4/21 / Disc 2 ; Quiz 10

$$\boxed{\begin{aligned} b_0 &= 5 \\ b_1 &= -5 \end{aligned}}$$

$b_n = b_{n-1} + 6b_{n-2}$

$x^n = x^{n-1} + 6x^{n-2} \Rightarrow x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$b_n^h = A \cdot 3^n + \boxed{B(-2)^n}$ 3 -2

(a) $b_n = b_{n-1} + 6b_{n-2} + \boxed{7^n}$

$b_n^p = c \cdot 7^n$

(b) $\boxed{n^2} 7^n$

$(\underline{a} + \underline{b}n + \underline{c}n^2) \cdot 7^n$

(c) $+ 12n \; 7^n$

Same

$$\boxed{b_n^p = a + bn}$$

$$+(-2)^n$$

$$b_n^p = nc\cdot(-2)^n$$

2. <u>guess</u> : $b_n^p = a + bn$

$$b_n = b_{n-1} + 6b_{n-2} + 12n$$

$$a+bn = a+b(n-1)+6(a+b(n-2))+12n$$

n: $\cancel{b} = \cancel{b} + 6b + 12$

const: $\cancel{a} = \cancel{a} - b + 6a - 12b$

$$-6b = 12$$

$$\boxed{b = \frac{12}{-6} = -2}$$

$$13b = 6a \qquad a = \frac{13b}{6} = \frac{-26}{6} = \frac{-13}{3}$$

$$b_n^p = -\frac{13}{3} - 2n$$

Last time: Graphs

Isomorphisms, degree sequence

Walk, open/closed/trail,

path/circuit/cycle

Connected graphs

Connected components

More subtle connectedness!

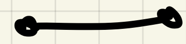
Vertex connectivity of $G = \kappa(G)$

min # vertices whose removal disconnects G

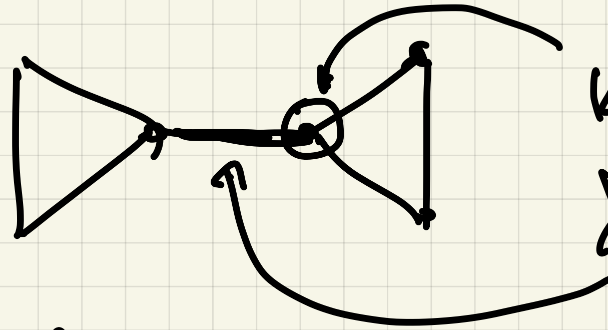
$\boxed{\text{unless } G \cong K_n, \ \kappa(G) = n-1}$

Edge connectivity of $G = \lambda(G)$

min # edges whose removal disconnects G



Ex (a)

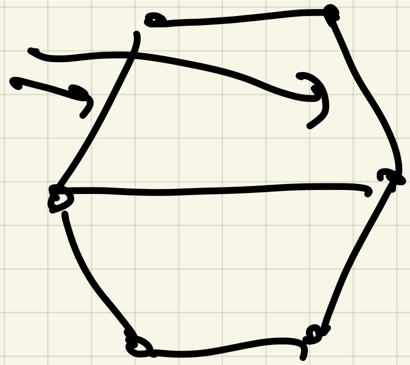


$\kappa(G)=1$

$\lambda(G)=1$

$\delta(G)=2$

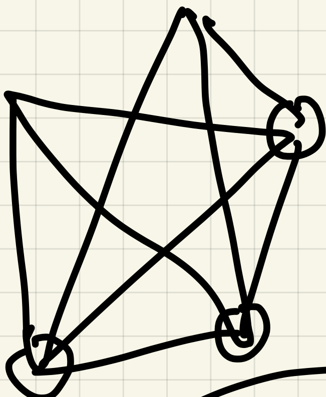
(b)



$\kappa(G)=2$ $\delta(G)=2$

$\lambda(G)=2$

(c)

