9/18 Discrete:

Exam → Sept. 25
Next Wednesday

Review sheet

Quiz 5 avg 87%
med 90%

x	y	$x \wedge y$	$y \to x$	$\neg(y \to x)$	$(x \wedge y) \vee \neg(y \to x)$
T	T	T	T	F	T
T	F	F	T	F	F
F	T	F	F	T	T
F	F	F	T	F	F

$$(x \wedge y) \vee \neg(y \to x) \equiv y$$

2. y

Insight: From properties:

$$(x \wedge y) \vee \neg(y \to x) \overset{\text{defn}}{\equiv} (x \wedge y) \vee \underline{\neg(\neg y \vee x)}$$

DeMorgan

$$= (x \wedge y) \vee (\neg(\neg y) \wedge \neg x) =$$

$$(x \wedge y) \vee (y \wedge \neg x) =$$

$$(y \wedge x) \vee (y \wedge \neg x) = \text{distrib}$$

$$y \wedge (\underbrace{x \vee \neg x})$$

$$y \wedge T = y$$

<u>Last time:</u> Sets S

Cardinality $|S|$

Set builder notation

$x \in A$ $\quad (x \notin A)$

$A \subseteq B$ $\quad (A \subset B$ means $A \subseteq B$ but $A \neq B)$

$A = B$

Ex0 We listed all subsets of $A = \{a, b, c, d\}$

$\phi = \{\}, \{a\}, \{b\}, \{c\}, \{d\},$

$\{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\},$

$\{c,d\}$

$\{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}$

$\{a,b,c,d\}$

Defn: If A is a set, the <u>power set of A</u> is the set of all subsets of A,

Notation $P(A)$ (<u>text</u>: 2^A)

Ex 1 $A = \phi$ $P(A) = \{\phi\}$

(a) $|P(A)| = 1$

(b) $A = \{a\}$, $P(A) = \{\{a\}, \phi\}$

(c) $A = \{a, b\}$

$P(A) = \{\{a,b\}, \{a\}, \{b\}, \phi\}$

Pattern $|P(A)| = 2^{|A|}$

Why?

Here's a way to label the subsets of

$A = \{a_1, a_2, \ldots, a_n\}$

For $S \subseteq A$, I'll label S by string of 0s & 1's

$b_1 \ldots \quad b_n$

$$b_k = \begin{cases} 0 & a_k \notin S \\ 1 & a_k \in S \end{cases}$$

Ex1 $\quad A = \{a_1, a_2, a_3\}$

subset	label
$\emptyset$	000
$\{a_1\}$	100
$\{a_2\}$	010
$\{a_3\}$	001
$\{a_1, a_2\}$	110
$\{a_1, a_3\}$	101
$\{a_2 a_3\}$	011
$\{a_1, a_2, a_3\}$	111

There's a 1-1 corresponce

$$\left\{ \begin{array}{c} \text{subsets} \\ \text{of } A \end{array} \right\} \longleftrightarrow \left\{ \begin{array}{c} \text{bit strings} \\ \text{of length } n \end{array} \right\}$$

count 2^n by multiplication principle

<u>Ex 2</u> T/F

$A = \{1, 2, 3, 4, 5\}$

(a) $2 \in A$ T

(b) $\{2\} \in A$ F

(c) $\phi \in A$ F

(d) $\phi \subseteq A$ T

(e) $\phi \in P(A)$ T

(f) $\{2\} \in P(A)$ T

(g) $\{2\} \subseteq P(A)$ F

(h) $\{\{2\}\} \subseteq P(A)$ T

(i) $\{1, \{1,2\}\} \subseteq A$ F

(j) $\{1, \{1,2\}\} \subseteq P(A)$ F

(k) $\{\{1\}, \{1,2\}\} \subseteq P(A)$ T

§11 Quantifiers

Defn Existential quantifier

is $\boxed{\exists}$ it means "There exists"

Ex1 $a \in \mathbb{Z}$ is odd if

(a) $\exists c \in \mathbb{Z}$ (i) $a = 2c+1$

such that

(b) $x \in \mathbb{Z}$ is composite if

$\exists y \in \mathbb{Z} : 1 < y < x$ and $y \mid x$

Ex1 $a|b$ if $\exists c \in \mathbb{Z} : b = ac$

Ex2 $\exists n \in \mathbb{N} : n^2 = 169$

true : $n = 13$

$13^2 = 169$ ✓

Ex3 $\exists p \in \mathbb{N} : p > 1000$ and p is prime

T/F?

$p = 1001 = 7 \cdot 11 \cdot 13$ not

$p = 1002$

$p = 1003 = 17 \cdot 59$

$p = 1004$

1005

⋮

$p = 1009$ is prime

$\{1, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31\}$

do not divide 1009

Ex 4 $\exists n \in \mathbb{N} : 2 | n$ and $3 | n$ and $4 | n \ldots$ and $10 | n$

proof T: consider $n = 10!$

smallest such n is

$$n = 8 \cdot 5 \cdot 9 \cdot 7 = 2520$$

Defn The universal quantifier is $\boxed{\forall}$ means "for every" or "for all"

Ex 5 $p \in \mathbb{Z}$ is prime if

$p > 1$ and $\forall d > 0$

$d | p \Rightarrow d = 1$ or $d = p$

Ex 6 $\forall x, y \in \mathbb{Z}$, $\underbrace{x \text{ odd and } y \text{ even}}_{A} \Rightarrow \underbrace{x + y \text{ odd}}_{B}$

Proof: Let x be odd and y be even.

Then $\exists c \in \mathbb{Z} : x = 2c+1$

and $\exists d \in \mathbb{Z} : y = 2d$

Therefore $x + y =$

$(2c+1) + 2d =$

$\therefore$

$\underline{2(c+d)+1}$ b/c

$c+d \in \mathbb{Z}$ b/c $c, d \in \mathbb{Z}$

$\therefore$ $x+y$ is odd.

$\therefore$ therefore

$\because$ = because / since

Negating quantifiers

Ex] A: There's a man with three ears

Symbolically:

A: $\exists$ man : man has 3 ears

$\neg A$: There's no man with 3 ears

also / $\boxed{\text{for each man } m, \; m \text{ does not have 3 ears}}$

This holds in general:

If $P(x)$ is a condition on x

[$P(m)$: m has 3 ears]

$\exists x\, P(x)$ A

$\forall x (\neg P(x))$ $\quad\quad\quad$ $\neg A$

Punchline: $\neg(\exists x\, P(x)) \equiv \forall x (\neg P(x))$

Similar story with universal quantifier

$$\neg(\forall x\, P(x)) \equiv \exists x (\neg P(x))$$

Ex 8 negate:

A: $\forall x \in \mathbb{Z}$ $\underbrace{x \text{ is odd}}_{P(x)}$ $\quad$ F

$\neg A$:

$\neg(\forall x \in \mathbb{Z}\ x \text{ odd}) \equiv$ $\quad$ T

$\exists x \in \mathbb{Z}\ \neg(x \text{ odd})$

Ex 9

$\forall x \in \mathbb{Z}$ x is odd or x is even

$\forall x \in \mathbb{Z}$ x odd $\vee$ x even (true)

$\neg \forall x \in \mathbb{Z}$ (x odd $\vee$ ~~o~~x even)

$\exists x \in \mathbb{Z}$ $\neg$(x odd $\vee$ x even)

DeMorgans

$\exists x \in \mathbb{Z}$ x not odd $\wedge$ x not even

$\exists x \in \mathbb{Z}$ $\neg$(x odd) $\wedge$ $\neg$(x even)