Quiz 13

1. $a_0 = 0,\ a_1 = 1,\ a_n = 7a_{n-1} - 12a_{n-2}$

(a) $a_2 = 7a_1 - 12a_0 = 7(1) - 12(0) = 7$

$a_3 = 7a_2 - 12a_1 = 7(7) - 12(1) = 49 - 12 = 37$

(b) <u>Claim</u> $a_n = 4^n - 3^n$ for all $n \geq 0$

<u>Proof</u>: By strong induction on n

<u>Base</u> <u>$n=0$</u> $4^0 - 3^0 = 1 - 1 = 0 = a_0$ ✓

<u>$n=1$</u> $4^1 - 3^1 = 4 - 3 = 1 = a_1$ ✓

<u>Step</u> Assume $a_k = 4^k - 3^k$ for $0 \leq k \leq n$

(NTS $a_{n+1} = 4^{n+1} - 3^{n+1}$)

Then $a_{n+1} = 7a_n - 12a_{n-1} =$

$7(4^n - 3^n) - 12(4^{n-1} - 3^{n-1}) =$

$7 \cdot 4^n - 12 \cdot 4^{n-1} - 7 \cdot 3^n + 12 \cdot 3^{n-1} =$

$(\underbrace{7 \cdot 4}_{28} - 12)4^{n-1} - (\overbrace{7 \cdot 3}^{21} - 12)3^{n-1} =$

$16\, 4^{n-1} - 9 \cdot 3^{n-1} = 4^2 \cdot 4^{n-1} - 9 \cdot 3^{n-1} =$

$4^{n+1} - 3^{n+1}$ ✓