4/16/ Disc2

3

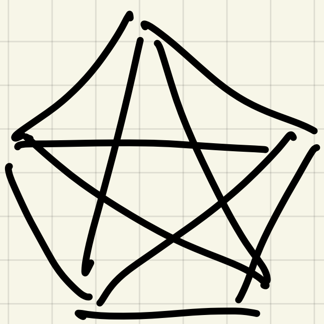
Last time

Graphs

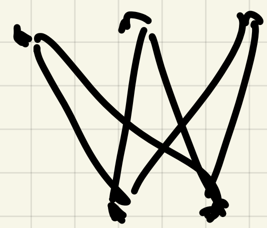
Simple (no self-loops, no // edges)

Special graphs $K_n, C_n, K_{n,m}$

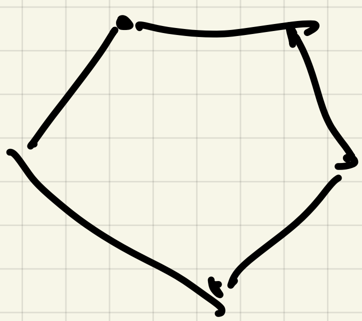
K_5



$K_{2,3}$



C_5



Q_n = vert n-bit strings

Q_2

$$\begin{tikzcd}[arrows=dash] 01 \arrow[r] \arrow[d] & 11 \arrow[d] \\ 00 \arrow[r] & 10 \end{tikzcd}$$

edges if exactly one bit changes

Adjacency matrix

Adjacency list

__Graph isomorphism__ $f: G \to G'$

$f: V_G \to V_{G'}$ bijection on vertices

s.t. $\{a,b\}$ edge in $G \iff$

$\{f(a), f(b)\}$ edge in G'

__Notation:__ $G \cong G'$

Degree sequence

$G \cong G' \Rightarrow G, G'$ have same degree sequence

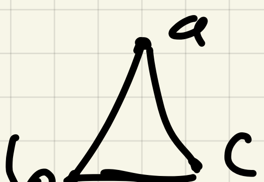
same total degree

__Ex)__ $K_3 \cong C_3$

$f(a)=1$

$f(b)=2$

$f(c)=3$



Degree sequence 2 2 2

Total degree 6

$$\begin{array}{c} a \\ b \\ c \\ \\ \end{array}\begin{array}{c}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \\ \begin{matrix} a & b & c \end{matrix}\end{array}$$

