9/23/ Discrete:

Exam 1 Wednesday

Review sheet

Covers §3-9

Quiz 6:

$$257 \quad \boxed{\underset{10}{\_}\ \underset{10}{\_}\ \underset{10}{\_}\ \underset{10}{\_}}$$

0-9

a) All possible 10^4

b) $6\ \underset{10}{\_}\ \underset{10}{\_}\ \underset{10}{\_}$ $\quad 10^3$

c) No repetition

$\underset{10}{\_}\ \underset{9}{\_}\ \underset{8}{\_}\ \underset{7}{\_}$

$$10 \cdot 9 \cdot 8 \cdot 7 = \frac{10!}{6!} = 10_4$$

(d)

$$\overline{10}\ \overline{9}\cdot\overline{9}\cdot\overline{9} = 10\cdot 9^3$$

2. $$\prod_{k=1}^{n}\frac{k}{k+1}$$ $\underline{\underline{n=1,2,3}}$

(a)

$\underline{n=1}$ $$\prod_{k=1}^{1}\frac{k}{k+1} = \frac{1}{1+1} = \frac{1}{2}$$

$\underline{n=2}$ $$\prod_{k=1}^{2}\frac{k}{k+1} = \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3} = \frac{1}{3}$$

$\underline{n=3}$ $$\prod_{k=1}^{3}\frac{k}{k+1} = \frac{1}{\cancel{2}}\ \frac{\cancel{2}}{\cancel{3}}\ \frac{\cancel{3}}{4} = \frac{1}{4}$$

$k=1$ $k=2$ $k=3$

(b) $$\prod_{k=1}^{n}\frac{k}{k+1} = \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{\cancel{3}}\ \frac{3}{4}\cdots\frac{K}{n+1}$$

$$\frac{1}{n+1}$$

Last time: § 1.1

important

$\exists$ there exists

$\forall$ for all

quantifiers

$\ni$: such that

$\therefore$ therefore

$\because$ since / because b/c

Negations:

$$\neg(\exists x \; P(x)) \equiv \forall x \; \neg P(x)$$

$$\neg(\forall x \; P(x)) \equiv \exists x \; \neg P(x)$$

More examples

Ex1 Negate and assess truth

(a) $\exists x \in \mathbb{N}\ \exists y \in \mathbb{N}\ y > x$

Read: $\exists x \in \mathbb{N} (\exists y \in \mathbb{N} : y > x)$

(true: $x = 5, y = 10$)

Negate:

$\neg(\exists x \in \mathbb{N} (\exists y \in \mathbb{N}\ y > x))$

$\forall x \in \mathbb{N}\ \neg(\exists y \in \mathbb{N}\ y > x)$

$\forall x \in \mathbb{N}\ \forall y \in \mathbb{N}\ \boxed{\neg\, y > x}$

$\forall x \in \mathbb{N}\ \forall y \in \mathbb{N}\ y \leq x$

(b) $\exists x \in \mathbb{N}\ \forall y \in \mathbb{N}\ x < y$

false

$\neg(\exists x \in \mathbb{N}\ \forall y \in \mathbb{N}\ x < y)$

$\forall x \in \mathbb{N}\ \neg(\forall y \in \mathbb{N}\ x < y)$

$\forall x \in \mathbb{N}\ \exists y \in \mathbb{N}\ \underline{\neg(x < y)}$

$\forall x \in \mathbb{N}\ \exists y \in \mathbb{N}\ \ x \geq y$

<u>true</u>

(c) $\forall x \in \mathbb{N}\ \exists y \in \mathbb{N}\ x < y$ T

$\neg(\forall x \in \mathbb{N}\ \exists y \in \mathbb{N}\ x < y)$

$\exists x \in \mathbb{N}\ \forall y \in \mathbb{N}\ x \geq y$ F

(d) $\forall x \in \mathbb{N}\ \forall y \in \mathbb{N}\ \ x < y$ F

$\neg \forall x \in \mathbb{N}\ \ \forall y \in \mathbb{N}\ x < y$

$\exists x \in \mathbb{N}\ \exists y \in \mathbb{N}\ \ y \geq x$ T

(e) $\forall x \in \mathbb{N}\ \underbrace{\exists y \in \mathbb{N}\ \ x > y}_{T}$, F

