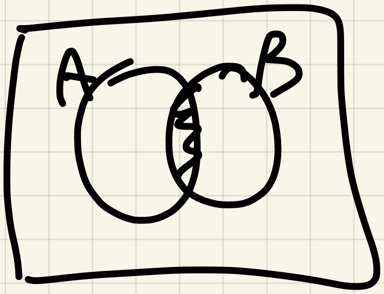
$|P(A)| = 2^{|A|}$

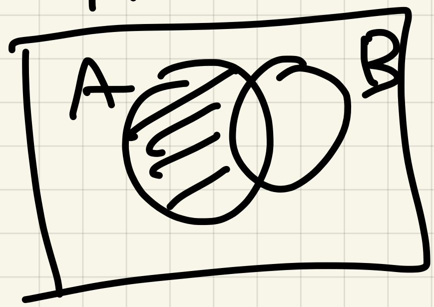
Set operations:

Given sets A, B can form new sets:

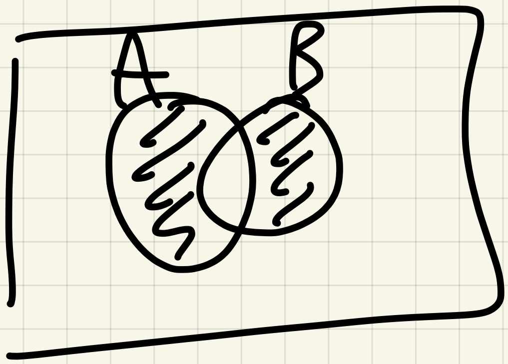
$A \cup B$ $\qquad$ $A \cap B$ $\qquad$ $A - B$







Symmetric difference:

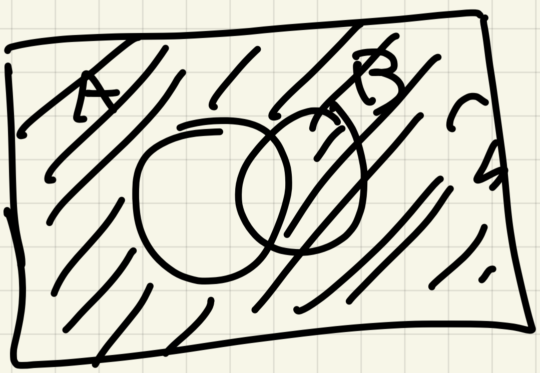


Scheinerman

$A \,\Delta\, B$

For us:

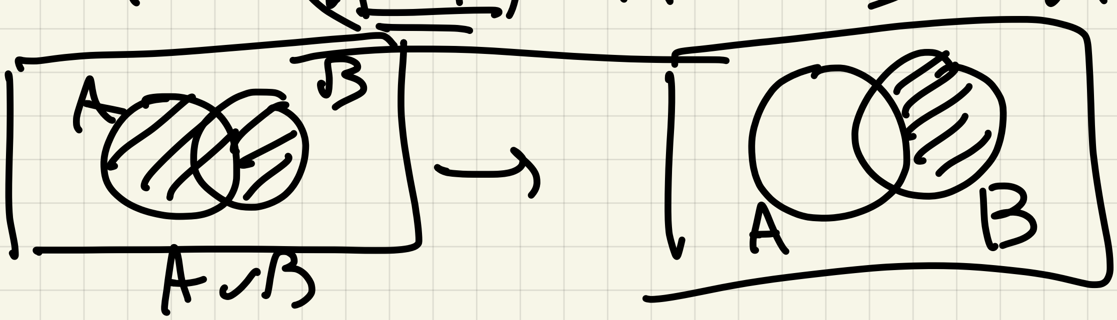
$$(A-B)\cup(B-A) = A \oplus B$$



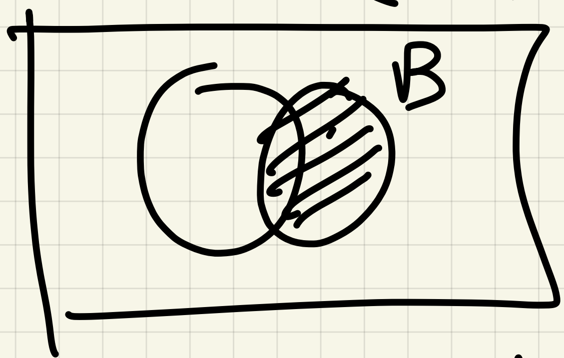
complement of A

$\bar{A}$ (or A^c)

<u>Ex2</u> Is $(\underline{A\cup B}) - A = B$??



