9/4| Discrete

Quiz 2

1. If x, y, z are odd integers

(a) then $x^2+y^2+z^2$ is odd

(b) If x, y, z are Consecutive integers, ~~then~~

$$3 \mid (x+y+z)$$

If x is an integer, then

$$3 \mid (x+(x+1)+(x+2))$$

2. (a) A $x^2 < y^2$

B $0 \le x < y$

$B \Rightarrow A$ ✓ / $A \Rightarrow B$? ✗

<u>No</u>: $y = -2, \quad x = 1$

$A \Leftrightarrow B$ ✗

(b) A. $\quad xy = y$

$B \quad x = 1$ or $y = 0$

$B \Rightarrow A$ ✓

$A \Rightarrow B$?, $\quad xy = y \Rightarrow$

$xy - y = 0$

$y(x-1) = 0 \Rightarrow$

$y = 0$ or $x = 1$ ✓

$A \Rightarrow B$

$\therefore$ also $A \Leftrightarrow B$

<u>Last week</u>

§5 Proofs

§6 counterexamples

§7 Boolean Algebra
(George Boole, 1854)

Applications: { Logic statements
computer circuit design

What is it?

Algebra: { expressions
$y^2 - x^2 = (y-x)(y+x)$
evaluation
$y = 7, x = 5$
operations
$+ / \cdot / - / \div$

Boole's Idea:

Use expressions with operations $\wedge$ / $\vee$ / $\neg$

$\wedge$	$\vee$	$\neg$
and	or	not
conjunction	disjunction	negation

with variables x, y, z - -

and evaluate at ~~$x = T$ $y =$~~

values T = true

F = false

Basic operations

x	y	and $x \wedge y$	or $x \vee y$	$\neg x$	$\neg y$
T	T	T	T	F	F
T	F	F	T	F	T
F	T	F	T	T	F
F	F	F	F	T	T

Evaluate

Ex) $\boxed{(x \wedge y) \vee (\neg x \vee y)}$ ← Boolean expression

at $x = T,\ y = F$

Evaluate

$$\underbrace{(T \wedge F)} \vee (\neg T \vee F)$$

$$F \vee \underbrace{(F \vee F)}$$

$$F \vee F = F$$

Easy!

Can compute all outcomes at once with a table!

x	y	$x \wedge y$	$\neg x$	$\neg x \vee y$	$(x \wedge y) \vee (\neg x \vee y)$
T	T	T	F	T	T
T	F	F	F	F	F
F	T	F	T	T	T
F	F	F	T	T	T

Defn Two expressions are **logically equivalent** if they have the same outputs in truth table.

Ex2 $(x \wedge y) \vee (\neg x \wedge y)$ is logically equivalent to $\neg x \vee y$

Why?

x	y	$\neg x$	$\neg x \vee y$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

Same

Notation

$$(x \wedge y) \vee (\neg x \vee y) \equiv ?x \vee y$$

Common non-basic operations

(a) $x \rightarrow y$ (models if-then)

(b) $x \leftrightarrow y$ (models if and only if)

(c) $x \oplus y$ exclusive or

$x \veebar y$

(d) $x \bar{\wedge} y$ not and

x nand y (csci)

Table

x	y	$x \rightarrow y$	$x \leftrightarrow y$	$x \oplus y$	$x \bar{\wedge} y$
T	T	T	T	F	F
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	T	F	T

Ex3 (a) $(x \to y) \equiv (\neg x \vee y)$

(b) $(x \leftrightarrow y) \equiv \neg(x \oplus y)$

(c) $(x \leftrightarrow y) \equiv (x \to y) \wedge (y \to x)$

Hope true;

Check

x	y	$x \to y$	$y \to x$	$(x \to y) \wedge (y \to x)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Ex4 Is $(x \to y) \wedge (y \to z) \equiv (x \to z)$?

More intense,

3 variables

x	y	z	$x \to y$	$y \to z$	$(x\to y)\wedge(y \to z)$	$x \to z$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	F	T
T	F	F	F	T	F	F
F	T	T	T	T	T	
F	T	F	T	F	F	
F	F	T	T	T	T	
F	F	F	T	T	T	

000 001 010 011 100 101 110 111

binary 0–7

not same ,

$\therefore$ not logically equivalent

Ex 5 Find a simple expression logically equivalent to

$$((x \vee y) \wedge (x \vee \neg y)) \wedge \neg x$$

x	y	$x \vee y$	$\neg y$	$x \vee \neg y$	$(x \vee y) \wedge (x \vee \neg y)$	$\neg x$
T	T	T	F	T	T	F
T	F	T	T	T	T	F
F	T	T	F	F	F	T
F	F	F	T	T	F	T

F
F
F
F

F

$\equiv$ Simple