take $x = 0$, false

$\neg(\forall x \in \mathbb{N}\ \exists y \in \mathbb{N}\ x > y)$

$\exists x \in \mathbb{N}\ \forall y \in \mathbb{N}\ x \leq y$ T

T $x = 0$

Ex2 Calc I - Real

$\lim_{x \to c} f(x) = L$ How close

"We can f(x) arbitrarily close

to L by choosing

x close enough to c"

not $x \neq c$

How close

$\forall \varepsilon > 0\ \exists \delta > 0\ \forall x\ 0 < |x - c| < \delta \Rightarrow$

$|f(x) - L| < \varepsilon$

Negation

$$\neg\, \underline{\forall \varepsilon>0}\ \exists \delta>0 : \forall x\in\mathbb{R}\ \ 0<|x-c|<\delta \Rightarrow |f(x)-L|<\varepsilon$$

$$\exists \varepsilon>0\ \forall \delta>0\ \exists x\in\mathbb{R}$$

$$\underbrace{\neg\big(\underbrace{0<|x-c|<\delta}_{A} \Rightarrow \underbrace{|f(x)-L|<\varepsilon}_{B}\big)}_{}$$

$$A \Rightarrow B$$

$$0<|x-c|<\delta \ \wedge\ |f(x)-L|\geq\varepsilon$$

§12 Set operations

If A, B are sets

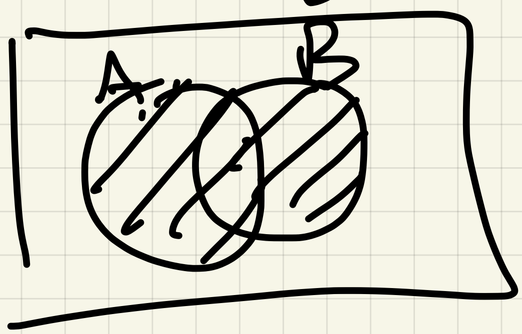
$$A\cup B = \{x : x\in A \text{ or } x\in B\}$$

__union__

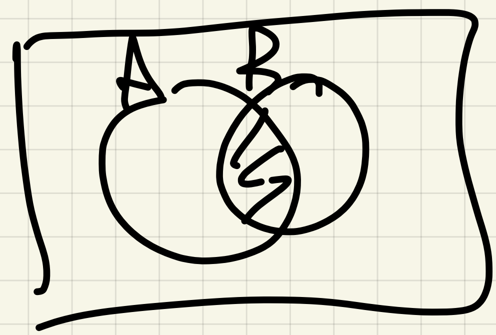
$$A\cap B = \{x : x\in A \text{ and } x\in B\}$$

__intersection__

Venn diagrams



$A \cup B$



$A \cap B$

Ex Playing cards

A = set of face cards

B = cards with red suit ♡ ◇

$|A| = 12$ $|B| = 26$

$|A \cap B| = 6$

$|A \cup B| = 32$

Properties:

1. $A \cup B = B \cup A$, $A \cap B = B \cap A$
2. $A \cup (B \cup C) = (A \cup B) \cup C$

$$A \cap (B \cap C) = (A \cap B) \cap C$$

③ $A \cup \phi = A \qquad A \cap \phi = \phi$

④ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Very similar to properties from Boolean algebra (Thm 2.2)

$\cup / \cap \qquad \vee / \wedge$

Ex Prove that

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Normally, prove $\boxed{\text{LHS} \subseteq \text{RHS}}$ and $\text{RHS} \subseteq \text{LHS}$

Let $x \in A \cap (B \cup C)$.

