2/17 | Disc 2

Exam 1 $\rightarrow$ Wednesday 2/26

Quiz 4

24 crayons to 6 kids

1. $\underset{\text{crayon 1}}{\underline{6}}\ \underset{2}{\underline{6}}\ \cdots\ \_\ \ \underset{24}{\underline{6}} \Rightarrow 6^{24}$

(Not 24^6 : this is # ways to assign 1 crayon to each kid)

2. 5^{24} (Harry avoided)

3. $6^{24} - 5^{24}$

4. $\underset{r}{\underline{5}}\ \underset{b}{\underline{5}}\ \underset{y}{\underline{5}}\ \underbrace{\underline{6}\ \underline{6}\ \cdots\ \underline{6}}_{\text{other crayons}} \Rightarrow 5^3 \cdot 6^{21}$

5. $\dfrac{24!}{(4!)^6} = \binom{24}{4}\binom{20}{4}\binom{16}{4}\binom{12}{4}\binom{8}{4}\binom{4}{4}$

6. $\binom{24}{3}\binom{20}{4}\cdots\binom{8}{4}\binom{4}{4} =$

$\frac{24!}{3!(4!)^5}$ # ways Jim gets red crayon,

so $\frac{24!}{(4!)^6} - \frac{23!}{3!(4!)^5}$.

Last time:

PHP: If h boxes contain p pigeons and $p>h$, then some box contains at least 2 pigeons.

But we can expect better:

Ex) If 50 pigeons placed in 4 boxes, then some box has at least 13!

Why? Suppose not. I.e. each box has < 13, or ≤ 12.

12	12
12	12

Then total # pigeons is at most $48 < 50$

$\Rightarrow\Leftarrow$

This argument proves

<u>General PHP</u>: If n boxes contain p pigeons, then some box contains at least $\lceil \frac{p}{n} \rceil$.

<u>Ex2</u> A drawer is filled with socks of colors Blue, Red, Green, Yellow.

(a) How many socks must be taken to ensure at least a pair of same color?

<u>Answer</u> $p = 5 > 4 = n =$ sock colors

$\boxed{B \mid R \mid G \mid Y}$

<u>Note</u> $\left\lceil \frac{4}{4} \right\rceil = 1$ not enough,

$\left\lceil \frac{5}{4} \right\rceil = \lceil 1.25 \rceil = 2$ enough,

(b) How many to ensure 3 of one color?

<u>Answer</u> $p = 9$, $\left\lceil \frac{9}{4} \right\rceil = 3$, but

$p = 8$ not enough as $\left\lceil \frac{8}{4} \right\rceil = 2$

(could have 2 of each color for total of 8)

(c) How many to ensure 15 of a color?

$\left\lceil \frac{57}{4} \right\rceil = 15$, but $\left\lceil \frac{56}{4} \right\rceil = 14$, so

need $p \geq 57$

<u>Note</u> Instinctive answer $4 \cdot 15 = 60$ is wrong.

<u>In general</u>: to assure that one of h boxes has at least d, need $p > h(d-1)$, i.e.

$$p \geq h(d-1)+1$$

<u>Ex 3</u> How many SSNs needed to ensure that at least two have same last 4 digits?

$$\underline{10}\,\underline{10}\,\underline{10}\,\underline{10} \to 10^4 = \overset{10001}{\text{boxes}} = h$$

$p = \#$ people (SSNs)

Need $p \geq 10001$

(a) $$\left\lceil \frac{10001}{10000} \right\rceil = \lceil 1.0001 \rceil = 2,$$ $p = 10000$ not enough.

(b) To ensure 3 have same last 4 digits? 20001

(c) " " " " 250 have same?

2490001

Ex 4 Among any 6 integers from 1 to 10, there are 2 subsets with same sum.

subsets from set of size 6 is

$$2^6 = 64$$

possible sums?

$$1 \le \text{sum} \le 60$$

Atmost 60.

$\underbrace{60}_{\text{sums}} < \underbrace{64}_{\text{subsets}} \Rightarrow$ at least 2 subset have same sum

Ex 5 6 basketball players have heights between 6 + 7 feet. Must two have height < 2 inches apart?

No heights could be 6'0", 6'2", 6'4", 6'6", 6'8", 6'10"

How about 7 players?

No, add 7'0" to list

How about 8 players?

YES make boxes

$$\left.\begin{array}{l} [6'0", 6'2") \\ [6'2", 6'4") \\ \vdots \\ [6'10", 7'0) \\ \{7'0\} \end{array}\right\}$$

7 boxes, 8 players

8 > 7 ⇒ 2 within 2 inches

Ch 6

A binary relation between sets A and B is a subset $R \subset A \times B$

Notation: $(a,b) \in R$ abbrev. aRb

spoken: a is related to b

Ex) $A = B =$ all people

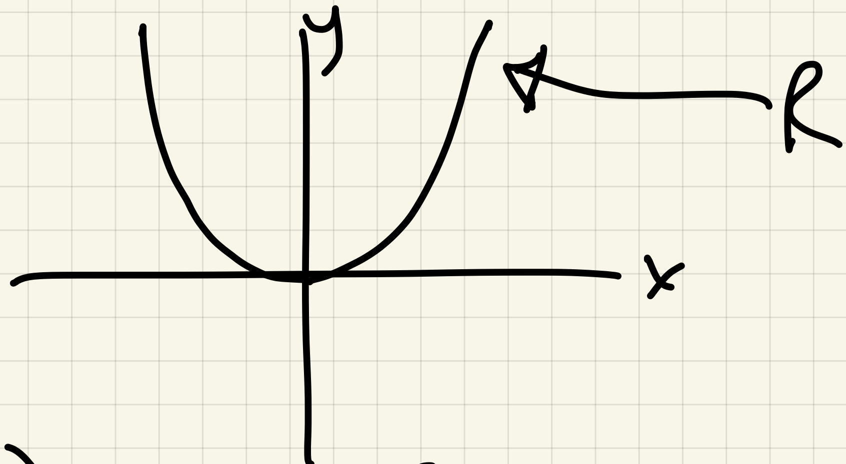
aRb if a related to b

is a relation!

Ex2 If $f: \mathbb{R} \to \mathbb{R}$ is a function,

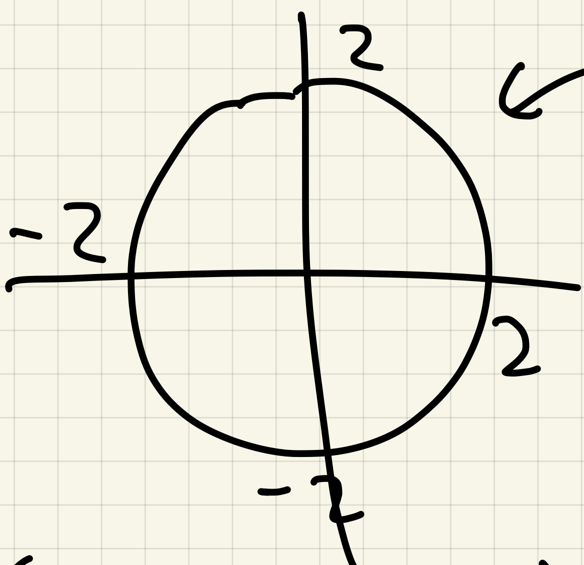
(a) say $A = B = \mathbb{R}$, xRy if $y = f(x)$

e.g. $f(x) = x^2$, then $R \subset \mathbb{R} \times \mathbb{R}$ is the graph $\{(x,y) : y = x^2\}$



(b) $A = B = \mathbb{R}$, xRy if $x^2+y^2=4$

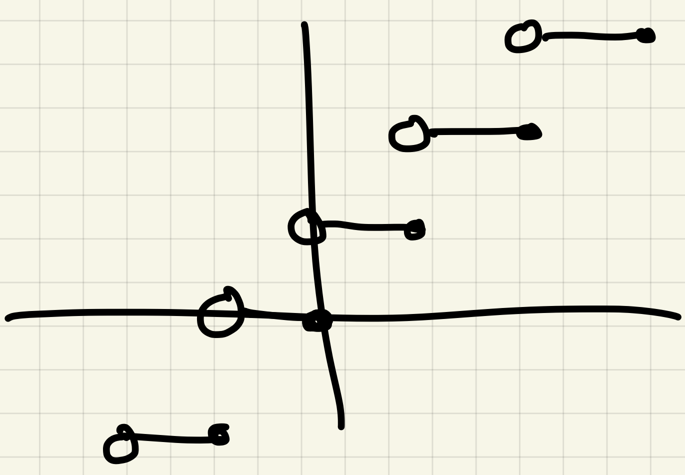
is relation



R is a circle

(Not a function)

(c) $A = \mathbb{R}$, $B = \mathbb{Z}$, xRy if $y = \lceil x \rceil$



Ex 3 $\quad A =$ TCU students

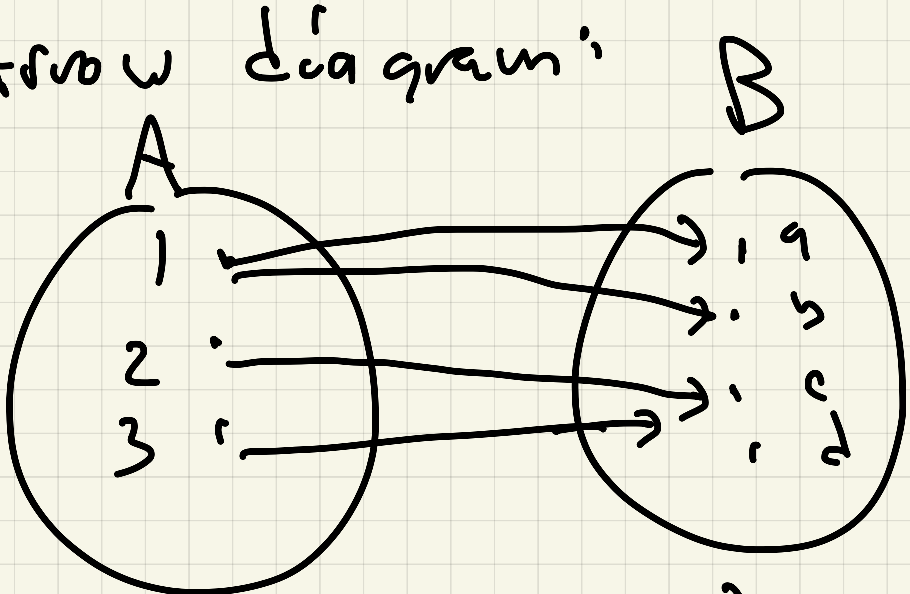
$B =$ courses

$R \subset A \times B$: aRb if a has taken b.

Ways to describe relations:

① Use words/formula (as above)

② Arrow diagram:



$(1Ra, 1Rb, 2Rc, 3Rc)$