4/14/ Disc 2

Quiz 9

avg 84
med 100

1. $b_0 = 1,\ b_1 = 2$

$$\boxed{b_n = 5b_{n-1} - 6b_{n-2}}$$

(a) $$x^n = 5x^{n-1} - 6\underline{x^{n-2}}$$

$$x^2 = 5x - 6$$

$$\underline{x^2 - 5x} + 6 = 0$$

$$(x-2)(x-3) = 0 \quad x = 2, 3$$

(b)

Gen'l solution

$$b_n = A \cdot 3^n + B \cdot 2^n$$

(c) $$b_2 = 5\underbrace{b_1}_{2} - 6\underbrace{b_0}_{1}$$

$$10 - 6 = 4$$

2. $b_n = b_{n-1} + 6b_{n-2}$

$$x^n = x^{n-1} + 6x^{n-2}$$

$x^2 = x + 6$ $\qquad x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$$\boxed{x = 3, -2}$$

(a) $b_n = A \cdot 3^n + B(-2)^n$

$$\boxed{\begin{aligned} b_0 = 1 &= A + B \\ b_1 = 2 &= 3A - 2B \end{aligned}}$$

$2 = 2A + 2B$

$2 = 3A - 2B$

$4 = 5A \quad 0$

$A = 4/5,$

$B = 1/5$

so $\boxed{b_n = 4/5 \cdot 3^n + 1/5(-2)^n}$

(c) $\boxed{b_2 = b_1 + 6b_0 = 2 + 6 \cdot 1 = 8}$

Also: $b_2 = 4/5 \cdot 3^2 + 1/5(-2)^2$

$= \frac{4}{5} \cdot 9 + 1/5 \cdot 4$

$\frac{36}{5} + \frac{4}{5} = \frac{40}{5} = 8$ ✓

Ch 13 : Graphs

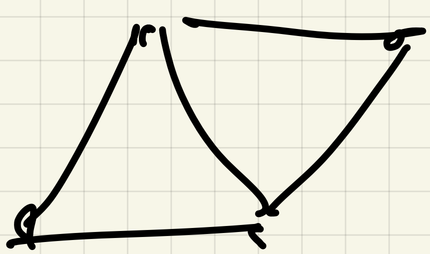
$$G = (V_G, E_G)$$

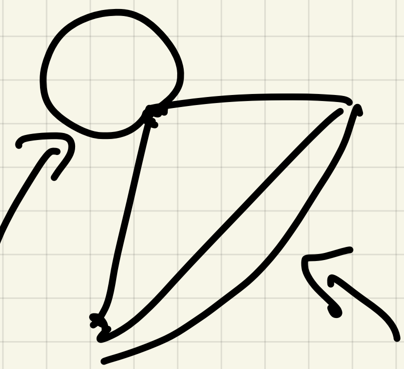
vertices edges

Simple graph

no // edges

no self-loop





vertex-degree

parallel edges

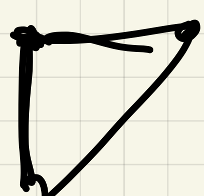
self-loop

$$\text{Total degree} = \sum_{v \in V_G} \deg(v) = 2|E_G|$$

d-regular graph

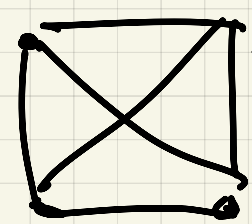
subgraph

2-regular



~~3-regular~~

$d = 3$ regular



$H \subset G$

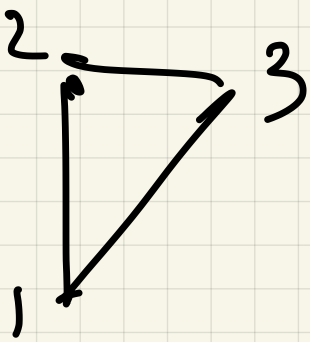
$V_H \subset V_G$

$E_H \subset E_G$

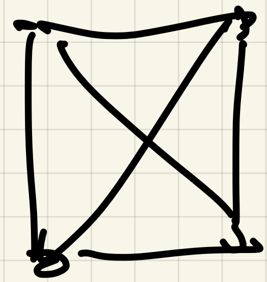
1 Special Graphs:

