Quiz 4b

For $x \in \mathbb{Z}$, $\underbrace{3 \mid x}_{A} \iff \underbrace{3 \mid 7x}_{B}$

Proof

A⇒B: Let $x \in \mathbb{Z}$ and assume $3 \mid x$. Then there is $c \in \mathbb{Z}$ such that $x = 3c$. Therefore $7x = 7(3x) = 21x = 3(7x)$ and $7x$ is an integer because $x \in \mathbb{Z}$, so $3 \mid 7x$.

B⇒A Let $x \in \mathbb{Z}$ and assume $3 \mid 7x$. Then there is $d \in \mathbb{Z}$ such that $7x = 3d$. Therefore $\underline{x = 7x - 6x} = 3d - 6x = 3(d - 2x)$ and $d - 2x \in \mathbb{Z}$ b/c $d, x \in \mathbb{Z}$, so $3 \mid x$, proving that B ⇒ A

Note could also use $x = 3(5x) - 2(7x)$, many possible proofs here.