

9/18 Discrete:

(Exam) → Sept, 25

Next Wednesday

Review sheet

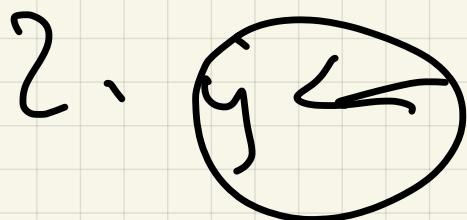
Quiz 5

avg 87%

med 90%

x	y	$x \wedge y$	$y \rightarrow x$	$\neg(y \rightarrow x)$	$(x \wedge y) \vee \neg(y \rightarrow x)$
T	T	T	T	F	T
T	F	F	F	F	F
F	T	F	T	T	T
F	F	F	T	F	F

$$(x \wedge y) \vee \neg(y \rightarrow x) \equiv y$$



Insight: From properties:

$$\begin{aligned}
 & \text{Defining } \neg x = \underline{\gamma(\neg x)} \quad \text{defn} \\
 & (x \wedge y) \vee \gamma(\underline{\gamma(\neg x)}) \equiv (x \wedge y) \vee \gamma(\underline{\gamma y \vee x}) \\
 & \equiv (x \wedge y) \vee (\gamma(\gamma y) \wedge \gamma x) = \\
 & (x \wedge y) \vee (y \wedge \gamma x) = \\
 & (\gamma \wedge x) \vee (\gamma \wedge \gamma x) = \text{defn} \\
 & \gamma \wedge (\underbrace{x \vee \gamma x}) \\
 & \gamma \wedge T = \textcircled{Y}
 \end{aligned}$$

Let time: Sets S
Containity $|S|$

Set builder notations

$$\begin{array}{ll}
 x \in A & (x \notin A) \\
 A \subseteq B & \left(\begin{array}{l} A \subset B \text{ means} \\ A \subseteq B \text{ but} \\ A \neq B \end{array} \right) \\
 A \approx B
 \end{array}$$

Ex 0 We listed all subsets

of $A = \{a, b, c, d\}$

$\emptyset = \{\}, \{a\}, \{b\}, \{c\}, \{d\},$

$\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\},$

$\{c, d\}$

$\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$

$\{a, b, c, d\}$

Defn: If A is a set,

the power set of A

is the set of all subsets

of A ,

Notation $P(A)$ (test: 2^A)

$$\underline{\text{Ex}} \quad A = \emptyset \quad P(A) = \{\emptyset\}$$

$$(a) \quad |P(A)| = 1$$

$$(b) \quad A = \{a\}, \quad P(A) = \{\{a\}, \emptyset\}$$

$$(c) \quad A = \{a, b\}$$

$$P(A) = \{\{a, b\}, \{a\}, \{b\}, \emptyset\}$$

$$\underline{\text{Pattern}} \quad |P(A)| = 2^{|A|}$$

Why?

It gives a way to label
the subsets of

$$A = \{a_1, a_2, \dots, a_n\}$$

For $S \subseteq A$, will label S
by string of 0s & 1's

b_1 . . . b_n

$$b_k = \begin{cases} 0 & a_k \notin S \\ 1 & a_k \in S \end{cases}$$

Ex

$$A = \{a_1, a_2, a_3\}$$

subset	label
\emptyset	0 0 0
$\{a_1\}$	1 0 0
$\{a_2\}$	0 1 0
$\{a_3\}$	0 0 1
$\{a_1, a_2\}$	1 1 0
$\{a_1, a_3\}$	1 0 1
$\{a_2, a_3\}$	0 1 1
$\{a_1, a_2, a_3\}$	1 1 1

There's a 1-1 correspondence

$\{ \text{subsets} \} \hookrightarrow \{ \text{bit strings of length } n \}$

↑
count
 2^r by
multiplication
principle

Ex2

T/F

$$A = \{1, 2, 3, 4, 5\}$$

(a) $2 \in A$ T

(b) $\{2\} \in A$ F

(c) $\emptyset \in A$ F

(d) $\emptyset \subseteq A$ T

(e) $\emptyset \in P(A)$ T

(f) $\{2\} \in P(A)$ T

(g) $\{2\} \subseteq P(A)$ F

(h) $\{\{2\}\} \subseteq P(A)$ T

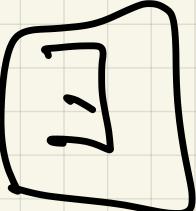
$$(i) \quad \{1, \{1, 2\}\} \subseteq A \quad F$$

$$(ii) \quad \{\{1, \{1, 2\}\}\} \subseteq P(A) \quad F$$

$$(iii) \quad \{\{\{1\}, \{1, 2\}\}\} \subseteq P^2(A) \quad T$$

|| Quantifiers

Defn Existential quantifier

is  it means "There ;,
exists"