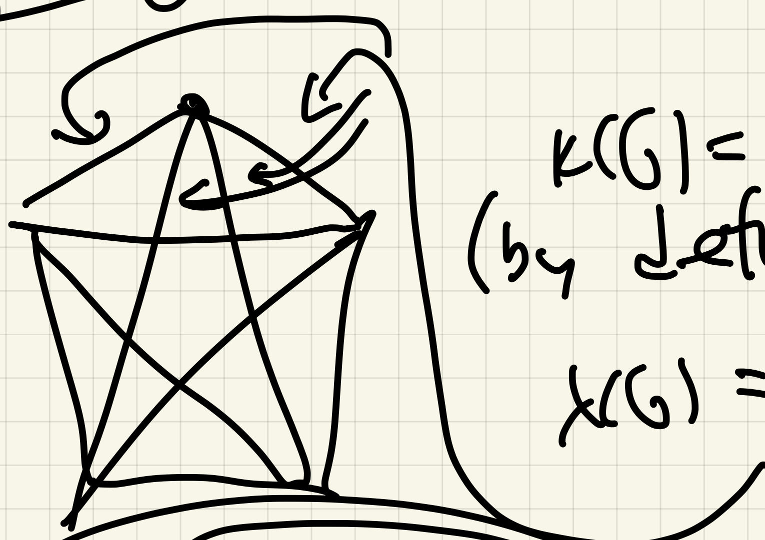


$\kappa(G)=3$ $\delta(G)=3$

$\lambda(G)=3$

(d)

K_5

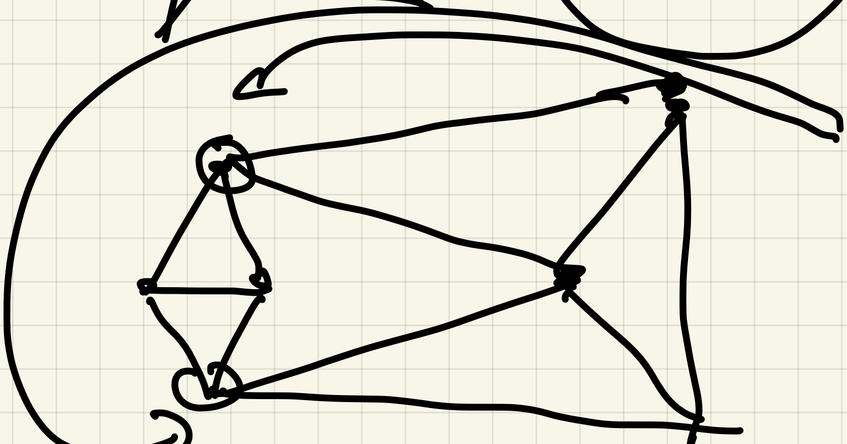


$\kappa(G)=4$ $\delta(G)=4$

(by definition)

$\lambda(G)=4$

(e)



$\kappa(G)=2$

$\lambda(G)=3$

$\delta(G)=3$

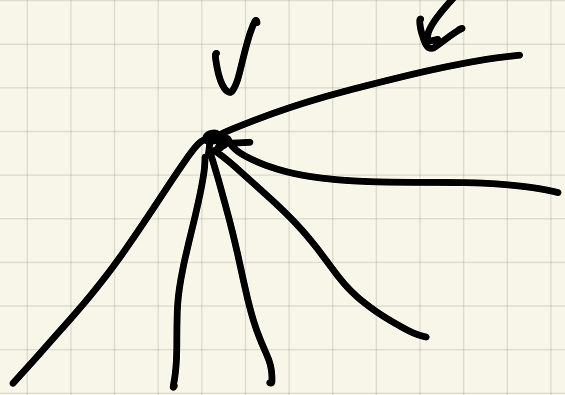
Theorem: $G = (V, E)$

Let $\delta(G) = \min\{\deg(v) : v \in V\}$

Then $\kappa(G) \leq \delta(G)$

$\lambda(G) \leq \delta(G)$

Proof $\deg(v) = \delta(G)$ # edges is $\delta(G)$



① removing edges incident to v disconnects $G \Rightarrow$

$\lambda(G) \leq \delta(G)$

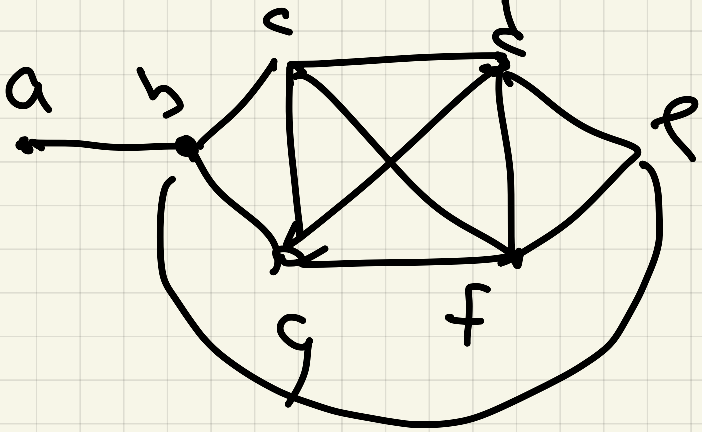
② removing vertices adjacent to v disconnects $G \Rightarrow$

