Exam 3 A

$\boxed{\#1}$ (a) $b_n = -5b_{n-1} - 4b_{n-2} \Rightarrow x^n = -5x^{n-1} - 4x^{n-2}$

$\Rightarrow x^2 + 5x + 4 = (x+4)(x+1) = 0 \Rightarrow x = -4, -1$

(b) $b_n = A(-4)^n + B(-1)^n$; $b_0 = 2$ $b_1 = 2 \Rightarrow$

$$\begin{array}{rl} & 2 = A + B \\ + & 2 = -4A - B \\ \hline & 4 = -3A \end{array} \Rightarrow A = -4/3,\ B = 10/3$$

$b_n = -4/3(-4)^n + 10/3(-1)^n$

$b_2 = -5(2) - 4(2) = -18 = \frac{-64+10}{3}$ ✓

$b_3 = -5(-18) - 4(2) = 82 = \frac{256-10}{3}$ ✓

(c) Guess $b_n = (a+bn)\cdot 2^n$: sub:

$(a+bn)\cdot 2^n = -5(a+b(n-1))2^{n-1} - 4(a+b(n-2))2^{n-2} + n2^n$

$4(a+bn) = -10(a+b(n-1)) - 4(a+b(n-2)) + 4n$

$n: 4b = -10b - 4b + 4 \Rightarrow b = 4/18 = 2/9$

$4a = -10a + 10b - 4a + 8b \Rightarrow 18a = 18b \Rightarrow$

$a = b = 2/9$, $b_n = (2/9 + 2/9\,n)\cdot 2^n$

$\boxed{\#2}$ (a) $A4^n + Bn4^n + Cn^2 4^n + D(-3)^n$

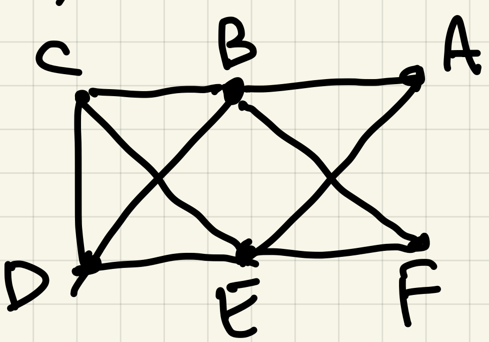
(b) $a_n = A\,7^n$ (c) $n^3(a+bn)4^n$

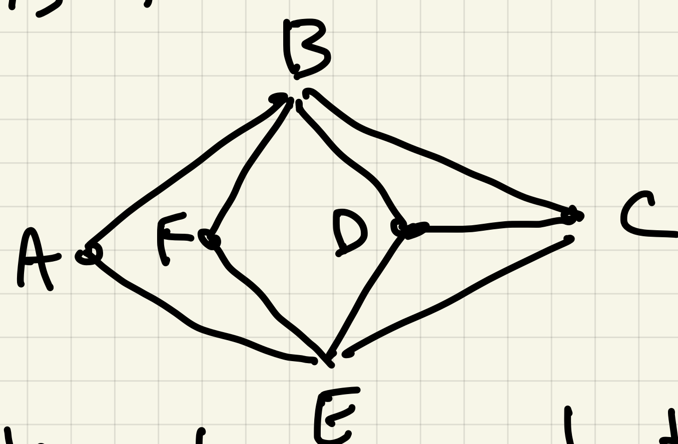
(d) $A + Bn + Cn^2 + Dn^3$

$\boxed{\#3}$ (a) Not isomorphic, degree sequences not same, $4,3,3,3,3,2 \neq 4,4,3,3,2,2$

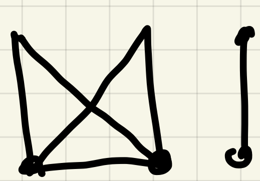
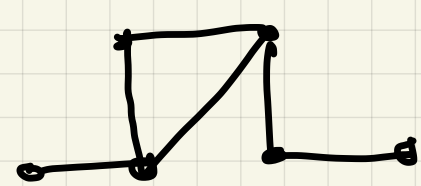
(b) Not isomorphic, G contains a subgraph isomorphic to $K_3 \cong C_3$, but $H \cong K_{3,3}$ does not.

(c) $G \cong H$ is true:





$\boxed{\#4}$ (a) Impossible, because total degree 13 is odd

(b)  or 

many possible

(c) $n(n-1)$ b/c K_n is $(n-1)$-regular

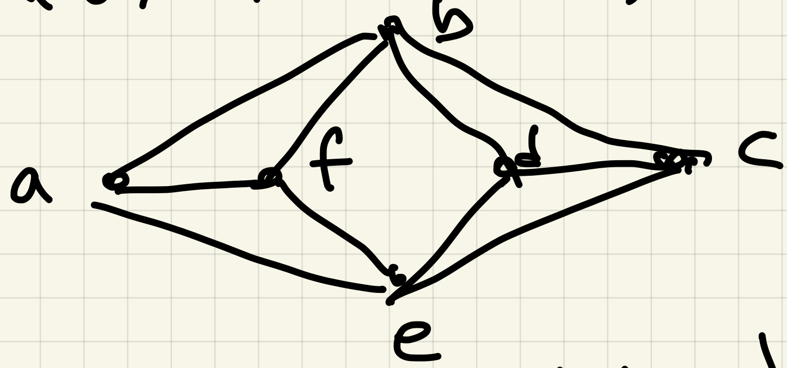
(d) $5+6-1=10$ vertices (1 shared)

(e) $10+15=25$

(f) $\kappa(G)=1$ (remove shared vertex)

(g) $\lambda(G)=4$ (using the K_5)

$\boxed{\#5}$



(a)
$$\begin{array}{c|cccccc|}
a & 0 & 1 & 0 & 0 & 1 & 1 \\
b & 1 & 0 & 1 & 1 & 0 & 1 \\
c & 0 & 1 & 0 & 1 & 1 & 0 \\
d & 0 & 1 & 1 & 0 & 1 & 0 \\
e & 1 & 0 & 1 & 1 & 0 & 1 \\
f & 1 & 1 & 0 & 0 & 1 & 0 \\
\hline
 & a & b & c & d & e & f
\end{array}$$

(b) 4, 4, 3, 3, 3, 3

(c) $\kappa(G)=2$ (remove b + e)

(d) $\lambda(G)=3$ (remove $(a,b), (a,f), (a,e)$)

(e) $\langle a, b, f, e, d, b, c, e, a, f\rangle$ $l=9$

(f) $\langle b, a, e, f, b, d, e, c, b\rangle$ $l=8$

(g) $\langle a, b, f, e, d, c\rangle$ $l=5$

(h) $\langle a, b, c, d, e, f, a\rangle$ $l=6$