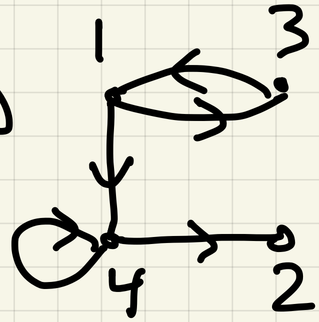
Exam 2 (A)

$\boxed{1}$ (a) There are $10 \times 10 \times 26 = 2600$ ATM codes possible. There are 3000 customers (pigeons), place them in 2600 holes by ATM code. Since $p = 3000 > h = 2600$, some hole = ATM code has at least 2 customers.

(b) $2600 \cdot 3 + 1 = 7801$

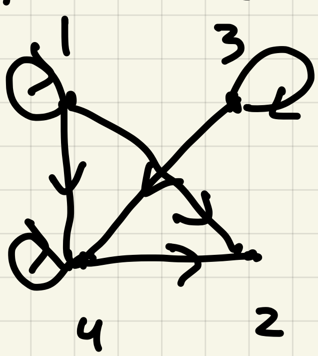
$\boxed{2}$ (a) R:



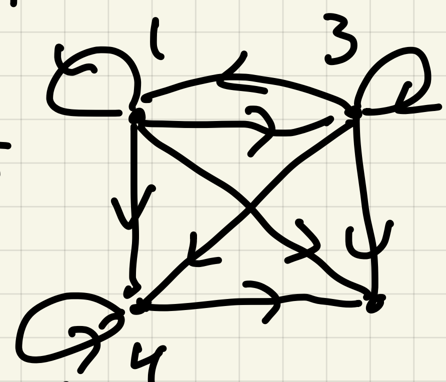
(b) $\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$

(c) (i) <u>no</u>, $\neg 2R2$ (ii) <u>no</u> $4R4$, (iii) <u>no</u> $1R4$ but $\neg 4R1$ (iv) <u>no</u> $1R3 \wedge 3R1$, but $1 \neq 3$ (v) <u>no</u> $1R4 \wedge 4R2$ but $\neg 1R2$

(d) R^2



(e) R^+



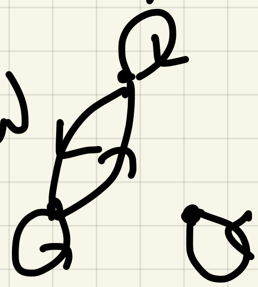
$\boxed{3}$ (a) b (3) (b) c (3)

(c) $\langle b, c, a, b, c, a, b, b \rangle$ many possible

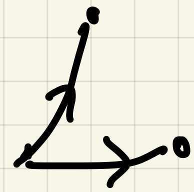
(d) $\langle f, e, c, d, a, b, c, b, b \rangle$ (e) $\langle f, e, c, d, a, b \rangle$

(f) $\langle b, b, c, d, a, b \rangle$

4 (a)



(b)



(many possible)

5 (a) $4(R \circ S)z \Rightarrow \exists w: 4Sw \wedge wRz \Rightarrow$

$\exists w: \ w < 4 \ \wedge \ z = w^2 \Rightarrow$

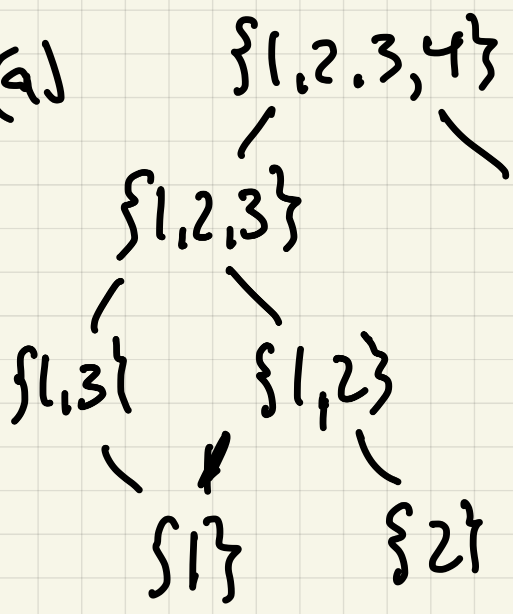
$w = 0,1,2,3 \Rightarrow z \in \{0,1,4,9\}$

(b) $4(S \circ R)z \Rightarrow \exists w: 4Rw \wedge wSz \Rightarrow$

$\exists w: w = 4^2 = 16 \ \wedge \ 16 > z \Rightarrow$

$z \in \{0,1,2, \ldots 15\}$

6 (a)



(b)

max: $\{1,2,3,4\}$

mins: $\{1\}, \{2\}, \{4\}$

(c) Not total,

$\neg \{1\} R \{2\} \wedge \neg \{2\} R \{1\}$

(d) 20 edges: didn't grade this.

7 (a) Need to show $xRy \wedge yRz \Rightarrow xRz$.

so assume $(a,b) R (c,d)$ and $(c,d) R (e,f)$.

x y y z

Then $2a-b=2c-d$ and $2c-d=2e-f$ by defn of R, so $2a-b=2c-d=2e-f \Rightarrow$ $2a-b=2e-f \Rightarrow (a,b)R(e,f)$ by defn

$\therefore (a,b)R(c,d) \wedge (c,d)R(e,f) \Rightarrow (a,b)R(e,f)$.

(b) $[(4,7)] = \{(a,b) : 2a-b = 2(4)-7 = 1\}$

$(-3,-7) \in [(4,7)]$ since $2(-3)-(-7)=1$ ✓

$\boxed{8}$ <u>not</u> reflexive, antireflexive <u>yes</u>, symmetric <u>no</u>, antisymmetric <u>yes</u> transitive <u>yes</u>.