Ex: K_n = complete graph of n vertices

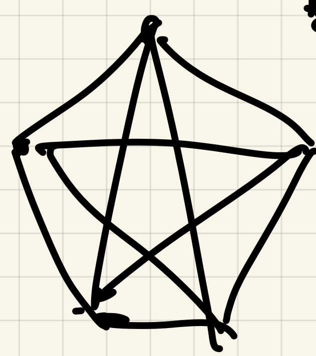
K_3



$K_4 =$



K_5

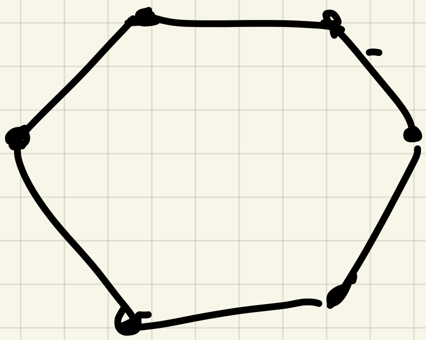


K_n: $(n-1)$-regular:

$$\sum_{v \in V_{K_n}} \deg(v) = \sum_{1}^{n} (n-1) = n(n-1) = 2|E|$$

$$|E| = \frac{n(n-1)}{2} = \binom{n}{2}$$

② C_n = cycle on n vertices

C_6  $\deg v = 2$

2-regular

Total degree $= 2n \Rightarrow |E| = n$

③ <u>Complete</u> <u>bipartite</u> <u>graph</u>

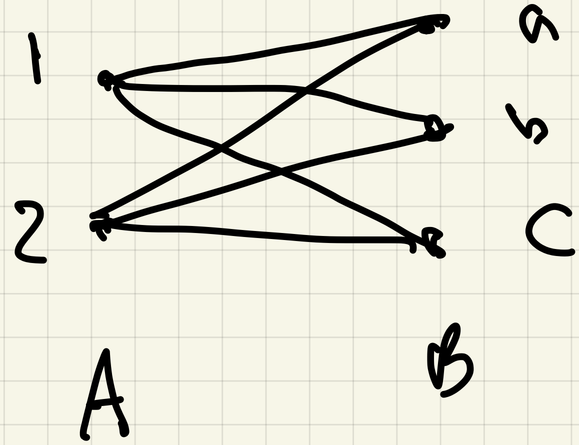
$K_{n,m}$ Vertex set

$$V = A \cup B$$

$|A| = n \quad |B| = m$

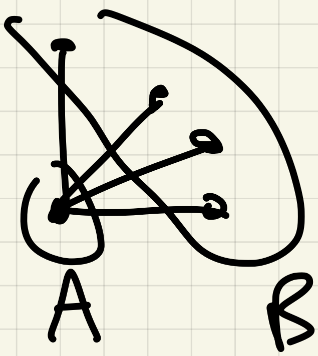
1 Edge between each vertex $v \in A$ and $w \in B$