$x \in A \cap (B \cup C) \iff$

$x \in A \wedge \underbrace{x \in (B \cup C)}_{} \iff$

$\underline{\underline{x \in A}} \wedge (\underline{\underline{x \in B}} \vee \underline{\underline{x \in C}})$

$a \wedge (b \vee c)$

$\Updownarrow$ Thm 7.2

$(a \wedge b) \vee (a \wedge c)$

$(\underbrace{x \in A \wedge x \in B}_{}) \vee (x \in A \wedge x \in C)$

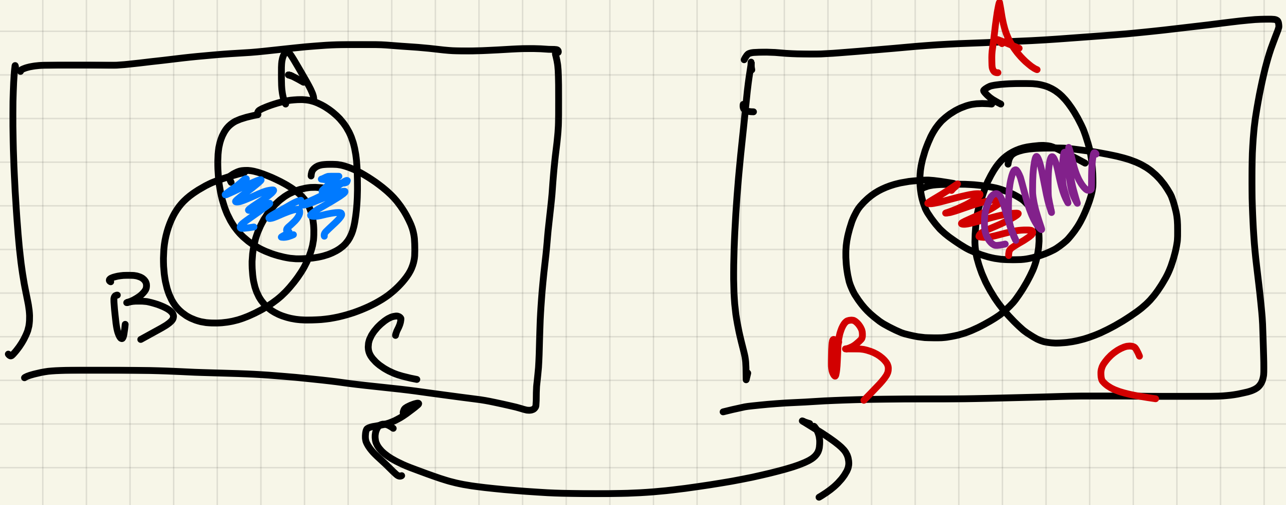
$(x \in A \cap B) \vee (x \in A \cap C) \iff$

$x \in (A \cap B) \cup (A \cap C)$

<u>Can visualize with</u>

<u>Venn diagrams</u>

$A \cap (B \cup C)$ $\qquad$ $\underline{\underline{(A \cap B)}} \cup \underline{\underline{(A \cap C)}}$



same shaded region

<u>Defn</u>: Let A, B be sets.