$\kappa(G) \leq \delta(G)$

Ex: compute δ for Ex1 (a)-(e)

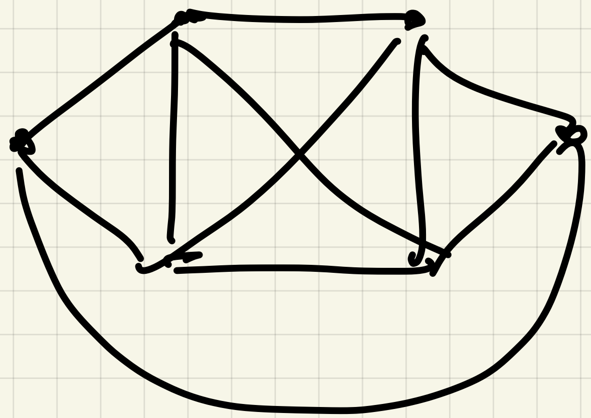
__Ex 2__

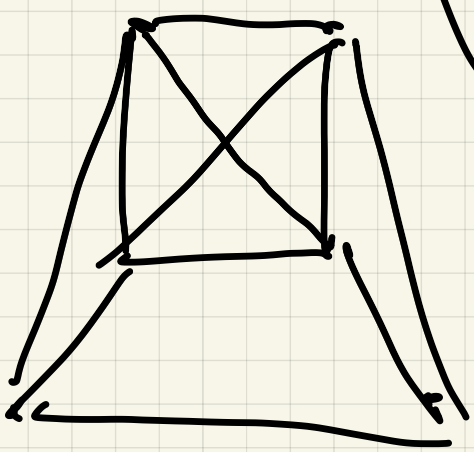
(a)



__Easy__ $\delta(G) = \lambda(G) = \kappa(G) = 1$

(b)