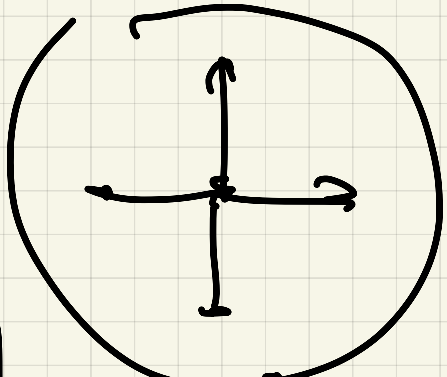
$K_{2,3}$



Not regular unless $m = n$

$K_{1,4}$





<u>How many edges</u>: Total degree

$$\sum \deg(v) = \sum_{v \in A} \deg(v) + \sum_{v \in B} \deg(v)$$

$$= n \cdot m + m \cdot n$$

$$= 2nm$$

$$\Rightarrow |E| = nm$$

E (4) n-dimensional hypercube

Q_n 2^n vertices

label with n-bit strings

2 vertices are adjacent

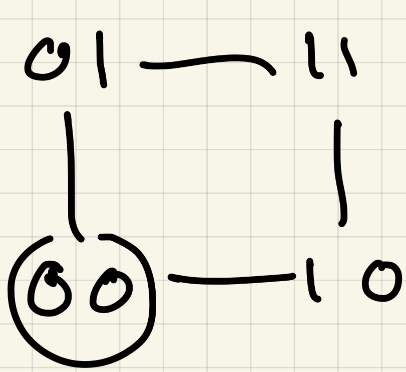
$\Updownarrow$

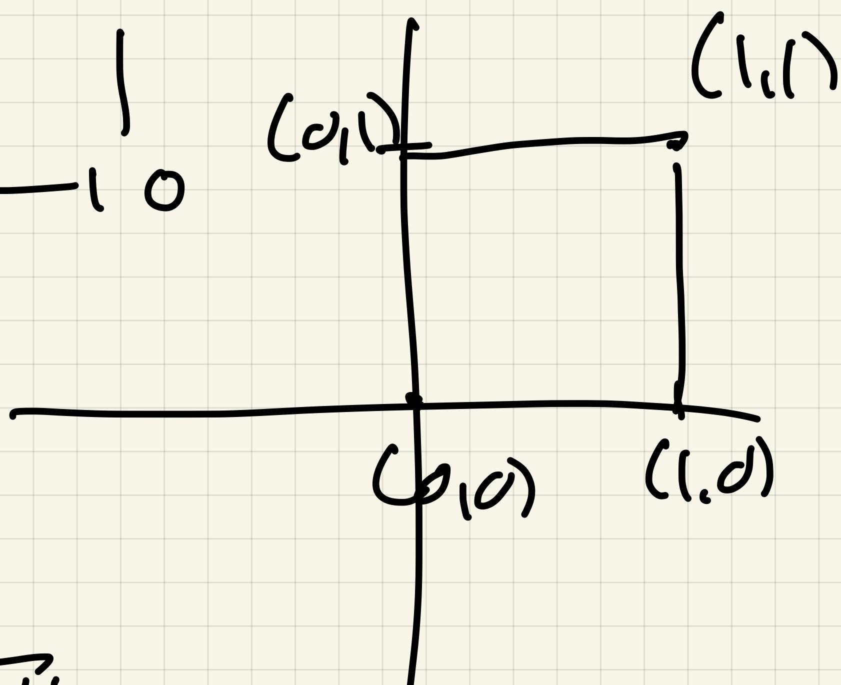
bit strings differ in exactly one position/digit