K_3 Adj. matrix

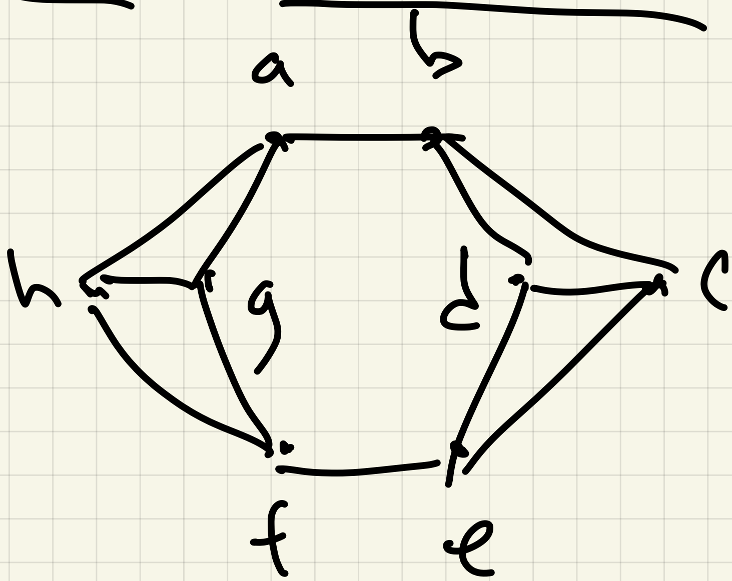
$\boxed{a} \to \boxed{b, c}$

$\boxed{b} \to \boxed{a, b}$

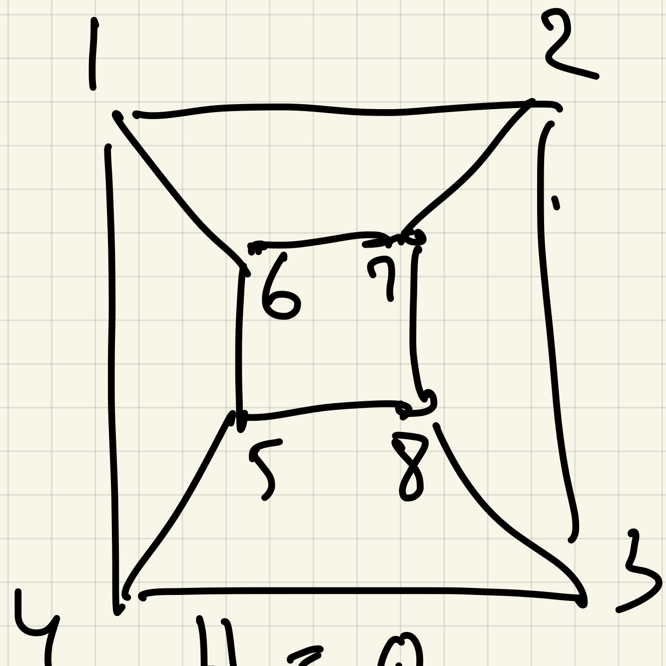
$\boxed{c} \to \boxed{a, b}$

Adjacency list

Ex 2 Is $G \cong H$



G



$H = Q_3$

Both 3-regular, total degree 24
8 vertices

Degree sequence 3 3 3 3 3 3 3 3

No : G has subgraph $\cong C_3$
H does not.

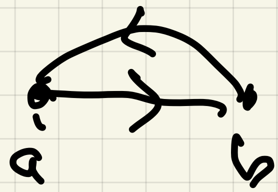
§13.4 Let $G=(V,E)$

Walk : sequence of vertices
$\langle v_0, v_1 \ldots - v_n \rangle$ where
(v_i, v_{i+1}) edge
length $= n$

open if $v_0 \neq v_n$
closed if $v_0 = v_n$

Types of walks !

Trail open walk,
edges different:



$\langle a, b, a \rangle$
trail

$\langle a, b, a \rangle$
not trail

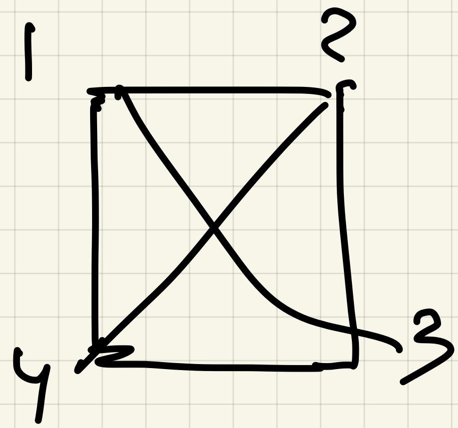
$\langle a, b \rangle = \langle b, a \rangle$

<u>Path</u>: Trail with distinct vertices

<u>Circuit</u>: Closed walk with distinct edges

<u>Cycle</u> Circuit with distinct vertices EXCEPT first = last

<u>Ex</u> K_4



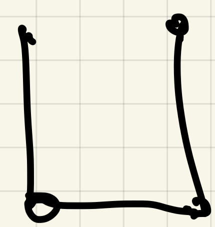
(a) $\langle 1, 2, 3, 4, 2, 3, 1, 4 \rangle$
open walk length 7