The <u>set difference</u>

$$A - B = \{x : x \in A \wedge x \notin B\}$$

<u>Symmetric difference</u>

$$A \,\Delta\, B = (A - B) \cup (B - A)$$

<u>Ex</u> In a deck of cards

A = face cards

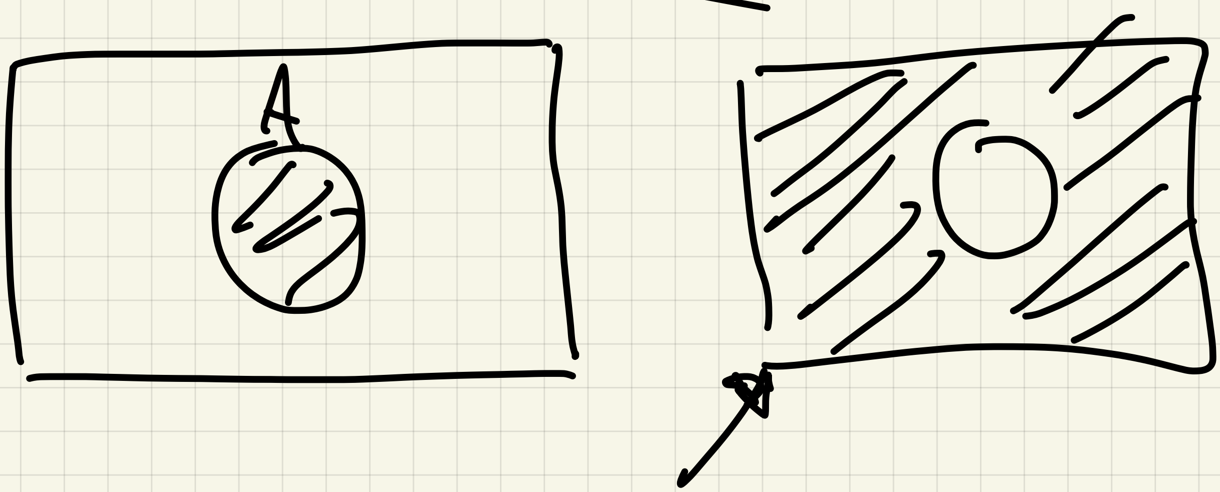
B = red suit cards

$$A - B = \{\underbrace{J, Q, K}_{\spadesuit}, \underbrace{J, Q, K}_{\clubsuit}\}$$

$B - A$ = red cards not face cards

$$|B - A| = 20$$

$$(A \,\Delta\, B) = (A - B) \cup (B - A)$$

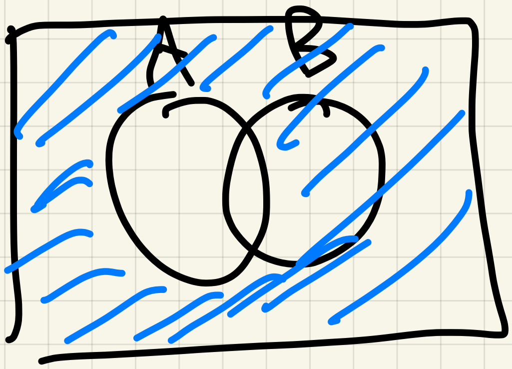


Complement of A =

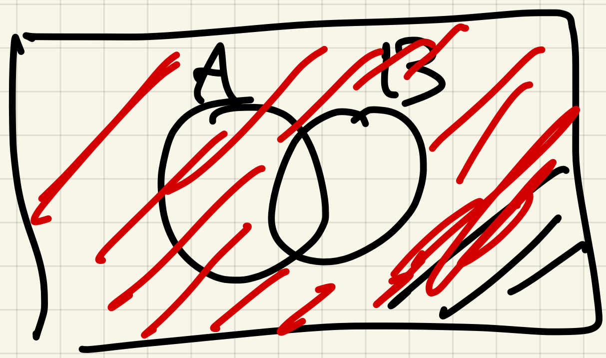
$$\{x : x \notin A\} = \bar{A}$$

Use Venn diagram to

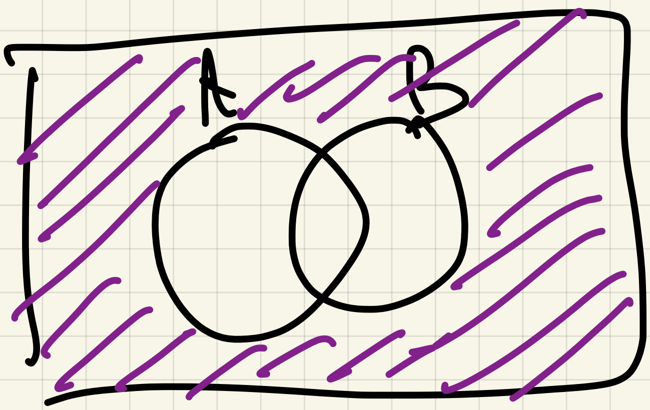
Describe $\bar{A} \cap \bar{B}$



$\bar{A}$



$\bar{B}$



$$\bar{A} \cap \bar{B} = \overline{(A \cup B)}$$

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