$(A \cup B) - A$



<u>Answer:</u> not in general:

<u>Eg.</u>

$A = \{1\}$

$B = \{1\}$

$(A \cap B \neq \phi)$

but sometimes true:

$A = \{1\}$ $\quad B = \{2\}$

<u>Products:</u>

<u>Ex1</u> $\quad A = \{a, b, c\}$ $\quad B = \{1, 2\}$

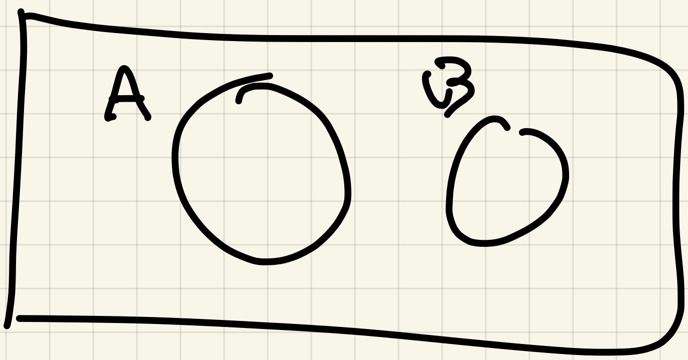
$$A \times B = \{(a,1), (a,2) \\ (b,1)\ (b,2) \\ (c,1)\ (c,2)\}$$

$|A \times B| = |A| \times |B|$

$B \times A$ (3.1–3.4, 3.6)

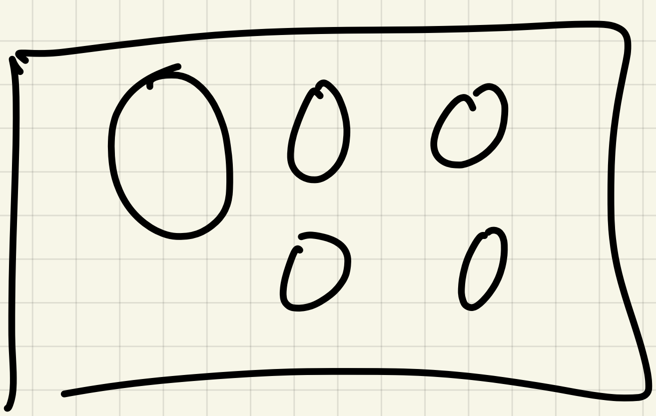
3.7 Partitions

Two sets A & B are **disjoint** if $A \cap B = \phi$



$A_1, \ldots, A_n$ are **pairwise disjoint**

if $A_i \cap A_j = \phi$ for all $i \neq j$



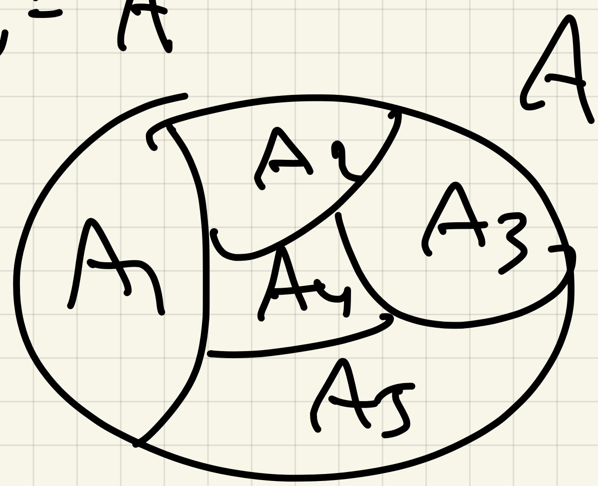
set

Definition: A **partition of A** is a collection of nonempty subsets A_i so that

① A_i are pairwise disjoint

② $\bigcup A_i = A$

Idea:



