2/24/ Disc 2

Exam 1 Wednesday
Review sheet
HW solutions visible

Quiz 5

14 cookies 4 types

CC | PB | R | S

1. $\binom{14+4-1}{4-1} = \binom{17}{3}$

2. 10 cookies 4 types

$\binom{10+3}{3} = \binom{13}{3}$

3. 10 cook 3 types

$\binom{12}{2}$

4. $\boxed{\geq 4 \text{ CC}}$ $\boxed{\geq 4 \text{ PB}}$

CC d PB

$$\binom{13}{3} + \binom{13}{3} - \binom{9}{3}$$

$$|CC \cup PB| = \underline{\underline{|CC|}} + \underline{\underline{|PB|}} - \underline{\underline{|CC \cap PB|}}$$

6 cookies
4 types $\rightarrow$ $\binom{6+3}{3}$

5. $|CC \cup PB \cup R| =$
$\geq 4 \quad \geq 4 \quad \geq 4$

$$\underbrace{|CC| + |PB| + |R|}_{3 \cdot \binom{13}{3}} - 3\binom{9}{3} + \binom{5}{3}$$

$|CC \cap PB| - |CC \cap R| - |PB \cap R|$

$|CC \cap PB \cap R|$ 2 cook
4 typ

<u>10.9.2</u> 15 cookies
6 types

(d) $\binom{15+5}{5} = \binom{20}{5}$ ←

(e) # with at most 2 sugar

2 approaches:

(1) Complement:

$\# \geq 3$ sugar $\binom{17}{5}$

Ans $\binom{20}{5} - \binom{17}{5}$

(2) Enumerate:

0 sugar # $\binom{19}{4}$

1 sugar $\binom{18}{4}$

2 sugar $\binom{17}{4}$

?? Why?

$$\binom{19}{4} + \binom{18}{4} + \binom{17}{4}$$

d $\binom{20}{5} - \binom{19}{5} + \binom{19}{5} - \binom{18}{5} + \binom{18}{5} - \binom{17}{5}$

$$\underbrace{\binom{20}{5} - \binom{19}{5}}_{=\binom{19}{4}} + \underbrace{\binom{19}{5} - \binom{18}{5}}_{=\binom{18}{4}} + \underbrace{\binom{18}{5} - \binom{17}{5}}_{=\binom{17}{4}}$$

Pascal's Δ

<u>10.10.3</u> 5 same books to 20 kids

at most 1 to each kid,

(a) $\Rightarrow \binom{20}{5}$

(b) books different:

$$\binom{20}{5} \cdot 5! = P(20, 5)$$

(c) Books same, no restrictions

"Putting 5 pigeons into 20 boxes"

$$\binom{5+20-1}{20-1} = \binom{24}{19}$$

10.10.4 $\quad$ 100 students

(a) $\quad$ 10 prizes

A, B

(a) 2 ways: (1) $\binom{100}{10} - \binom{98}{10}$

not given to A or B

Inc

(2) $\binom{99}{9} + \binom{99}{9} - \binom{98}{8}$

A gets $\quad$ B gets

(b)

$P(100,10) - P(98,10)$

10.11.1 $\quad \{a,b,c\}$ $\quad 3^9$

length 9-strings

(a) # 1st or last char is a

$A_1 = \underline{a}\ \underbrace{\_ \quad \quad \_}_{8}$ $\quad 3^8$

$A_2 : \underbrace{\_\ \_ \quad \quad \_}\ \underline{a}$ $\quad 3^8$

$A_1 \cap A_2$ $\boxed{\underline{a}\ \underbrace{\_\ \_\ \_\ \_}_{7}\ a \quad 3^7}$

$3^8 + 3^8 - 3^7$

(b) 8 consec a

$\underline{a}\ \_ \quad \underline{a}\ \underline{?}$ $\quad 3$

$\underline{?}\ \underline{a} \quad \underline{a}$ $\quad 3$

$\boxed{a\ a \quad a \quad 1}$

$3+3-1 = 5$

(d) A

1st char = last char OR

1st a or last a

B C

$|B| = |C| = 3^8$ (a)

$|A| = 3^8 = 3 \cdot 3^7$

$\_ \underbrace{\_ \_ \; \_ \;\; \_}_{7} \; \_$

same

$A \cap B = \underline{a} \underbrace{\_ \, . \, . \; \_}_{7} \underline{a}$ 3^7

$B \cap C = \underline{a} \; . \; \_ \quad \underline{a}$ 3^7

$A \cap C = \underline{a} \; . \; . \quad \underline{a}$ 3^7

$A \cap B \cap C = \underline{a} \underbrace{. \; . \; .}_{7} \; a$ 3^7

$\underset{A}{3^8} + \underset{B}{3^8} + \underset{C}{3^8} - 3^7 - 3^7 - 3^7 + 3^7$

