2/19 Disc 2

- Exam 1 → Wednesday
- Review sheet
- Unhide HW solutions

Last Time PHP

Relations between sets

$R \subset A \times B$

$(a,b) \in R$ write aRb

"a is related to b"

Ex 0: $f: A \to B$ function

$R = \{(a,b) : f(a) = b\}$

$A = B = \mathbb{R}$ Calc I settings

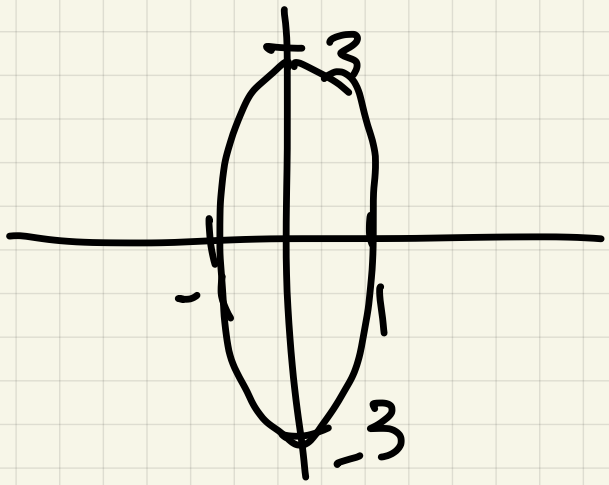
R = Graph of $y = f(x)$

How to represent a relation?