Ex 3 $A = \{1, 2, 3\}$

Find partitions:

① $A_1 = \{1, 2, 3\}$

② $A_1 = \{1, 2\}$, $A_3 = \{3\}$

③ $A_1 = \{1, 3\}$, $A_3 = \{2\}$

④ $A_1 = \{2, 3\}$ $A_3 = \{1\}$

(5) $A_1 = \{1\}$, $A_2 = \{2\}$, $A_3 = \{3\}$,

<u>Ex4</u> $A = \mathbb{Z}$

$$A_k = \{z \in A : z \equiv k \bmod 5\}$$

$$A_0 = \{0, \pm 5, \pm 10 \ldots\}$$

$$A_1 = \{1, 6, 11, 16, 21, 26 \ldots, -4, -9, -14 \ldots\}$$

$$A_2 = \{2, 7, \ldots\}$$

$$A_3 = \{3, 8, -2, -7 \ldots\}$$

$$A_4 = \{4, 9, 14, -1, -6, \ldots\}$$

<u>Ex5</u> A = deck of cards

$$= \{\spadesuit, \diamondsuit, \clubsuit, \heartsuit\} \times \{A, 2, 3 \ldots 10, J, Q, K\}$$

$A_\spadesuit$ $A_\diamondsuit$ $A_\clubsuit$ $A_\heartsuit$