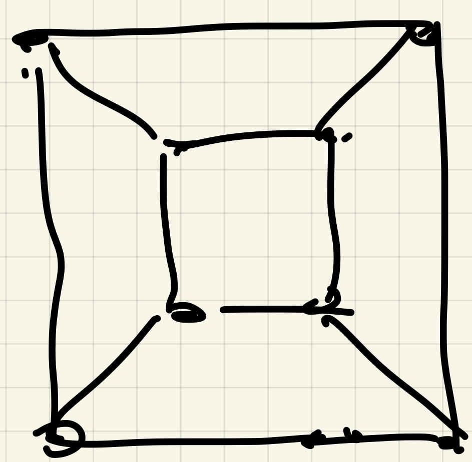


__Thm__: $\kappa(G) \le 3$

$\lambda(G) \le 3$

Appears $\kappa(G) = 3 = \lambda(G)$

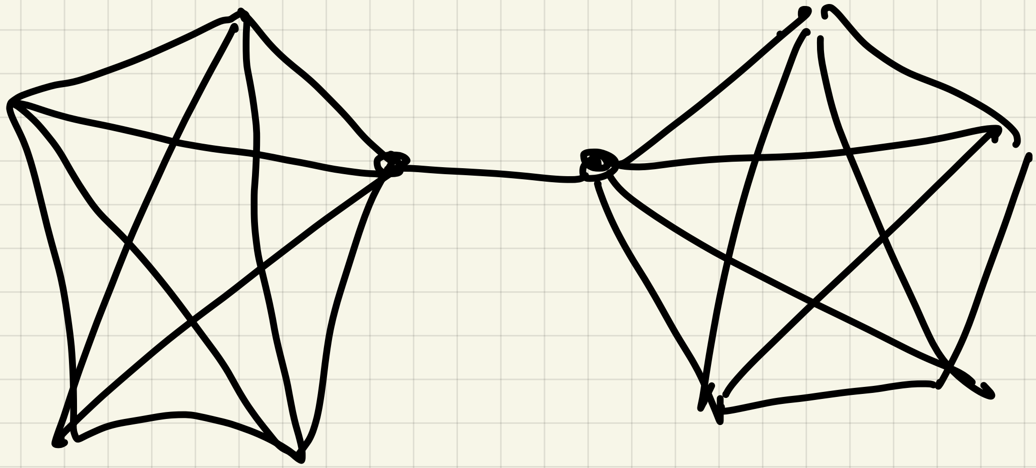
(c) Q_3



$\delta = 3$

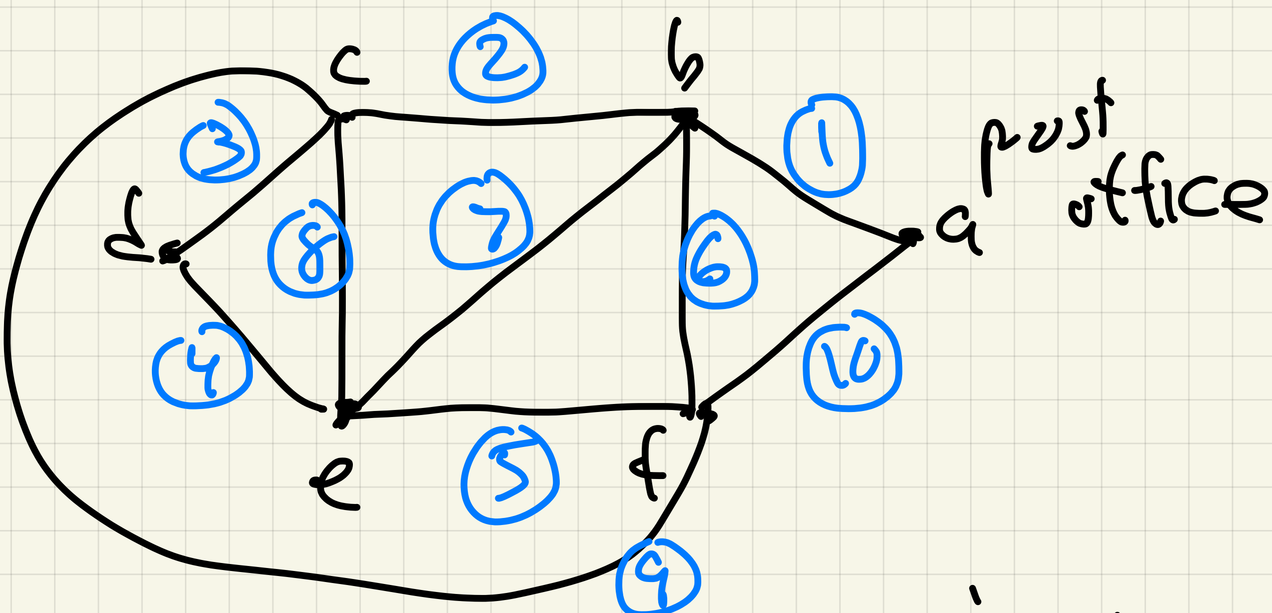
$\underline{\kappa = \lambda = 3}$

(d)



§13.6

Ex0 **Problem**: You're the postman, want to deliver mail efficiently;



Most ~~efficient~~ circuit:

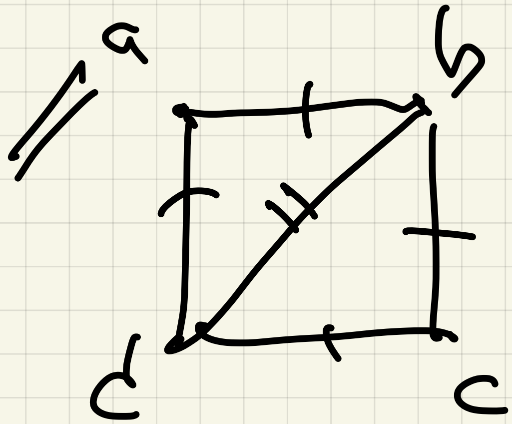
$\langle a, b, c, d, e, f, b, e, c, f, a \rangle$

Here there's an ideal solution, our walk crosses each edge exactly once.

<u>Definition</u>: An <u>Euler circuit</u> for a graph G is circuit that uses all edges.

