Exam 3

1 (a) $X^n = -4X^{n-1} - 3X^{n-2} \Rightarrow X^2 + 4X + 3 = 0$

$(x+3)(x+1)$

$\Rightarrow X = -3, -1$

(b) $b_n = A(-3)^n + B(-1)^n$,

$3 = A + B$

$+ \quad 2 = -3A - B$

$5 = -2A$

$A = -5/2, \quad B = 11/2$

$b_n = -5/2(-3)^n + 11/2(-1)^n$

$b_2 = -8 - 9 = -17 = \frac{-5(9) + 11}{2}$ ✓ $b_3 = 62$

(c) Guess $b_n = (a + bn)2^n$

$(a+bn)2^n = -4(a+b(n-1))2^{n-1} - 3(a+b(n-2))2^{n-2} + n2^n$

$4(a+bn) = -8(a+b(n-1)) - 3(a+b(n-2)) + 4n$

n: $4b = -8b - 3b + 4 \Rightarrow b = 4/15$

1: $4a = -8a + 8b - 3a + 6b \Rightarrow 15a = 14b \Rightarrow$

$a = \frac{14}{15}\left(\frac{4}{15}\right) = \frac{56}{225}$

$b_n = \left(\frac{56}{225} + \frac{4}{15}n\right) \cdot 2^n$

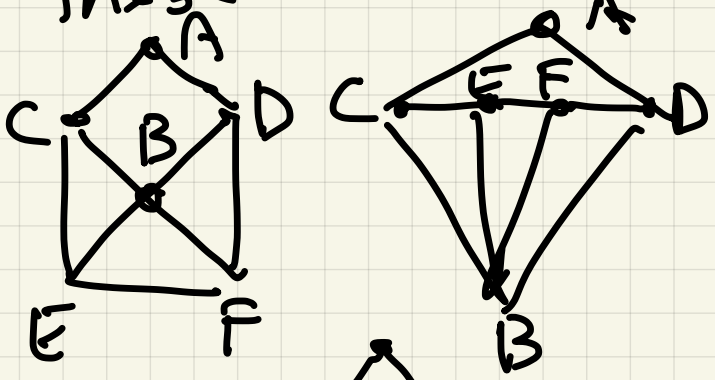
2 (a) $A5^n + Bn5^n + Cn^2 5^n + D(-2)^n$

(b) $A6^n$ (c) $n^3(a+bn)5^n$ (d) $a + bn + cn^2 + dn^3$

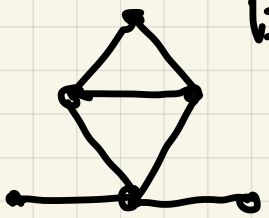
3 (a) not isomorphic, degree sequences different $4\,3\,3\,3\,3\,2 \neq 4\,4\,3\,3\,2\,2$

(b) Not isomorphic, G contains a K_3 subgraph, H does not.

(c) These are isomorphic



4 (a)



(b) Impossible, Total degree is odd.

(c) $n(n-1)$ b/c K_n is $(n-1)$-regular

(d) $7+4-1 = 10$ (one vertex shared)

(e) $21+6 = 27$

(f) $\kappa(G) = 1$ (remove shared vertex)

(g) $\lambda(G) = 3$ (Isolate K_4)

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(a)

	a	b	c	d	e	f
a	0	1	0	0	1	1
b	1	0	1	1	0	1
c	0	1	0	1	1	0
d	0	1	1	0	1	0
e	1	0	1	1	0	1
f	1	1	0	0	1	0

(b) 4 4 3 3 3 3

(c) $k(G) = 2$ (remove b & e)

(d) $\chi(G) = 3$ ($(a,f), (a,e), (a,b)$)

(e) $\langle a, b, f, e, c, b, d, e, a, f \rangle$ $\ell = 9$

(f) $\langle b, a, e, c, b, d, e, f, b \rangle$ $\ell = 8$

(g) $\langle a, b, c, d, e, f \rangle$ $\ell = 5$

(h) $\langle a, b, c, d, e, f, a \rangle$ $\ell = 6$