(A) Words / Equations

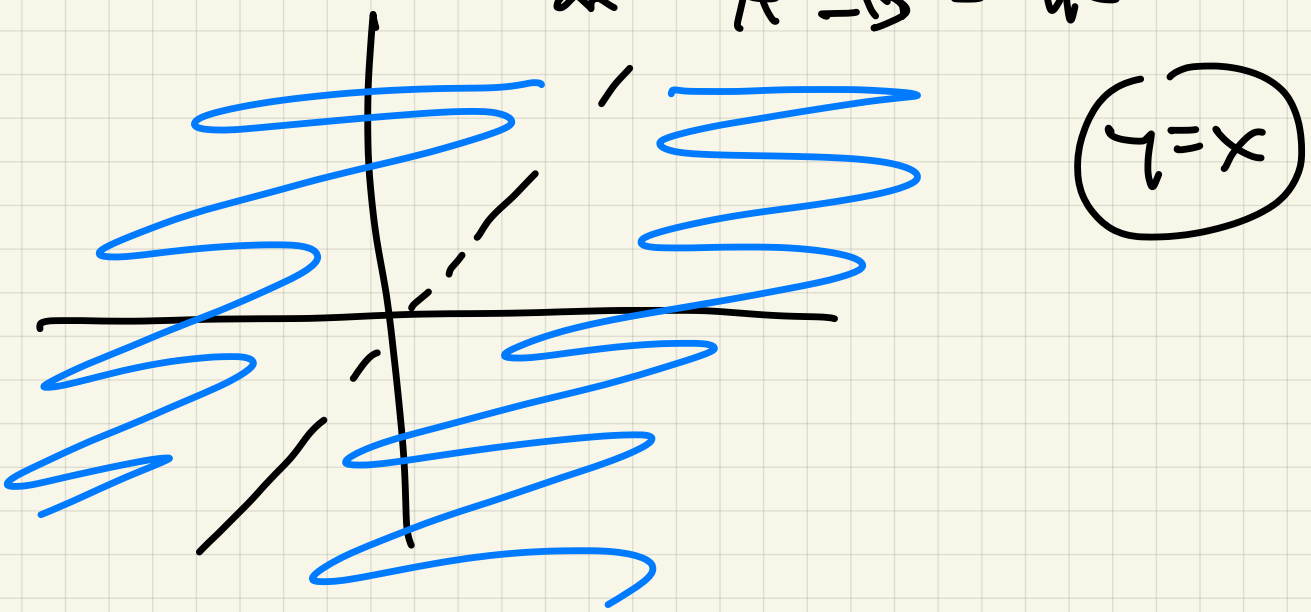
Ex! $A = B = \mathbb{R}$,

(a) xRy if $x^2 + \frac{y^2}{9} = 1$



(b) xRy if $x \neq y$

on $A = B = \mathbb{R}$

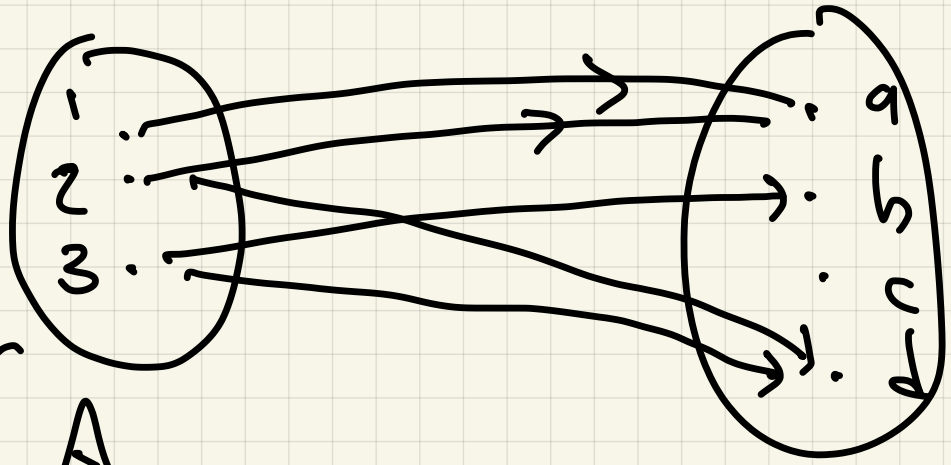


Ex2 $A = B =$ all people

xRy if y is taller than x

(B) Arrow diagram:

Ex3



$A = \{1, 2, 3\}$

$B = \{a, b, c, d\}$

$1Ra$ $2Ra$

(C) Can list $R \subseteq A \times B$

Ex3 $R = \{(1, a), (2, a), (2, d)\}$

$(3,b)$ $(3,d)\}$

① Matrix representation

Rectangular array of numbers

$$\begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$
$$a \quad b \quad c \quad d$$