<u>Ex2</u>

post office



Solution to postman problem has redundancy

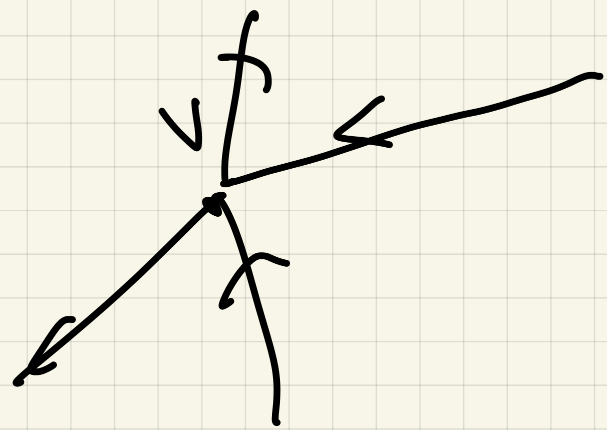
$\langle a, b, c, d, b, d, a \rangle$

<u>Observe</u>:

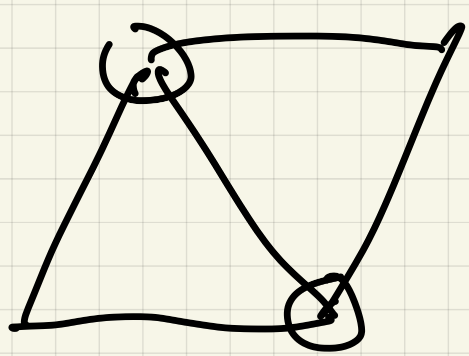
① If G has an Euler circuit, then G is connected.

② Every vertex has even degree.

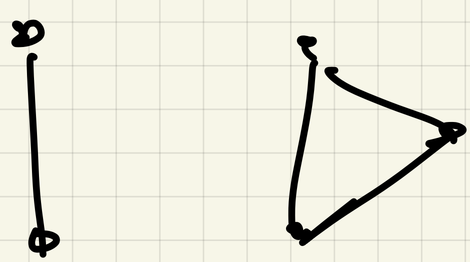
Idea each time the Euler circuit passes through a vertex it adds 2 edges to degree count.

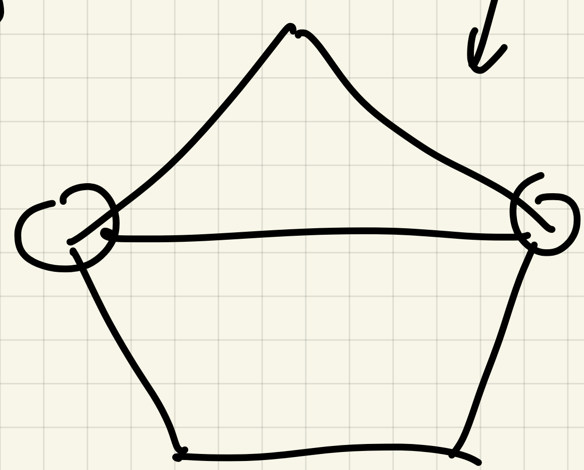


Ex3



no Euler circuit





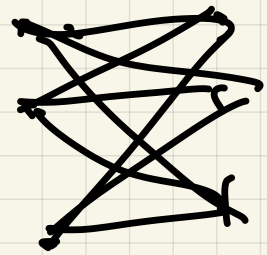
Ex 4: Other graphs:

K_4 3-regular, fails

K_6 5-regular

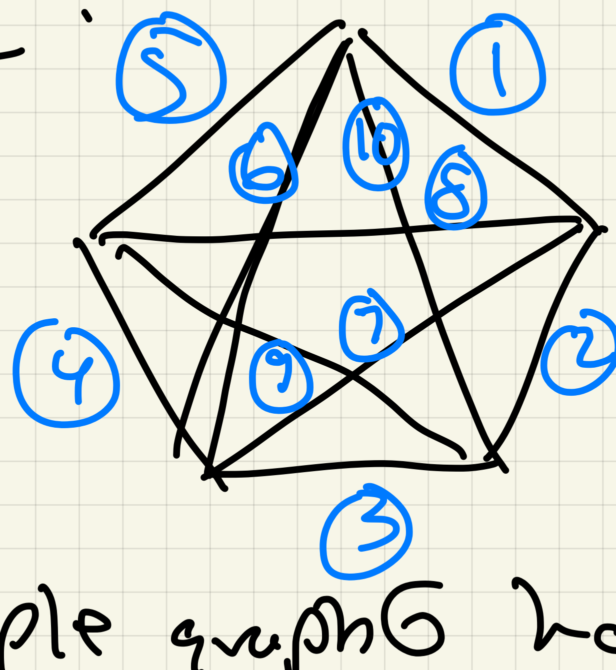
Q_3 Q_5

$K_{3,3}$



$K_{3,100}$

Ex 5 K_5? YES:



Q_4 ???

16 vertices

32 edges

Theorem: A simple graph G has an Euler circuit $\iff$ G connected and every

Vertex has even degree.

<u>Idea</u>: