9/11/ Discrete

Last time

Multiplication Principle

Consider length n lists consisting of $(a_1, \ldots a_n)$

$$\left.\begin{array}{l} m_1 \text{ choices for } a_1 \\ m_2 \text{ choices for } a_2 \\ \\ m_n \text{ choices for } a_n \end{array}\right\}$$

$\Downarrow$

$m_1 m_2 \ldots m_n$ choices for $(a_1 \ldots a_n)$

Ex 0 (a) How many license plates numbers or form 0-9

26	26	26	10	10	10	10
___	___	___	___	___	___	___

letter — digits

$$26^3 \cdot 10^4$$

(b) How many if no digit can be a 0?

$$26^3 \cdot 9^4$$

(c) How many if the letters are distinct?

$$26 \cdot 25 \cdot 24 \cdot 10^4$$

(d) How many if letters and digits are distinct?

$$\overline{26}\ \overline{25}\ \overline{24}\ \overline{10}\ \overline{9}\ \overline{8}\ \overline{7}$$

$26\cdot 25\cdot 24\cdot 10\cdot 9\cdot 8\cdot 7$

<u>Theorem</u>: The number of length r lists whose objects are taken from a set of size n is

$$\begin{cases} \boxed{n^r} & \text{if repetition is allowed} \\ n(n-1)(n-2)\cdots(n-r+1) & \text{if repetition } \underline{\text{not}} \text{ allowed} \end{cases}$$

<u>Book notation</u> n_r

<u>ex</u> $10_4 = 10\cdot 9\cdot 8\cdot 7$

$$\boxed{P(n,r) \quad \text{Calculator}}$$

Ex 2 Drama club has 30 members

(a) How many ways to elect 4 distinct members to be president / VP / treasurer / secretary
P V T S

$\overline{30}\,\overline{29}\,\overline{28}\,\overline{27} = 30_4 = \begin{matrix}30 \cdot 29 \cdot 28 \cdot \\ 27\end{matrix}$

(b) How many if ~~students~~ members can hold multiple office?

30^4

(c) How many if president & treasurer can be same, but no others.

Tricky: $\overline{P}\ \overline{V}\ \overline{T}\ \overline{S}\ \}$??

30 29 29 28 $\quad$ 1

30 30 29 28 $\left.\begin{matrix} \\ \\ \end{matrix}\right\}$??

P T V S

both tempting

<u>Two cases</u>: $P = T$ $\quad \dfrac{30}{P}\ \dfrac{1}{T}\ \dfrac{29}{V}\ \dfrac{28}{S}$

$P \neq T$ $\quad \dfrac{30}{P}\ \dfrac{29}{T}\ \dfrac{28}{V}\ \dfrac{27}{S}$

<u>correct</u>:

$$30 \cdot 29 \cdot 28 + 30 \cdot 29 \cdot 28 \cdot 27$$

<u>§9</u>

<u>Ex 1</u>: How many ways to make a list of length 20 from letters in alphabet

(a) with repetition and

(b) without repetition?

(a) 26^{20}

(b) $26_{20} = \underbrace{26 \cdot 25 \cdot 24 \cdot 23 \cdots 7}$

Defn For n in $\mathbb{N} = \{0, 1, 2, \ldots\}$

$$n! = \begin{cases} n(n-1)(n-2)\cdots(2)(1) & n > 0 \\ 1 & n = 0 \end{cases}$$

"n factorial"

$0! = 1$	$4! = 24$
$1! = 1$	$5! = 120$
$2! = 2$	$6! = 720$
$3! = 6$	$7! = 5040$

Ex 1 (b)

$$26 \cdots 7 = \frac{26!}{6!}$$

Remark: $n_r = \dfrac{n!}{(n-r)!}$

(a) Can define recursively:

$$\begin{cases} 0! = 1, & \text{for } n > 0, \\ n! = n \cdot (n-1)! \end{cases}$$

(b) $n!$ = # ways to make a list of length n from n objects with no repetitions

$\underline{n=0}$: One such list, $()$

so $0! = 1$.

$\underline{\text{Ex 2}}$ How many ways are there to order a deck of cards?

(a) $52! \cong 8.066 \times 10^{67}$

(b) How many ways to order cards if all cards from each suit are together?

$$\overset{13!}{\underbrace{\qquad\qquad}_{\diamondsuit}}\ \overset{13!}{\underbrace{\qquad}_{\clubsuit}}\ \overset{13!}{\underbrace{\qquad}_{\heartsuit}}\ \overset{13!}{\underbrace{\qquad}_{\spadesuit}}$$

$$4! \cdot 13! \cdot 13! \cdot 13! \cdot 13!$$

$$(\doteqdot 3.61 \times 10^{40})$$

(c) How many <u>ordered</u> 5-card hands can be made?

$$52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 = \frac{52!}{47!}$$

<u>Recall</u> Sum notation

$a_1 + \ldots \quad fan = \sum_{k=1}^{n} a_k$ ✓

<u>Ex1</u> $a_k = k \quad , n = 100$

$$\sum_{k=1}^{100} k = \underline{1+2+3+4+\ldots+100}$$

<u>Can compute</u>:

$$S = 1+2+3+\ldots+100$$
$$S = 100+99+98+\ldots+1$$

$$2S = \underbrace{101+101+\ldots+101}_{100 \cdot 101 =}$$

$$2S = 10{,}100$$

$$S = \frac{10{,}100}{2} = 5{,}050$$

Product notation:

$$\prod_{k=1}^{n} a_k = a_1 \cdot a_2 \cdot a_3 \cdots a_n$$

Ex 2 Compute the products:

(a) $\prod_{k=1}^{5} 2 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 = 32$

(b) $\prod_{k=1}^{n} 2 = 2^n$

(c) $\prod_{k=1}^{5} 2^k = 2^1 \cdot 2^2 \cdot 2^3 \cdot 2^4 \cdot 2^5 =$

$= 2^{15} = 32{,}768$

(d) $\prod_{k=1}^{n} 2^k = 2^1 2^2 2^3 \cdots 2^n$

$$= 2^{(1+2+3 \cdots + n)} =$$

$$= 2^{\left(\sum_{k=1}^{n} k\right)}$$

(e) $\prod_{k=1}^{100} k = 1 \cdot 2 \cdot \cdots \cdot 100$

$$= 100!$$

(f) $\prod_{k=1}^{n} k = n!$

(g) $\prod_{k=48}^{52} k = 48 \cdot 49 \cdot 50 \cdot 51 \cdot 52$

(h) $\prod_{k=-50}^{50} 2^{k} = 1$

$$2^{-50} \cdot 2^{-49} \cdot 2^{-48} \cdots 2^{49} \cdot 2^{50}$$

(i) $\prod_{-50}^{50} k = 0$

(j) $\prod_{k=-50}^{50} 2 = 2^{101}$?

$\underbrace{2 \cdot - 2}_{-50 \quad -1} \; \underset{0}{2} \; 2 \; - \; - \; \underset{50}{2}$