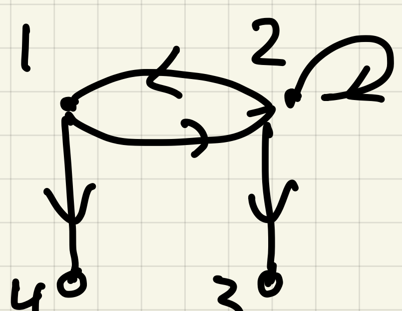
Exam 2 (B)

$\boxed{1}$ (a) There are $10 \times 10 \times 10 \times 26 = 26000$ ATM codes. Take the $p = 40000$ customers as pigeons, assign them to $h = 26000$ pholes by ATM code, PHP $\Rightarrow$ since $p = 40000 > h = 26000$, some hole has at least 2 pigeons, i.e. some ATM code goes with at least 2 customers.

(b) $26000 \times 2 + 1 = 52001$

$\boxed{2}$ (a)

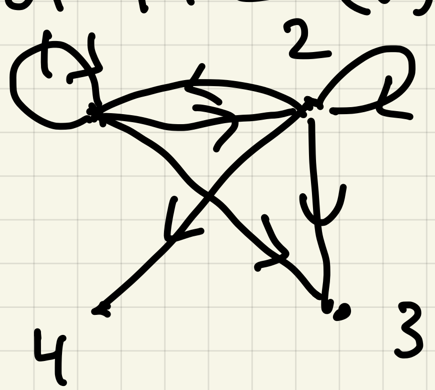


(b) $\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

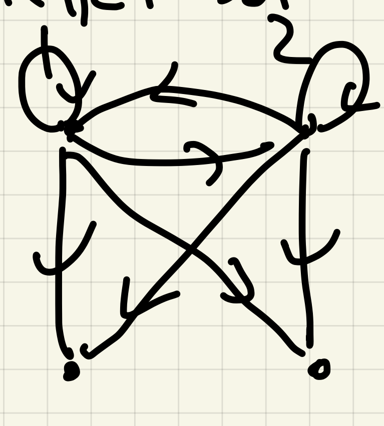
(c) (i) <u>no</u> $\neg 1R1$ (ii) <u>no</u> $2R2$

(iii) <u>no</u> $1R4$ but $\neg 4R1$ (iv) <u>no</u> $1R2 \wedge 2R1$, but $1 \neq 2$ (v) <u>no</u> $2R1 \wedge 1R4$ but $\neg 2R4$

(d)



(e)



$\boxed{3}$ (a) d (3) (b) a (3)

(c) $\langle a, b, d, a, b, d, a, a \rangle$ <u>many</u>

(d) $\langle a, b, c, d, a, a, d, e, f \rangle$

(e) $\langle a, b, c, d, e, f \rangle$ (f) $\langle a, b, c, d, a, a \rangle$

$\boxed{4}$ (a) $3(R \circ S)z \Rightarrow \exists w : 3Sw \wedge wRz \Rightarrow$

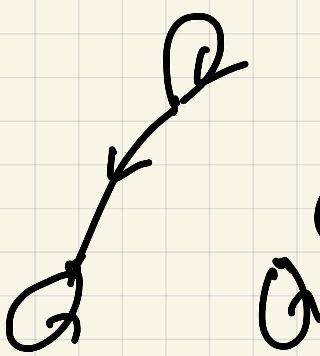
$w < 3 \wedge z = 2w^2 \Rightarrow w = 0, 1, 2,$

$z = \{0, 2, 8\}$

(b) $3(S \circ R)z \Rightarrow \exists w: 3Rw \wedge wSz \Rightarrow$

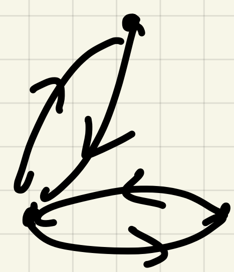
$w = 18 \wedge 18 > w \Rightarrow w \in \{0, 1, 2, \ldots 17\}$

$\boxed{5}$ (a)



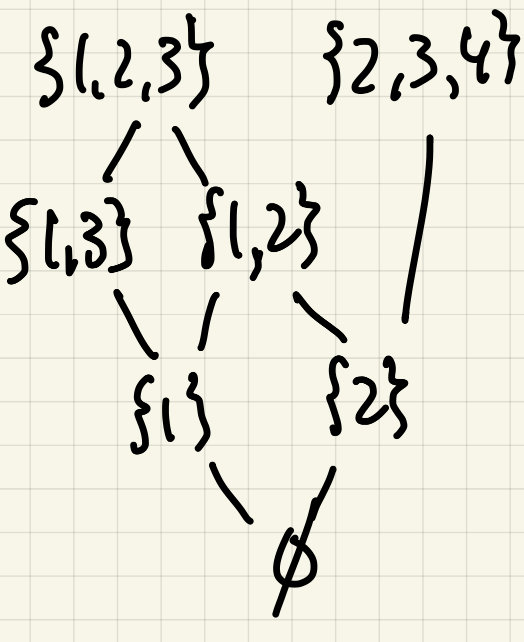
(many)

(b)



(many)

$\boxed{6}$ (a)



(b) <u>min</u>: $\emptyset$

<u>max</u> $\{1,2,3\}, \{2,3,4\}$

(c) <u>no</u> $\neg\{1\}R\{2\}$ and $\neg\{2\}R\{1\}$.

$\boxed{7}$ (a) Need $xRy \wedge yRz \Rightarrow xRz$.

Assume $\underset{x}{(a,b)} R \underset{y}{(c,d)}$ and $\underset{y}{(c,d)} R \underset{z}{(e,f)}$.

Then by defn $3a+b = 3c+d$ and $3c+d = 3e+f$,

so $3a+b = 3c+d = 3e+f \Rightarrow 3a+b = 3e+f \Rightarrow$

$(a,b) R (e,f)$ ✓

(b) $[(4,7)] = \{(a,b) : (a,b) R (4,7)\} =$

$\{(a,b) : 3a+b = 3(4)+7 = 19\}$

$(-3,-7) \notin [(4,7)]$ b/c $3(-3)+(-7) = -16 \neq 19$

$\boxed{8}$ R is reflexive, anti-symmetric, and transitive.