9/9 | Discrete   Quiz 3

1. The difference of two odd numbers is even

If x and y are odd, then $x-y$ is even.

Proof Let x, y be odd.

Then there are c, d integers so that $x = 2c+1$ and $y = 2d+1$,

Therefore $x-y = (2c+1)-(2d+1) =$

$2c+1-2d-1 = 2c-2d =$

$2(c-d)$

Since c, d ints, so $c-d$ int,

so $2(c-d)$ is even.

$x \overset{\shortparallel}{-} y.$

2. If x is pos int then $x^3 - 1$ is composite

(a) $(x-1)(x^2+x+1)$

ex : $\boxed{\begin{matrix} x=1 \\ x=2 \end{matrix}}$ $\Rightarrow x^3 - 1 = 0$

$\Rightarrow x^3 - 1 = 7$ prime

(b) If $x^2 < y^2$ then $x < y$

$x = 1, \; y = -2$

$x^2 < y^2$

$\quad \shortparallel$

$1 < 4$ ✓

Last time Boolean Algebra

Operations $\wedge / \vee / \neg$

variables x, y $\quad$ T or F

$\rightarrow$, $\leftrightarrow$, $\oplus$, $\bar{\wedge}$

logical equivalence

& truth tables

<u>Ex 0</u> $((x \vee y) \wedge (x \vee \neg y)) \wedge \neg x \equiv F$

An expression is a <u>contradiction</u> if it's log. equiv to F

An expression is a <u>tautology</u> if it's log. equiv to T

<u>Ex 1</u> Is $((x \rightarrow y) \wedge (x \rightarrow \neg y)) \rightarrow \neg x$ a tautology, contradiction, or neither

x	y	$x \to y$	$\neg y$	$x \to \neg y$	$(x \to y) \wedge (x \to \neg y)$
T	T	T	F	F	F
T	F	F	T	T	F
F	T	T	F	T	T
F	F	T	T	T	T

x	y	$\neg x$	$((x \to y) \wedge (x \to \neg y)) \to \neg x$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	T

tautology

In fact, truth table shows that

$$((x \to y) \wedge (x \to \neg y)) \leftrightarrow \neg x$$

is also a tautology

Basic properties

Thm 1 ① $x \wedge y = y \wedge x$ and
$x \vee y = y \vee x$ (commutative)

② $(x \wedge y) \wedge z = x \wedge (y \wedge z)$
$(x \vee y) \vee z = x \vee (y \vee z)$
(associative)

③ $x \wedge T = x$ $\quad x \vee F = x$
(identity)

④ $\neg(\neg x) = x$

⑤ $x \wedge x = x$ or $x \vee x = x$

⑥ $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$
$x \vee (y \wedge z) = \boxed{(x \vee y) \wedge (x \vee z)}$
(distributive)

⑦ $x \wedge (\neg x) = F$ $\quad x \vee (\neg x) = T$

⑧ $\neg(x \wedge y) = (\neg x) \vee (\neg y)$
$\neg(x \vee y) = (\neg x) \wedge (\neg y)$

De Morgan's laws

Ex Check ③, ⑧, ②

③

x	T	$x \wedge T$
T	T	T
F	T	F

same ✓

⑧ $\neg(x \vee y) = (\neg x) \wedge (\neg y)$

x	y	$x \vee y$	$\neg(x \vee y)$	$\neg x$	$\neg y$	$(\neg x) \wedge (\neg y)$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

same ✓

② $(x \vee y) \vee z = x \vee (y \vee z)$

x	y	z	$x \vee y$	$(x \vee y) \vee z$	$y \vee z$	$x \vee (y \vee z)$	
T	T	T	T	T	T	T	T
T	T	F			T		
T	F	T			T		
→ T	F	F			F	T	
F	T	T			T	T	
F	T	F			T		
F	F	T	F		T		
→ F	F	F	F	F	F	F	