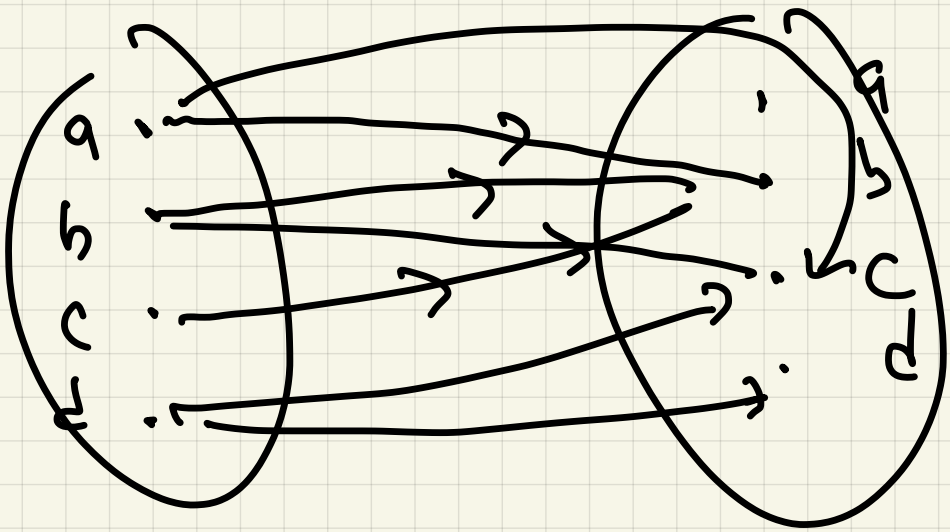
Put 1 in position (a,b) if aRb

otherwise 0

Ex3 $A = \{a,b,c,d\} = B$

$R = \{(a,b),(b,b),(b,c),(c,b),(a,c)$
$(d,c),(d,d)\}$

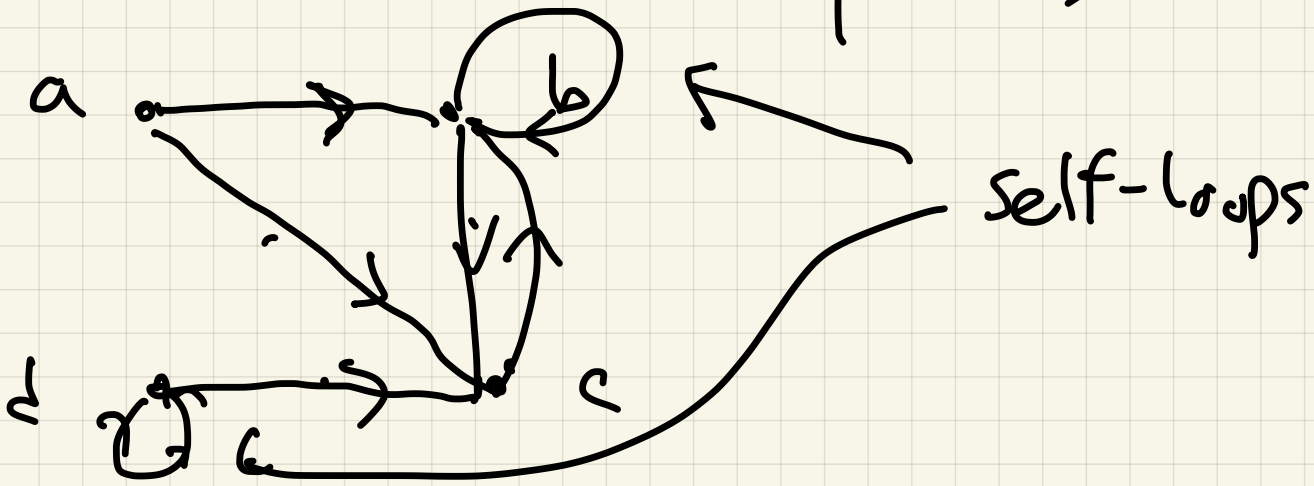
Arrow Diagram



matrix

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \\ \begin{array}{cccc} a & b & c & d \end{array}$$

mess, another possibility



6.2 Ex 4 $A = B = \{1,2,3\}$

xRy if $x \neq y$

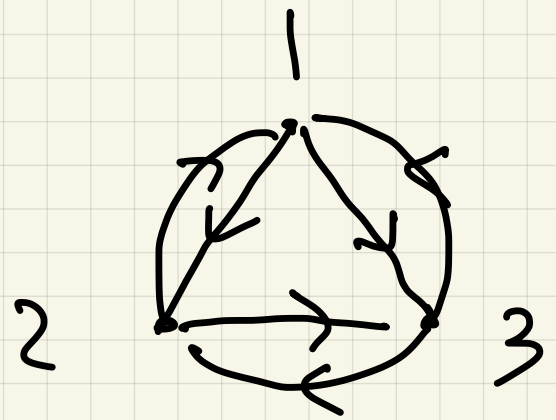
List $\{(1,2),(1,3),(2,1),(2,3),(3,1),(3,2)\}$

matrix

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$\quad\;\; 1 \;\; 2 \;\; 3$

digraph: $A = B$: use elements of $A = B$ as vertices, include directed from x to $y \iff xRy$