Ex 4

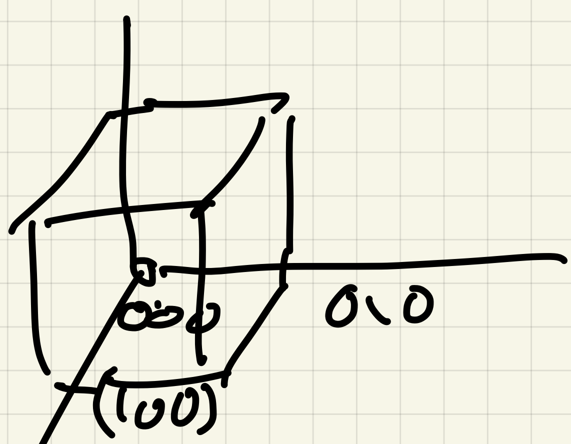
$n=2$

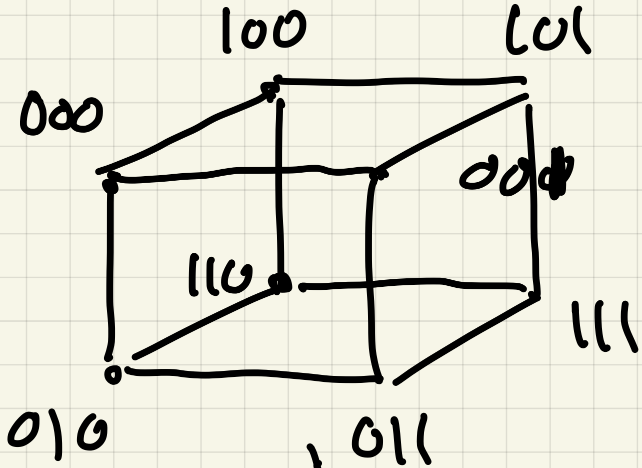
(a)

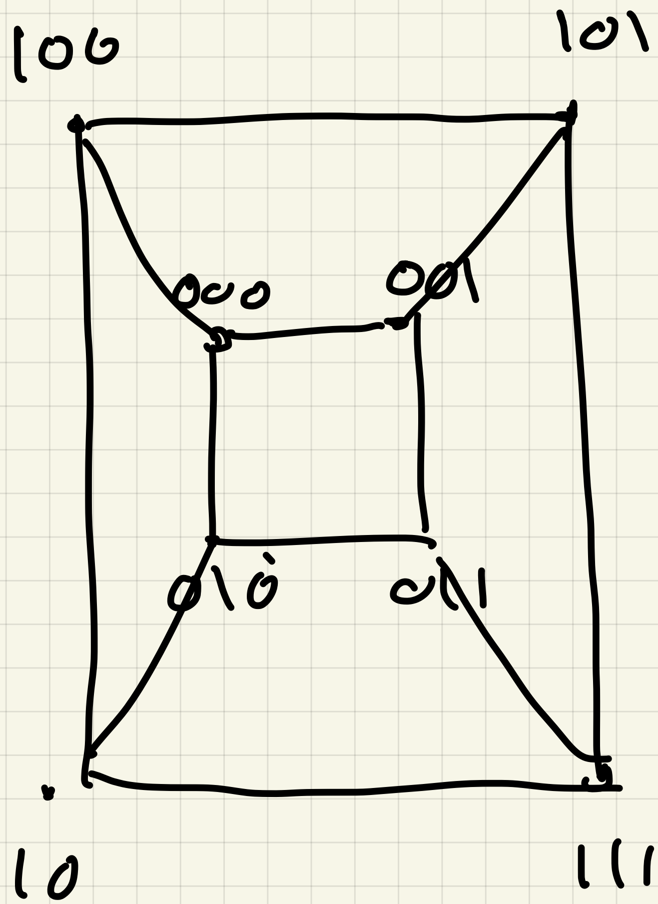




(b)







Q_n is n-regular

$\sum \deg(v)$

$2^n \cdot n = 2 \cdot |E|$

$\Downarrow$

$|E| = n \cdot 2^{n-1}$

13.2 Graph representation:

① Adjacency matrix

If vertices are $v_1 \ldots v_n$,

put 1 in position (i,j) if there's an edge from v_i to v_j

$$M = \begin{array}{c}
000 \\ 001 \\ 010 \\ 011 \\ 100 \\ 101 \\ 110 \\ 111
\end{array}
\begin{pmatrix}
0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 1 & 1 & & & & & \\
1 & 0 & 0 & & & & & \\
0 & 1 & 0 & & & & & \\
0 & 0 & 1 & & & & 0 & 1 \\
0 & 0 & 0 & & & & 1 & 0
\end{pmatrix}$$

$$\begin{array}{cccccccc} 000 & & & & & & & 111 \end{array}$$

<u>Note</u>: no 1's on diag (no self-loops)

all entries ≤ 1 (no parallel edges)

② <u>Adjacency list</u>:

<u>Idea</u>: for each vertex, list its neighbors:

000 → 001 010 100

001 → 101 011 000

010 → 110 000 011

011 → 111 001 010

100

101

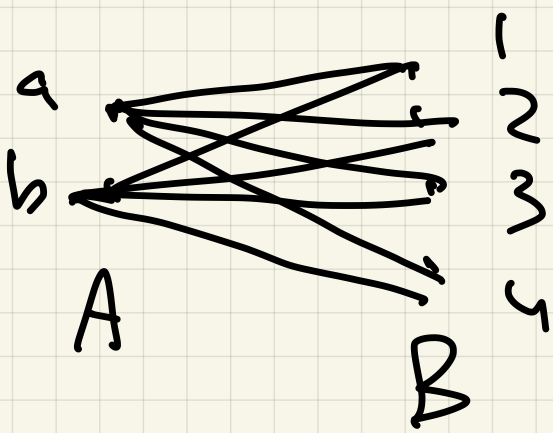
110

111

⋮

$\boxed{\text{Ex } 2}$

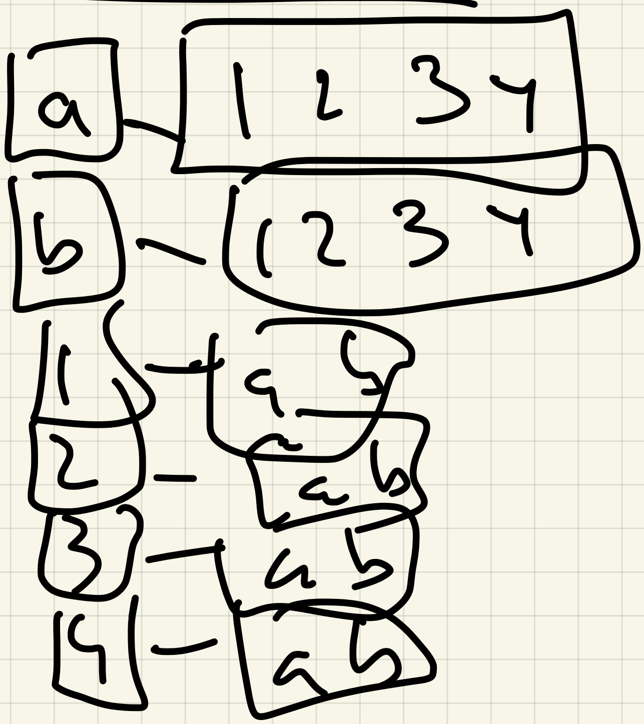
$K_{2,4} =$



$$M = \begin{array}{c|cc|cccc} & a & b & 1 & 2 & 3 & 4 \\ \hline a & 0 & 0 & 1 & 1 & 1 & 1 \\ b & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 & 0 & 0 \\ 4 & 1 & 1 & 0 & 0 & 0 & 0 \end{array}$$

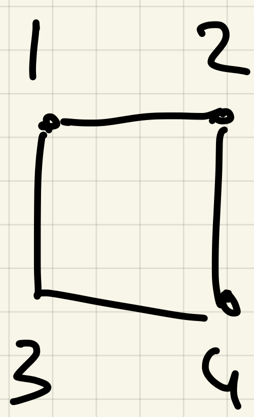
matrix

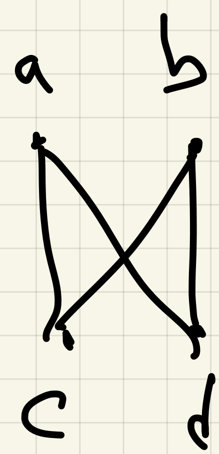
Adjacency list:



§13.3 Graph isomorphism

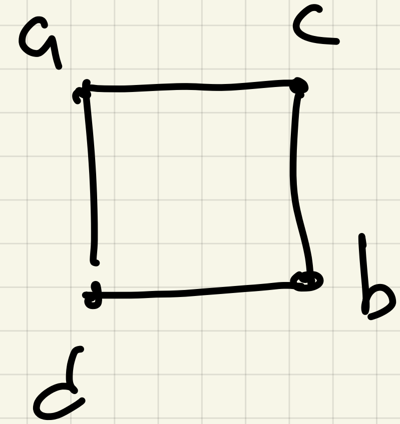
Ex1

H = 

G = 

$H \neq G$ b/c $V_H \neq V_G$
$E_H \neq E_G$

"Look same"