same ✓

Ex 2

We showed (Ex 0) that

$$((x \vee y) \wedge (x \vee \neg y)) \wedge \neg x = F$$

$$\| \quad 6$$

$$(x \vee (y \wedge \neg y)) \wedge \neg x$$

$$\| \quad 7$$

$$((x \vee F)) \wedge \neg x$$

$$\| \quad 3 \qquad 7$$

$$x \wedge \neg x = F$$

Ex 3

Show with Tand Mat

$$((x \to y) \wedge (x \to \neg y)) \to \neg x$$

is a tautology

$\boxed{x \to y \equiv \neg x \vee y}$ Save

$$(\underbrace{(x \to y)} \wedge (\underbrace{x \to \neg y})) \to \neg x$$

substitute

$$\neg(\underbrace{(\neg x \vee y) \wedge (\neg x \vee \neg y)}) \vee \neg x$$

distrib. 6

$$\neg(\neg x \vee (\underline{y \wedge \neg y})) \vee \neg x$$

7

$$\neg(\underline{\neg x \vee F}) \vee \neg x$$

3

$$\underline{\neg(\neg x)} \vee \neg x$$

4 7

$$x \vee \neg x = T$$

§8 Collections / Sets

A list is an ordered sequence of objects

Notation $(a_1, a_2, \dots a_n)$

a_i objects

n = length of list

Ex $(2, E, Q, 2, \text{Caitlyn Clark})$

Notes repitition OK

Empty list $()$

List of length 2 is called an ordered pair

Exs: $(4,7)$ $(7,4)$

not same

order matters!

<u>Ex)</u> How many ordered pairs (a_1, a_2) are there with integers

(a) $1 \le a_1 \le 10$
$1 \le a_2 \le 20$?

$$10\left\{\underbrace{\begin{matrix}(1,1) & (1,2) & (1,3) & \cdots & (1,20)\\ (2,1) & (2,2) & - & & (2,20)\\ \vdots & & & & \\ (10,1) & (10,2) & - & & (10,20)\end{matrix}}_{20}\right.$$

$$10 \times 20 = 200$$

(b) How many if a_1, a_2 are letters from alphabet?

(A, A) — — (A, Z)

(B, A)

$\vdots$

(Z, A) $26^2 = 676$

(c) Same as (b) but

$a_1 \neq a_2$

All the above except

(A, A) (B, B) — — (Z, Z)

$\underbrace{}_{26}$

$\therefore$ number $26 \cdot 26 - 26$

$= 676 - 26 = 650$

Another way: (a_1, a_2)

$26 \cdot 25 = 650$

Ideas

Multiple Principle:

Consider length 2 lists consisting of (a,b) where

m choices for a

n choices of b for each a

Then there ~~are~~ are $m \times n$ such lists.

Ex2 How many length 2 lists (a,b) are there with

(a) a,b in $\{1,2,3 \ldots - 20\}$

$20 \times 20 = 400$

(b) same as (a), but $a \neq b$

(a,b)

$20 \times 19 = 380$

(c) How many length 3 lists (a,b,c) with a,b,c in $\{1,2,\ldots 20\}$?,

$(a,b,c) = ((a,b),c)$

$400 \times 20 = 8000$

(d) How many with $a \neq b$

$((a,b),c)$

$380 \times 20 = 7600$

(e) (a,b,c) as above $a \neq b$, $b \neq c$

(a,b,c)

$20 \times 19 \times 19 = 7220$

(f) How many with $a \neq b, b \neq c, a \neq c$?

$$(a, b, c)$$

$$20 \times 19 \times 18 = \underline{\underline{6{,}840}}$$

<u>Multiple principle</u>

Consider length n lists consisting of $(a_1, \dots, a_n)$ where

$$\left.\begin{array}{l} m_1 \text{ choices for } a_1 \\ m_2 \text{ choices for } a_2 \\ \vdots \\ m_n \text{ choices for } a_n \end{array}\right\} \Rightarrow$$

$m_1 m_2 \dots m_n$ choices for $(a_1, \dots, a_n)$

(ex) How many license plate numbers are there?

$$\underbrace{\square\ \square\ \square}_{\text{Letter}}\ \underbrace{\square\ \square\ \square\ \square}_{\text{numbers}}$$

$$26 \times 26 \times 26 \times 10 \times 10 \times 10 \times 10$$

$$26^3 \cdot 10^4 =$$