(b) $\langle 1, 2, 3, 4, 1, 3 \rangle$
open walk, trail, $l = 5$,
not path

(c) $\langle 1, 2, 3, 4, 1 \rangle$ = cycle
circuit

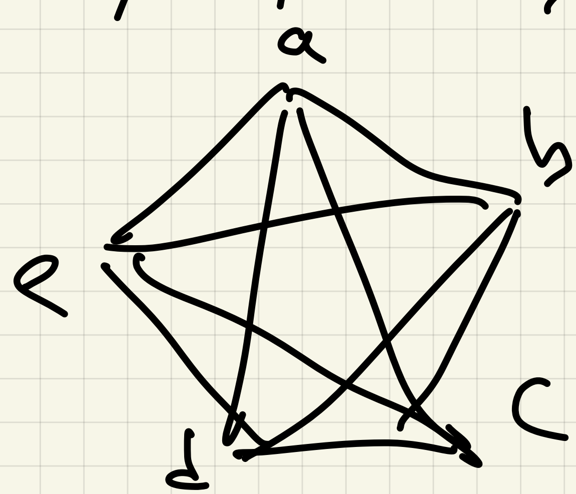
<u>longest trail</u>? $l = 5$

<u>longest path</u>: $l = 3$



longest circuit / cycle $l = 4$

<u>Ex 2</u> K_5



longest cycle $\ell = 5$

Longest path $\ell = 4$

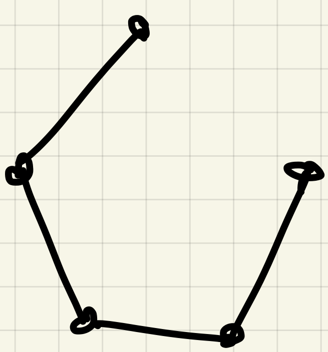
longest circuit? $\ell = 10$

$\langle a\ b\ c\ d\ e\ a\ d\ b\ e\ c\ a \rangle$

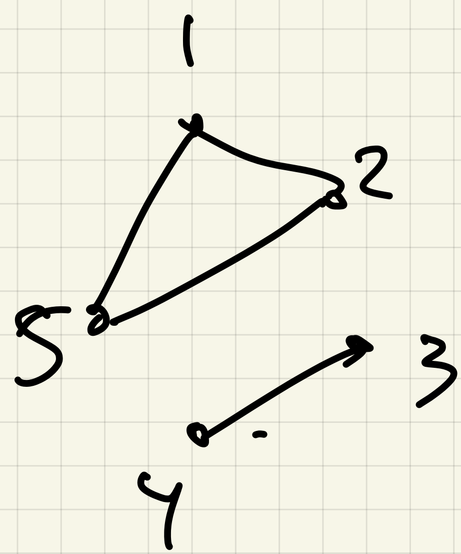
longest trail $\ell = 9$

§ 13.5

G



H



degree sequence is 2 2 2 1 1

but $G \not\cong H$.

G has trail length 4. H doesn't.

Also, G connected, If not.

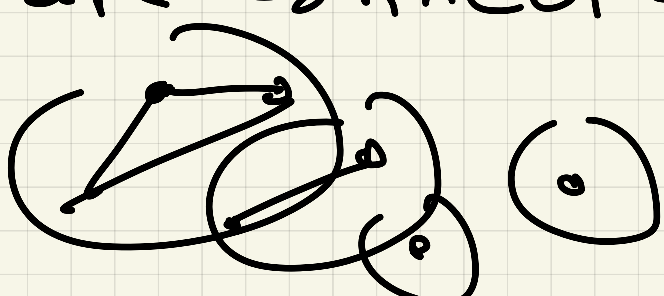
<u>Definition</u>: Two vertices v, w in a graph are <u>connected</u> if there's a path from v to w.

Graph G is <u>connected</u> if for each pair $v, w \in VG$, v connected to w.

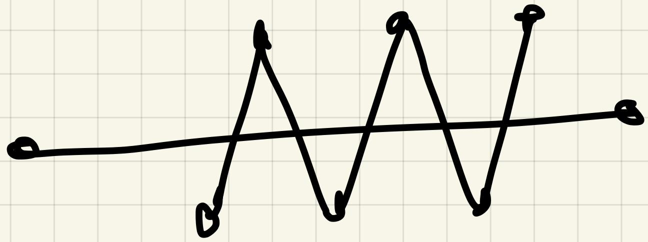
Otherwise G is <u>disconnected</u>.

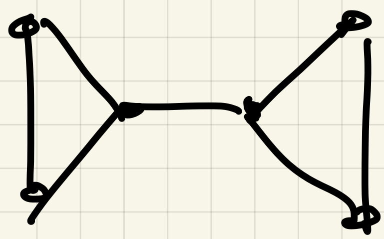
A <u>connected component</u> of G is a maximal set of vertices that are connected

<u>Ex</u>



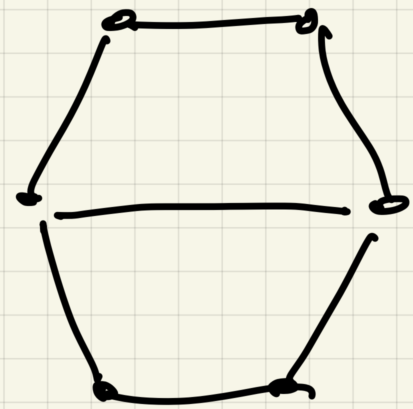
4 connected components





G

332222



H

same degree sequence

$G \not\cong H$

removing one edge disconnects G

removing an edge does not disconnect H

Connectivity

G is **k-vertex connected** if

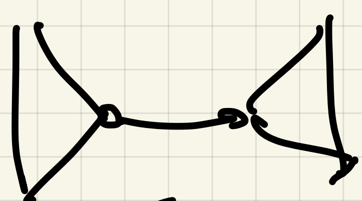
G has at least $k+1$ vertices and is connected after any $k-1$ vertices are removed

$\Updownarrow$

<u>Vertex connectivity of G</u> is the <u>largest</u> k with G k-connected.