$G =$ 

<u>Defn</u>: An <u>isomorphism</u>

$f : G = (V, E) \to G' = (V', E')$

<u>of graphs</u> is a

<u>bijection</u> $f : V \to V'$

such that $\forall\, x, y \in V$,

$$\boxed{(x, y) \in E \iff (f(x), f(y)) \in E'}$$

If such f exists, then

G is <u>isomorphic to G'</u>

Notation: $G \cong G'$

Ex 1: $f(1) = a$
$f(2) = c$
$f(3) = d$
$f(4) = b$

In fact, there are 8 isomorphism between G and H.

Graph isomorphism problem:

Given two finite graphs, G, H determine if $G \cong H$.

This problem is NP-~~hard~~ complete problem.

Thm: If $f: G \to G'$ is an

isomorphism, then $\forall v \in V_G$

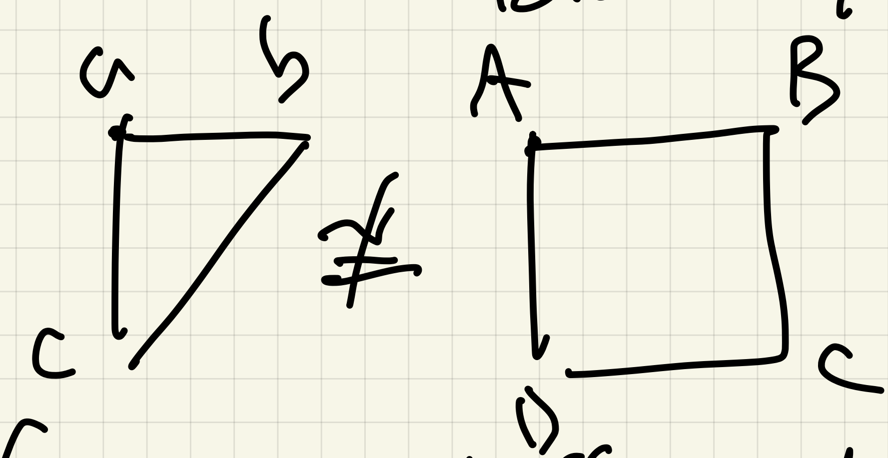
$$\deg v = \deg f(v)$$

Why? If $w_1, \ldots, w_d$ are neighbors of v, then

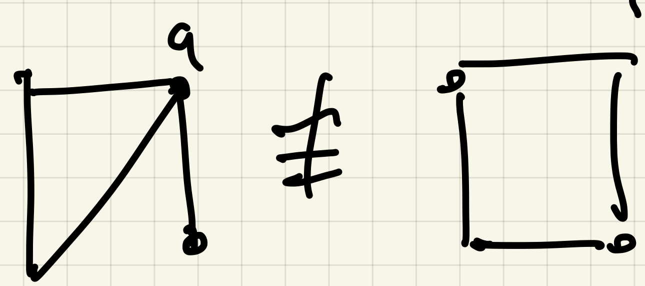
$f(w_1) \ldots f(w_d)$ are ~~deg~~ neighbors of $f(v)$

Cor: $G \cong G' \Rightarrow$ Total degree of G = total degree of G'

Ex:

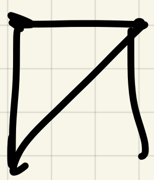
(a) 

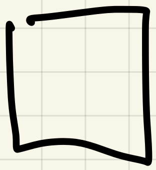
(# vertices different)

(b) G  G'

$\deg a = 3 \neq \deg v$ for any $v \in V_{G'}$

Definition: The degree sequence of graph G is a list of all degrees in non-increasing order:

G  deg seq 3 2 2 1

G'  degree 2 2 2 2