

## Quiz 4

$x$  is odd if and only if  $Sx$  is odd.

( $\Rightarrow$ ) Let  $x$  be odd. Then there is an integer  $c$  with  $x = 2c + 1$ .

Therefore  $Sx = S(2c + 1) = 10c + 5 = (10c + 4) + 1 = 2(5c + 2) + 1$  is odd, since  $c$  integer  $\Rightarrow 5c + 2$  integer.

( $\Leftarrow$ ) Let  $Sx$  be odd.

Then there is an integer  $d$  with

$$Sx = 2d + 1, \text{ so}$$

$$x = Sx - 4x = S(2d + 1) - 4x = 10d + 5 - 4x = 2(5d + 2 - 2x) + 1$$

is odd because  $x, d$  integers  $\Rightarrow 5d + 2 - 2x$  is an integer.