<u>Notation</u> $\kappa(G)$

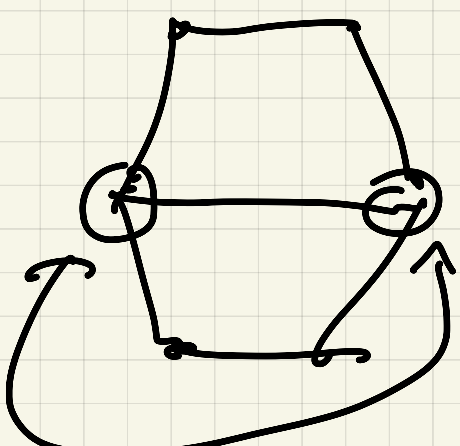
<u>Ex1</u>



G

$\kappa(G) = 1$

<u>Ex2</u>

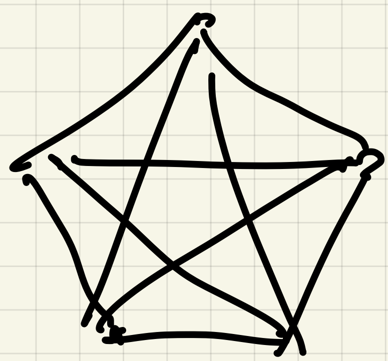


$\kappa(G) = 2$

removing these

disconnects the graph

$\underline{K_5}$



$\underline{\text{Weird}}$: by $\underline{\text{definition}}$

$\kappa(K_n) = n-1$

<u>Definition</u>: G is <u>k-edge-connected</u> if it is connected after removing any $k-1$ edges.

<u>Edge connectivity of G</u> is largest such k

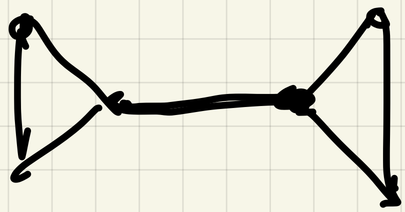
$\|$

(= min # edges whose removal disconnects G)

<u>Notation</u> $\lambda(G) =$ edge connectivity

__Ex__ What is $\kappa(G)$ & $\lambda(G)$

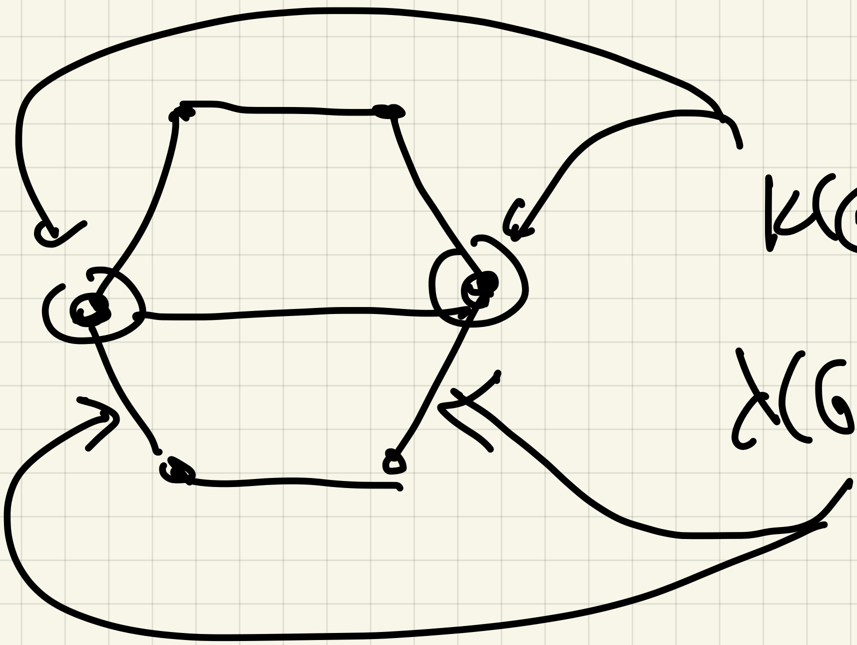
(a)



$\lambda(G) = 1$

$\kappa(G) = 1$

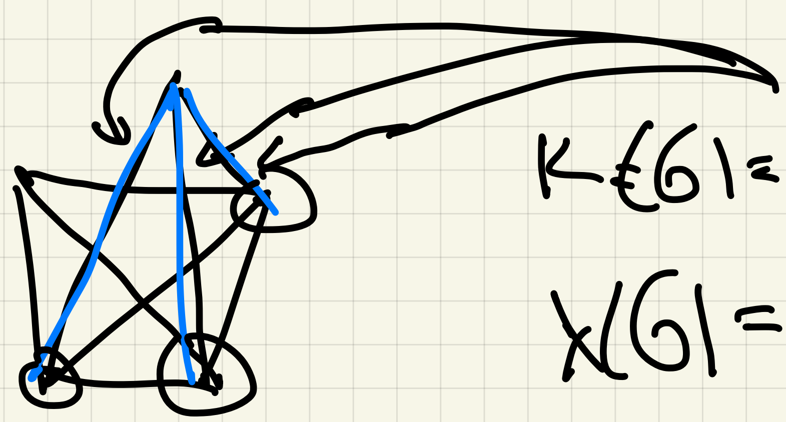
(b)



$\kappa(G) = 2$

$\lambda(G) = 2$

(c)



$\kappa(G) = 3$

$\lambda(G) = 3$