Properties of relations

$R \subseteq A \times A$

A = domain of relation

Defn: R is <u>reflexive</u> if

$aRa \quad \forall a \in A$

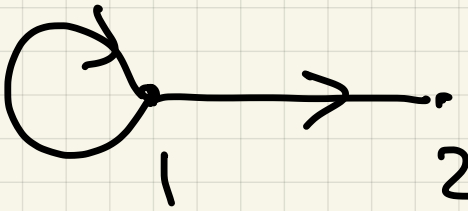
Ex NOT reflexive

Defn: R is <u>antireflexive</u> if

$\neg aRa \quad \forall a \in A$

Ex IS ANTIREFLEXIVE

Ex



reflexive? No b/c $\neg 2R2$

antireflexive? No
b/c |R|

Def R is **symmetric** if

$\forall a, b \in A$, $aRb \Rightarrow bRa$

Ex 4 is symmetric

Ex 5 is not symm:
|R2 but not R1

Equivalently $\forall a, b$, $aRb \Leftrightarrow bRa$

Equivalently $\forall a, b$, $aRb \wedge bRa$ both or neither

Rmk: R symmetric relation
matrix for R
M is symmetric $(M = M^T)$

Defn R is antisymmetric if

$\forall x \neq y, \neg xRy \vee \neg yRx$

$\equiv$

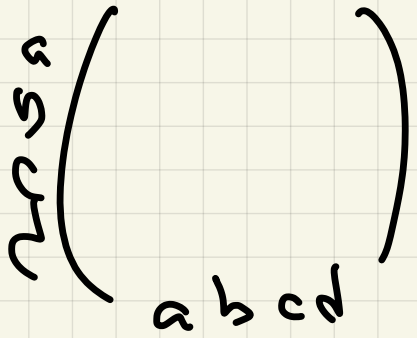
DeMorgan's $\neg(xRy \wedge yRx)$

Ex4 is not antisymmetric

Ex5 is antisymmetric

$\neg 2R1$

Rmk: in terms of matrix representation:



reflexive

$\Updownarrow$

1's on diagonal

antireflexive

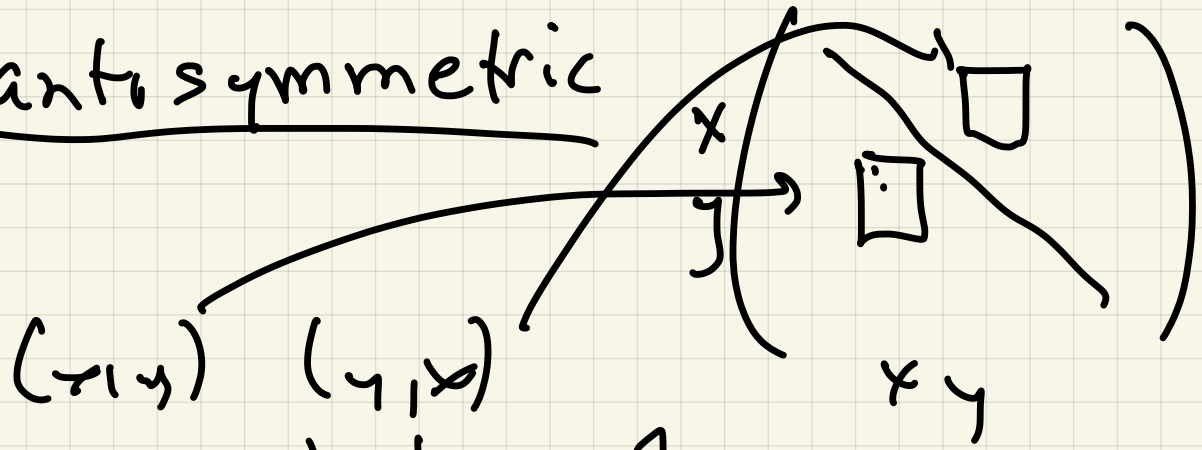
$\Updownarrow$

all 0s on diagonal

Symmetric

$A = A^T \iff$ reflection over the diagonal leaves matrix the same

antisymmetric



Not both 1s

$$\begin{array}{c} 1 \\ 2 \end{array}\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

1 2

Def: R is transitive if

$\forall a,b,c \in A,\ aRb \wedge bRc \Rightarrow aRc$

Ex 4 NOT transitive:

$1R2 \wedge 2R1$ but $\neg(1R1)$ fails

Ex 5 YES

Ex 4 $A = \mathbb{Z}_+ = \{z \in \mathbb{Z} : z > 0\}$

aRb if $a|b$

reflexive? $\forall a \in A$, aRa?

yes $a|a$: $a \cdot 1 = a$ ✓

antireflexive?

NO

Symmetric?, NO

$2|10$ but $10 \nmid 2$

2 R 10 but not R2

antisymmetric

yes

Know

$a|b \wedge b|a \Rightarrow a=b$

contrapos

$\neg(a|b \wedge b|a) \Leftrightarrow a \neq b$

$\neg a|b \vee \neg b|a$

$\neg aRb \vee \neg bRa$

Transitive:

$a|b \wedge b|c \Rightarrow a|c$??

(yes.)

$b = ar$ $c = bs$

$c = b \cdot s = ars \Rightarrow a|c$ ✓