

9/11/ Discrete

Last time

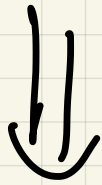
Multiplication Principle

Consider length n lists
consisting of (a_1, \dots, a_n)

m_1 choices for a_1

m_2 choices for a_2

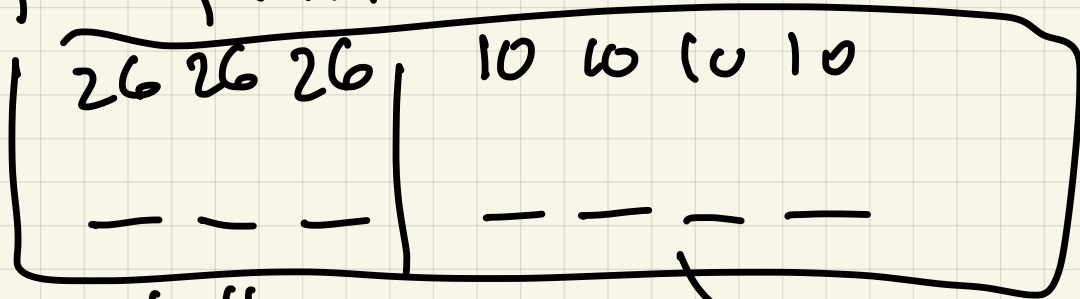
m_n choices for a_n



m_1, m_2, \dots, m_n choices for
 (a_1, \dots, a_n)

Ex 0 How many license
(a) plates numbers

at form 0-9



letters

digits

$$26^3 \cdot 10^4$$

(b) How many if no digit can be a 0?

$$26^3 \cdot 9^4$$

(c) How many if the letters are distinct?

$$26 \cdot 25 \cdot 24 \cdot 10^4$$

(d) How many if letters and digits are distinct?

$$\overline{26} \overline{25} \overline{24} \overline{10} \overline{9} \overline{8} \overline{7}$$

26.25-24.10.9.8.7

Theorem: The number of length r lists whose objects are taken from a set of size

$$\left. \begin{array}{l} n \\ \left\{ \begin{array}{l} \boxed{\begin{array}{c} n \\ r \end{array}} \\ n(n-1)(n-2)\dots(n-r+1) \end{array} \right. \end{array} \right\} \begin{array}{l} \text{if repetition is} \\ \text{allowed} \\ \text{repetition not} \\ \text{allowed} \end{array}$$

Book notation n_r

ex $10_4 = 10 \cdot 9 \cdot 8 \cdot 7$

$P(n, r)$ Calculator

Ex 2 Drama club has

30 members

(a) How many ways to elect
4 distinct members to be
president / VP / treasurer / secretary
P V T S

$$\overline{30} \overline{29} \overline{28} \overline{27} = 30^4 = 30 \cdot 29 \cdot 28 \cdot 27$$

(b) How many if ~~students~~ ^{members}
can hold multiple office?

$$30^4$$

(c) How many if president &
treasurer can be same,
but no others.

Tricky:

$$\overline{P} \overline{V} \overline{T} \overline{S} \{ ?? \}$$

$$\begin{array}{cccc} 30 & 29 & 29 & 28 & | \\ 30 & 30 & 29 & 28 & \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} ?? \\ ?? \end{array} \\ P & T & V & S & \end{array}$$

both tempting

Two cases: $P = T$

$$\begin{array}{cccc} \underline{30} & \underline{1} & \underline{29} & \underline{28} \\ P & T & V & S \end{array}$$

$P \neq T$

$$\begin{array}{cccc} \underline{30} & \underline{29} & \underline{28} & \underline{27} \\ P & T & V & S \end{array}$$

correct:

$$30 \cdot 29 \cdot 28 + 30 \cdot 29 \cdot 28 \cdot 27$$

§9

Ex 1: How many ways to make a list of length 20 from letters in alphabet

(a) with repetition and

(b) without repetition?

(a) 26^{20}

$$(b) \quad 26 \cdot 25 \cdot 24 \cdot 23 \cdot \dots \cdot 7$$

$$26_{20} \equiv \underbrace{\hspace{15em}}$$

Defn For $n \in \mathbb{N} = \{0, 1, 2, \dots\}$

$$n! = \begin{cases} \underline{n(n-1)(n-2) \dots (2)(1)} & n > 0 \\ 1 & n = 0 \end{cases}$$

"n factorial"

$$0! = 1$$

$$1! = 1$$

$$2! = 2$$

$$3! = 6$$

$$4! = 24$$

$$5! = 120$$

$$6! = 720$$

$$7! = 5040$$

Ex 1 (b)

$$26 \cdot \dots \cdot 7 = \frac{26!}{6!}$$

Remark: $n_r = \frac{n!}{(n-r)!}$

(a) Can define recursively:

$$\begin{cases} 0! = 1, & \text{for } n \geq 0, \\ n! = n \cdot (n-1)! \end{cases}$$