spades diamond clubs hearts

<u>Ex6</u> $A = \mathbb{R}$

$A_1 = \mathbb{R}^+ = \{r \in \mathbb{R}, r > 0\}$

$A_2 = \mathbb{R}^- = \{r \in \mathbb{R}, r < 0\}$

<u>Not</u> a partition

$A_1 \cup A_2 \neq \mathbb{R}$

but

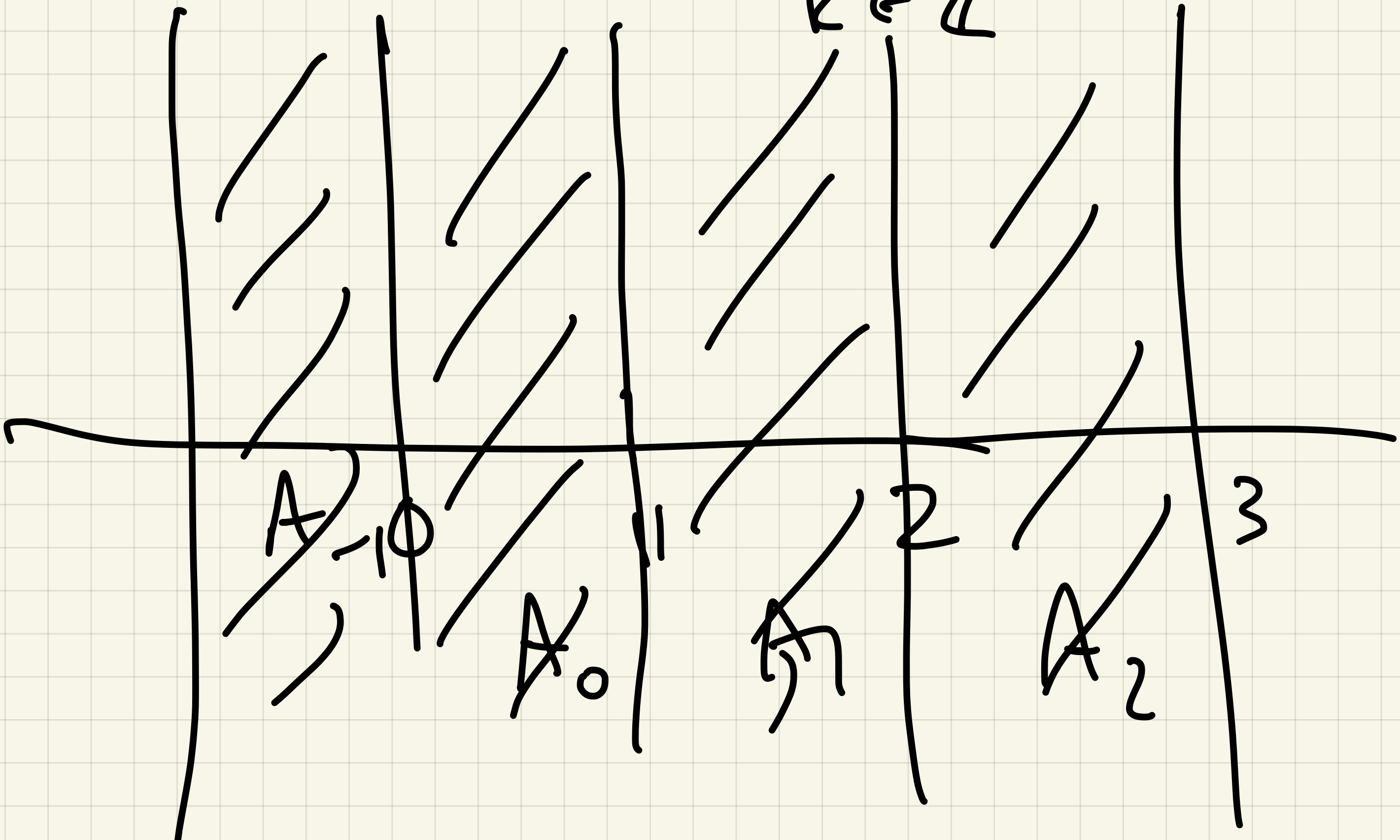
add $A_3 = \{0\}$

$A_1 A_2$ is partition

<u>Ex 7</u> $A = \mathbb{R} \times \mathbb{R}$

$A_k = \{(x,y) : k < x < k+1\}$

$k \in \mathbb{Z}$



Not a partition, but

$B_k = \{(x,y) : k \le x < k+1\}$

is a partition

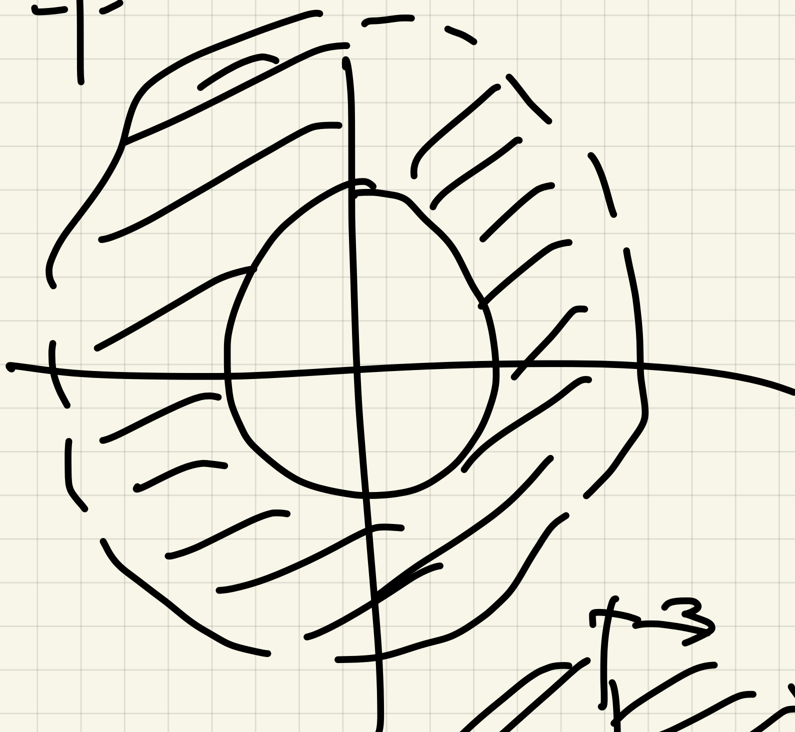
<u>Ex 8</u> $A = \mathbb{R} \times \mathbb{R}$

$A_k = \{(x,y) : k \le \sqrt{x^2+y^2} < k+1\}$

$k \in \mathbb{N}$

A_0

$A_1 =$



$A_3 =$