(b) Exactly 2a or 2b or 2c

$$\boxed{\begin{aligned}&\binom{9}{2}\cdot 2^7 + \binom{9}{2}\cdot 2^7 + \binom{9}{2}\cdot 2^7 \\ &- \binom{9}{2}\binom{7}{2}\cdot 1 - \binom{9}{2}\binom{7}{2} - \binom{9}{2}\binom{7}{2}\end{aligned}}$$

a b

$|A\cap B|$ $|A\cap C|$

$+ \quad 0$

$A\cap B\cap C$

<u>10.11.5 (e)</u> $\underline{F}\ \underline{M}\ \underline{D_1}\ \underline{D_2}\ \underline{S_1}\ \underline{S_2}$

orderings with M next to a daughter.

<u>PIE</u> : $\underbrace{MD_1}/\underbrace{D_1M}$ $\quad 2$

D_1 $\quad 2\cdot 5!$

MD_2/D_2M

D_2 $\quad 2\cdot 5!$

$D_1\cap D_2$: $\underbrace{D_1MD_2}/\underbrace{D_2MD_1}$ $\quad 2$

$2 \cdot 4!$

$\to 2 \cdot 5! + 2 \cdot 5! - 2 \cdot 4!$

Also **Complement**

$6! - \# \{ M \text{ not next to } D_1 \text{ or } D_2 \}$

ⓐ $\underline{M} \; \underline{\ } \; \underline{\ } \quad \underline{\ }$ $\leftarrow 3$ $\leftarrow 4!$ $\;$ $D_1 D_2$

ⓑ $\underset{3}{\underline{\ }} \; M \; \underset{2}{\underline{\ }} \; \underline{\ } \; \underline{\ } \; \underline{\ }$

ⓒ $\underline{\ } \; \underline{\ } \quad \underline{\ } \; M$

$$3 \cdot 4! + 3 \cdot 2 \cdot 3! + 3 \cdot 4!$$

Last time: Relation on set between A & B

6.1 $\quad R \subset A \times B$

6.2 $\quad A = B \qquad R \subset A \times A$

(A domain)

Digraph : (Matrix square)

Reflexive $aRa \quad \forall a \in A$

Antireflexive $\neg aRa \quad \forall a \in A$

symmetric $aRb \Rightarrow bRa \quad \forall a,b$

Anti-symmetric $\boxed{aRb \wedge bRa \Rightarrow a=b}$

transitive $aRb \wedge bRc \Rightarrow aRc$
$\forall a,b,c$ in A.

matrix

Ex) $A = \mathbb{N} = \{0,1,2\ldots\}$

aRb if $a < b$

reflexive aRa

NO $a < a$ false

antireflexive.

$\neg aRa$ true $\forall a \in A$

YES

Symmetry: $a<b \not\Rightarrow b<a$

NO

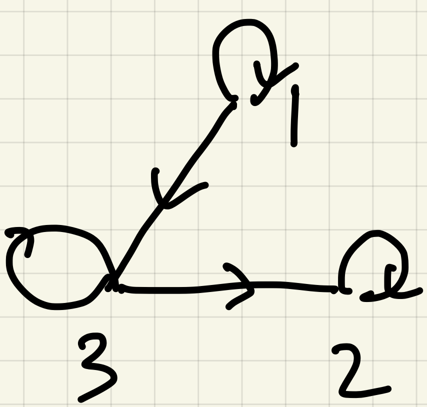
antisymmetry: $\underbrace{aRb \wedge bRa}_{\text{never happens}} \Rightarrow a=b$?

vacuous truth

transitive $aRb \wedge bRc \Rightarrow aRc$

yes

Ex2 $A = \{1,2,3\}$



$$M = \begin{array}{c c} 1 & \multirow{3}{*}{} \\ \end{array}\begin{array}{c}1\\2\\3\\ \\ \end{array}\begin{array}{c}\begin{pmatrix}1&0&1\\0&1&0\\0&1&1\end{pmatrix}\\ \begin{matrix}1&2&3\end{matrix}\end{array}$$

reflexive ✓

antireflexive ✗

symmetric ✗

antisymm ✓

transitive ?

NO : $1R3 \wedge 3R2$
but $\neg 1R2$

Ex3 Is there a relation R on $A = \{1, 2, 3\}$ such that :

(a) R reflexive and antireflexive?

$\forall a \; aRa$ $\qquad \forall a \; \neg aRa$

NO

(b) R is symmetric and antisymmetric ?

$\forall x, y$

Symm $xRy \Rightarrow yRx$

$x \neq y$

antisymm $\neg xRy \vee \neg yRx$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(c) R is not symmetric
and R is not anti-symmetric

$$\begin{pmatrix} \cdot & \square & \square \\ \square & \cdot & \square \\ \square & \square & \cdot \end{pmatrix}$$

not antisymm

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

not symm