9/30 Discrete ?

Exam 1

avg 81%
med 81%

150
90% 135
80% 120
105

$$\frac{6}{\;}\qquad\frac{3}{\;}\qquad\frac{4}{\;}\qquad\frac{4}{\;}$$

Comments:

$\boxed{\#1}$ a divides b if

there exists an integer c

$\exists$

with $b = ac$

$\boxed{\#2}$ (e) $B \to A$

$\boxed{x^2 = 0} \to x = 4$

vacuous truth (§5)

$\boxed{\#3}$ #2(a) ✓

#2(b) $\boxed{x \leftrightarrow y}$

$\boxed{(x \wedge y) \vee (\neg x \wedge \neg y)}$

T
F
F
T

$\boxed{\#4}$ If $\underbrace{x, y \text{ even}}_{A}$ then $\underbrace{4 \mid xy}_{B}$

Proof: Let x, y be even

Then $\exists\, c, d \in \mathbb{Z}$; $x = 2c$
$y = 2d$

$xy = (2c)(2d) = 4cd$

$c, d \in \mathbb{Z} \Rightarrow cd \in \mathbb{Z}$, so

$4 \mid xy = 4cd$
by defn

$\boxed{\#6}$ (b) $\boxed{n = 0}$

$\boxed{n=1}$

#7

a) 26^5

b) 25^5

c) $25^4 \cdot 5$

_ _ _ _ _

place rest   place H

d) # with <u>not</u> at least one H 25^5 (b)

so $26^5 - 25^5$

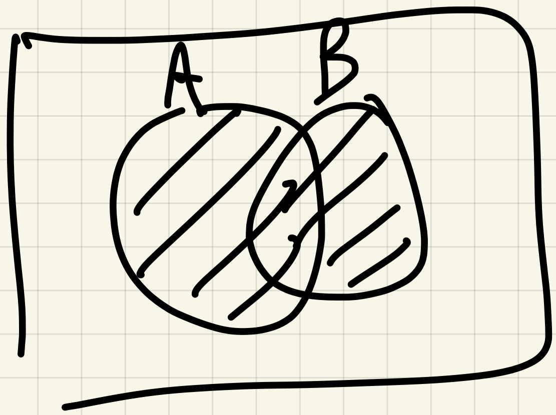
$$26_5 = \frac{26!}{21!}$$

8

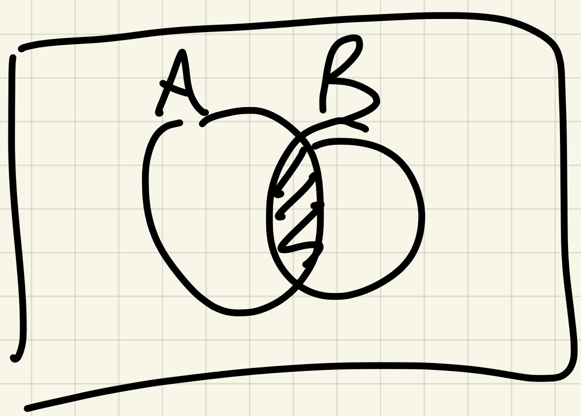
$$\prod_{k=1}^{3} \frac{(2k+1)}{(2k-1)} = \frac{3}{1} \cdot \frac{5}{\cancel{3}} \cdot \frac{7}{\cancel{5}} = 7$$

Last time: A, B sets, can form

Union $A \cup B$

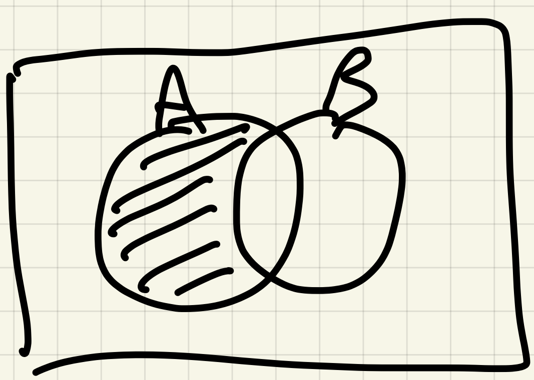


Intersection $A \cap B$

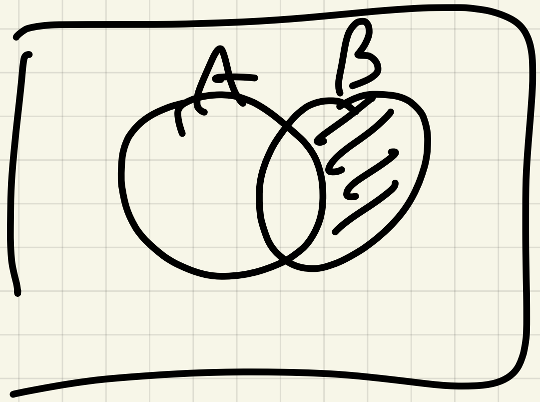


Set difference

$A - B$



$B - A$

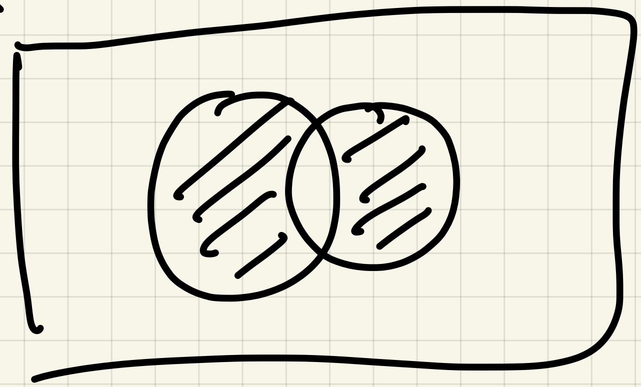


Symmetric difference

$A \triangle B$

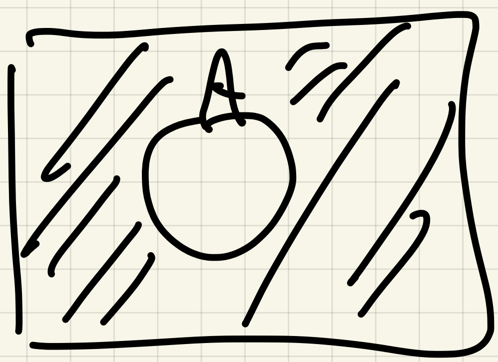
$\|$

$(A - B) \cup (B - A)$



Complement: $\{x : x \notin A\}$

$\bar{A}$



Need context

$A = \{a, b, c\}$

$\bar{A} = \{d, e, f, \ldots z\}$

$= \{d, e, \ldots \; z, A, \ldots \; Z\}$

Need a <u>universe</u> U for context:

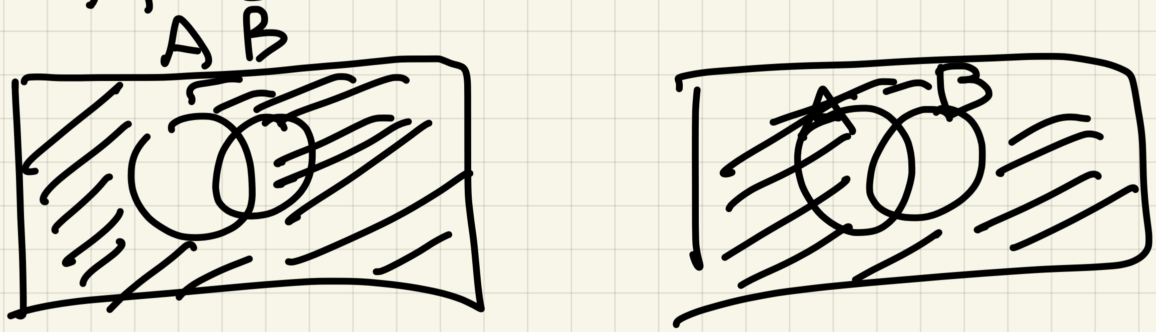
If $U = \{a, \ldots \; z\}$

then $\bar{A} = \{d, e, \ldots \; z\}$

<u>Note</u>: $\bar{A} = U - A$

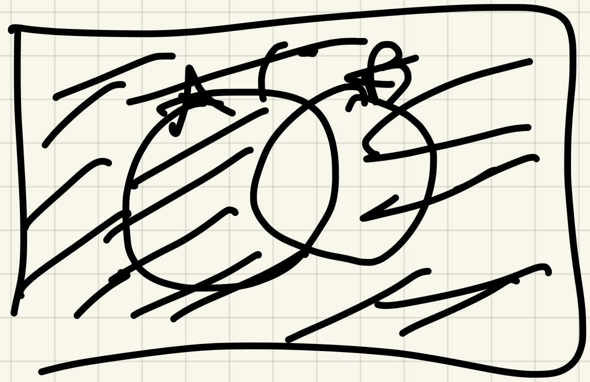
<u>Ex</u> Venn diagram suggests

$\bar{A} \cup \bar{B} = ??$



$\overline{A}$ $\overline{B}$

$\overline{A} \cup \overline{B} =$



$\uparrow$

$\overline{A \cap B}$

Resembles De Morgan's law:

$$\neg(x \wedge y) = \neg x \vee \neg y$$

Proof Claim: $\overline{A} \cup \overline{B} = \overline{A \wedge B}$

$$x \in \overline{A} \cup \overline{B} \iff \underline{x \in \overline{A}} \vee x \in \overline{B}$$

$$\neg(x \in A) \vee \neg(x \in B)$$

$\parallel$ Ch 7 De Morgan

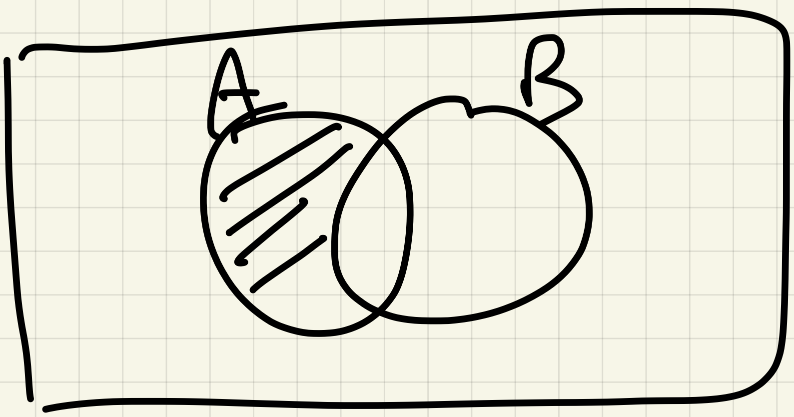
$$\neg(x \in A \wedge x \in B)$$

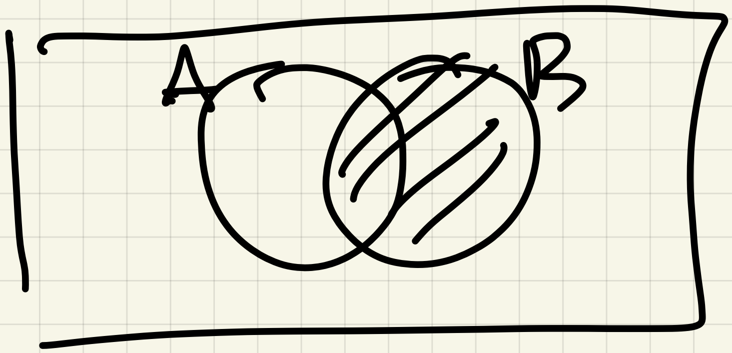
$$\neg(x \in A \cap B) \iff x \in \overline{A \cap B}$$

Ex2 Prove or disprove

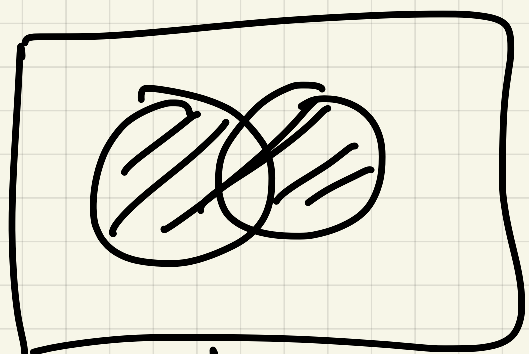
$(A - B) \cup B = A$????

Venn diagram:

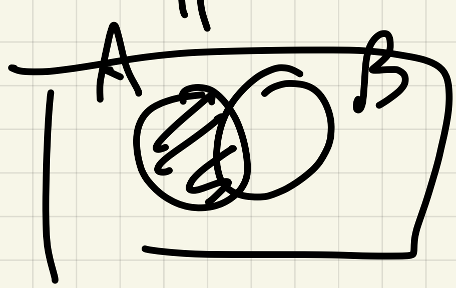




$(A - B) \cup B =$



Looks false:



Counterexample:

Any A, B with $B - A \neq \emptyset$

i.e.

$$A = \{1, 2\}$$
$$B = \{1, 3\}$$

$$\rightarrow B - A = \{3\}$$

$$(A - B) \cup B = \{1, 2, 3\}$$
$$\neq \quad \checkmark$$
$$A = \{1, 2\}$$

(b) Can correct false statement:

$$(A - B) \cup B = A \cup B$$

Cartesian product:

The Cartesian product of sets A and B is

Set of ordered pairs (a,b) with $a \in A$ and $b \in B$

$$\parallel$$

$$\{(a,b) : a \in A \wedge b \in B\}$$

$$\parallel$$

Notation: $A \times B$

Ex) $A = \{a,b\}$

$B = \{1,2,3\}$

(a) $A \times B = \{(a,1), (a,2), (a,3)$
$(b,1)\ (b,2), (b,3)\}$

(b) $A \times A = \{(a,a), (a,b), (b,a), (b,b)\}$

(c) $B \times A = \{(1,a), (1,b), (2,a), (2,b), (3,a)\ (3,b)\}$

<u>Ex 2</u> $A = \{\spadesuit, \diamondsuit, \clubsuit, \heartsuit\}$

$B = \{A, 2, 3, 4, \ldots\ 9, 10, J, Q, K\}$

$A \times B = \{(\spadesuit, A), (\spadesuit, 2) \ldots\}$

$\quad \parallel$

Deck of cards

<u>Thm.</u> $|A \times B| = |A| \times |B|$

(Multipl. principle)

<u>Ex 3</u>

$$L = \{A, B, C, \ldots \quad Z\}$$

$$N = \{0, 1, \quad 9\}$$

$$|L \times L| = 26^2$$

$$L \times L \times L \times N \times N \times N \times N$$

set of license plate numbers

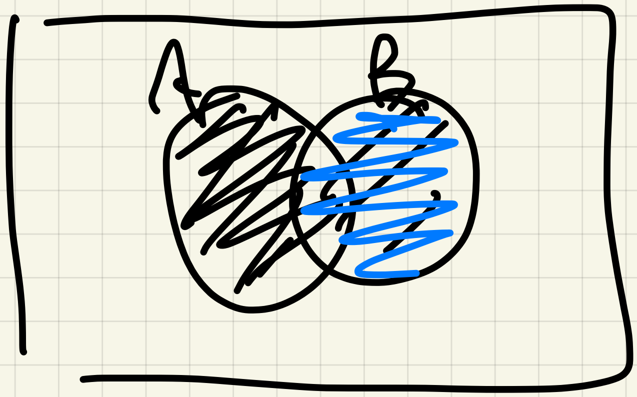
$$|L \times L \times L \times N \times N \times N \times N| =$$

$$\underbrace{|L| \times |L| \times |L|}_{26^3} \times |N| \times |N| \times |N| \times |N$$

$$26^3 \times 10^4$$

<u>Cardinality of union:</u>

$$|A \cup B| = |A| + |B|$$



$$|A \cup B| = |A| + |B| - |A \cap B|$$

<u>Inclusion - exclusion formula</u>.

Universe = deck of cards

<u>Ex 4</u> $A = \{$face cards$\}$

$B = \{$red suit cards$\}$

$|A| = 12$ $|B| = 26$

$|A \cap B| = 6$

$|A \cup B| = \{$cards that are face or red suit$\}$

$= |A| + |B| - |A \cap B|$

$= 12 + 26 - 6 = 32$

$C = A - B = \{$black suit face cards$\}$ =

$D = B - A = \{$red suit cards not face cards$\}$

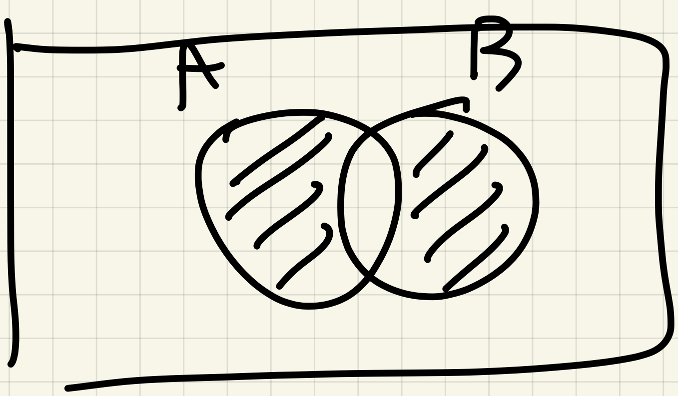
$|C| = 6 \qquad |D| = 20$

$|C \cup D| = |A \Delta B| =$

$= |C| + |D| - |C \cap D|$

$= 6 + 20 - 0 = 26$

<u>Definition</u>: A, B are <u>disjoint</u> if $A \cap B = \phi$



<u>More</u>

<u>Counting</u>

Jump $\rightarrow$ §17, 19

Summary of counting / cardinality

(a) $\left| \text{length } k \text{ list taken from } n \text{ objects} \right| = n^k$

(b) $\left| \text{length } k \text{ lists taken from } n \text{ objects no repetition} \right| =$

$$n_k = \frac{n!}{(n-k)!} = n(n-1)\ldots(n-k+1)$$

(c) $|A \times B| = |A| \times |B|$

(d) $|P(A)| = |2^A| = 2^{|A|}$

(e) $|A \cup B| = |A| + |B| - |A \cap B|$