Ex $a \in \mathbb{P}$ is odd if

Ex $\exists c \in \mathbb{Z} \text{ such that } a = 2c + 1$

(b) $x \in \mathbb{Z}$ is composite if

$\exists y \in \mathbb{Z} : 1 < y < x \text{ and } y \mid x$

61 $a \mid b$ if $\exists c \in \mathbb{Z} : b = ac$

Ex2 $\exists n \in \mathbb{N} : n^2 = 169$

true : $n = 13$

$$13^2 = 169 \checkmark$$

Ex3 $\exists p \in \mathbb{N} : p > 1000$ and
 p is prime

T/F?

$$p = 1001 = 7 \cdot 11 \cdot 13 \quad \text{not}$$

$$p = 1002$$

$$p = 1003 = 17 \cdot 59$$

$$\begin{array}{c} p = 1009 \\ 1005 \end{array}$$

$p = 1009$ is prime

$\{1, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31\}$
to not divide 1009

Ex 3 $\exists n \in \mathbb{N}$: $2|n$ and $3|n$

and $4|n$. --- and $10|n$

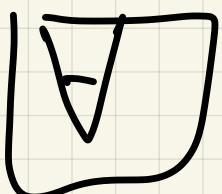
Pruf T: consider $n = \frac{10!}{2}$

smallest root n is

$$n = 8 \cdot 5 \cdot 9 \cdot 7 = 2620$$

Def The universal quantifier

is



means

"for every"
or "for all"

Ex 5 $p \in \mathbb{P}$ is prime if

$p > 1$ and $\forall d > 0$

$d|p \Rightarrow d=1$ or $d=p$

Ex 6 $\forall x, y \in \mathbb{Z}$, x odd and A

y even \Rightarrow

$x+y$ odd B

Proof: Let x be odd and
 y be even.

Then $\exists c \in \mathbb{Z} : x = 2c + 1$

and $\exists d \in \mathbb{Z} : y = 2d$

Therefore

$$\begin{aligned} x+y &= \\ (2c+1) + 2d &= \\ 2(c+d) + 1 & \xrightarrow{\text{b/c}} \\ c+d \in \mathbb{Z} \quad \text{b/c} & \quad c, d \in \mathbb{Z} \end{aligned}$$

$\therefore x+y$ is odd.

\therefore Therefore

\therefore because / since

Negating quantifiers

[Ex] A: There's a man with
three cars

Symbolically :

A: \exists man ; man has 3 cars

$\neg A$: There's no man w-h
3 cars

also / \forall each man m ,
 m does not have 3 cars

This holds in general:

If $P(x)$ is a condition on x

$[P(m) : m \text{ has 3 cars}]$

$\exists x P(x)$

A

$$\forall x (\neg P(x)) \quad \neg A$$

Punchline : $\neg(\exists x P(x)) \equiv \forall x (\neg P(x))$

Similar story with universal quantifier

$$\neg(\forall x P(x)) \equiv \exists x (\neg P(x))$$

Ex 8 negate:

A: $\forall x \in \mathbb{Z} \ x \text{ is odd}$ P(x) F

$\neg A$:

$$\neg(\forall x \in \mathbb{Z} x \text{ odd}) \equiv T$$

$$\exists x \in \mathbb{Z} \ \neg(x \text{ odd})$$

Ex 9

$\forall x \in \mathbb{Z}$ x is odd or x is even

$\forall x \in \mathbb{Z}$ x odd \vee x even (true)

$\neg (\forall x \in \mathbb{Z}) (x \text{ odd} \vee x \text{ even})$

$\exists x \in \mathbb{Z} \neg (x \text{ odd} \vee x \text{ even})$

$\exists x \in \mathbb{Z} x \text{ not odd} \wedge x \text{ not even}$

$\exists x \in \mathbb{Z} \neg(x \text{ odd}) \wedge \neg(x \text{ even})$