(b) $n!$ = # ways to make
a list of length n from
 n objects with no
repetitions

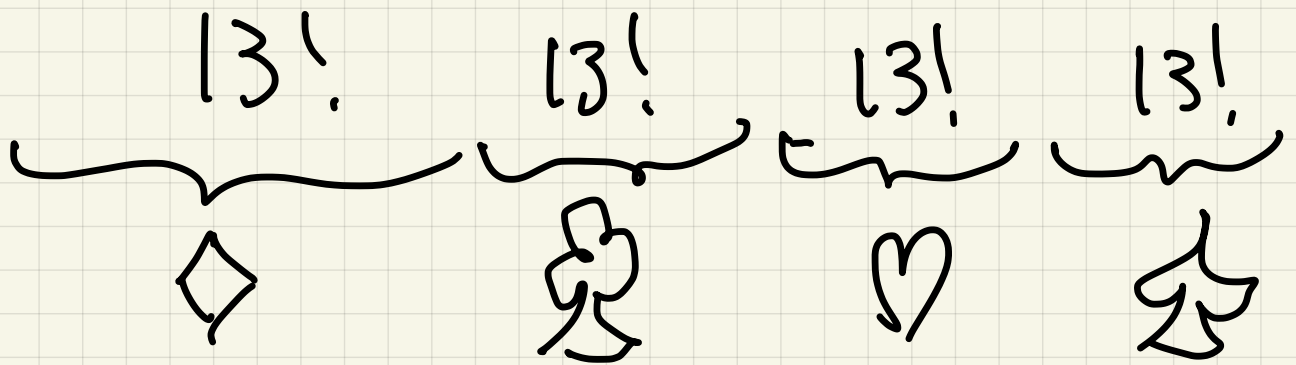
Ex 1: One such list,
()

$$\text{so } 0! = 1.$$

Ex 2 How many ways are
there to order a deck of
cards?

$$\text{(a)} \quad 52! \approx 8.066 \times 10^{67}$$

(b) How many ways to order cards if all cards from each suit are together?



$$4! \cdot 13! \cdot 13! \cdot 13! \cdot 13!$$

$$(\approx 3.61 \times 10^{40})$$

(c) How many ordered

5-card hands can be made?

$$52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 =$$

$$\frac{52!}{47!}$$

Recall Sum notation

$$a_1 + \dots + a_n = \sum_{k=1}^n a_k$$

Ex 1 $a_k = k$, $n = 100$

$$\sum_{k=1}^{100} k = \underline{1 + 2 + 3 + 4 + \dots + 100}$$

Can compute:

$$S = 1 + 2 + 3 + \dots + 100$$

$$S = 100 + 99 + 98 + \dots + 1$$

$$2S = \underbrace{101 + 101 + \dots + 101}_{100 \text{ terms}}$$

$$100 \cdot 101 =$$

$$2S = 10,100$$

$$S = \frac{10,100}{2} = 5,050$$

Product notation:

$$\prod_{k=1}^n a_k = a_1 \cdot a_2 \cdot a_3 \cdot \dots \cdot a_n$$

Ex 2 Compute the products:

$$(a) \prod_{k=1}^5 2 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$$

= 32

$$(b) \prod_{k=1}^n 2 = 2^n$$

$$(c) \prod_{k=1}^5 2^k = 2^1 \cdot 2^2 \cdot 2^3 \cdot 2^4 \cdot 2^5 =$$
$$= 2^{15} = 32,768$$

$$(d) \prod_{k=1}^n 2^k = 2^1 2^2 2^3 \dots 2^n$$

$$= \sum_{k=1}^n (1+2+\dots+k) = \sum_{k=1}^n \left(\sum_{j=1}^k j \right)$$

(e) $\prod_{k=1}^{100} k = 1 \cdot 2 \cdot \dots \cdot 100 = 100!$

(f) $\prod_{k=1}^n k = n!$

(g) $\prod_{k=48}^{52} k = 48 \cdot 49 \cdot 50 \cdot 51 \cdot 52$

(h) $\prod_{k=-50}^{50} 2^k = 1$

$$\underbrace{2^{-50} \cdot 2^{-49} \cdot 2^{-48} \cdot \dots \cdot 2^{49} \cdot 2^{50}}_1$$

(i) $\prod_{k=-50}^{50} k = 0$

$$(j) \quad \sum_{k=-50}^{50} 2 = 2^{101} \quad ?$$

$$\underbrace{2 \dots 2}_{-50} \quad \underbrace{2 \quad 2 \quad 2}_{-1 \quad 0} \quad \dots \quad 2$$