10/28 Discrete:

Exam 2
avg 84%

150	
	7
135	
	4
120	
	5
105	
	1

1. $A = \{1, 2, 3\}$

$\emptyset \subseteq A$? T

$\{\emptyset\} \subseteq A$? F

2. (d) (e) (f) size

$|A \cup B| = |A| + |B| - |A \cap B|$

$9 + 11 = 4 = 16$

4. $\neg(x < y) \equiv x \geq y$

$\neg(x \leq y) \equiv x > y$

6. 52 cards 4 suits

13 A, 2 ... 10, J Q K

(a) $\binom{52}{4} \neq (52 \cdot 51 \cdot 50 \cdot 49)$

(b) $\binom{13}{4}$

(c) $\underbrace{\binom{4}{1}}_{= 4 \text{ suites}} \cdot \binom{13}{4}$

(d) $\underbrace{13 \cdot \binom{4}{3}}_{} \cdot \overbrace{12 \cdot 4}^{48}$

fnt suit #

7. _ _ _ _ _ _

1

A - Z

(a) 26^6

(b) _ _ _ _ _ _

$\binom{6}{3} \cdot 25^3$

place A

(c) No H : 25^6

(d) No H OR No E

S_H S_E

$|S_H \cup S_E| =$

$|S_H| + |S_E| - |S_H \cap S_E|$

$25^6 + 25^6 - 24^6$

(e) $|S_H \cup S_E \cup S_R| =$

$$|S_H| + |S_E| + |S_R|$$
$$- |S_H \cap S_E| - |S_H \cap S_R| - |S_E \cap S_R|$$
$$+ |S_H \cap S_E \cap S_R|$$

$$3 \cdot 25^6 - 3 \cdot 24^6 + 23^6$$

Exam 3 →

<u>Last time</u> { induction proofs.
{ strong induction.

Fibonacci #s

<u>Functions</u>: $f: A \longrightarrow B$

domain codomain

$f(a) = b$

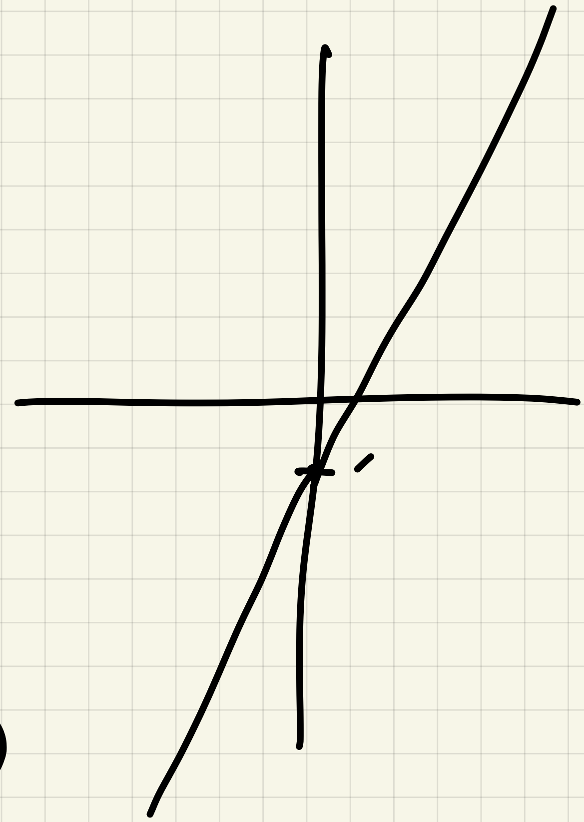
$$\text{Graph} = \{(a,b) \mid f(a) = b\}$$

$$\operatorname{Im} f = \{f(a) \mid a \in A\}$$

Ex 0 **Calculus:**

(a) $f : \mathbb{R} \to \mathbb{R}$

$f(x) = 2x - 1$

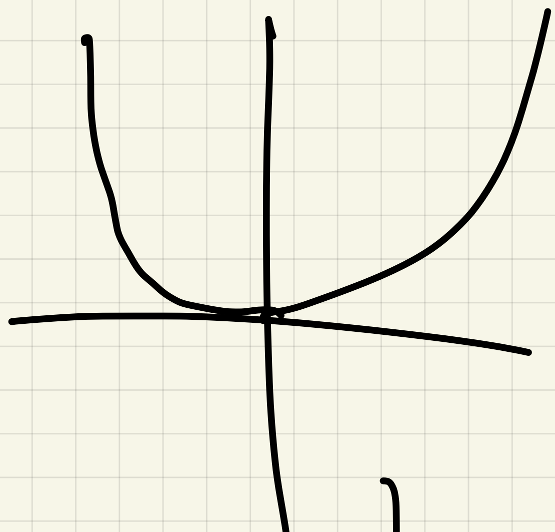


$\operatorname{Im} f = \mathbb{R}$

$= (-\infty, \infty)$

(b) $f : \mathbb{R} \to \mathbb{R}$

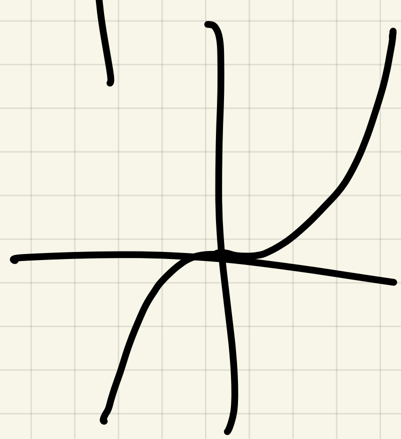
$f(x) = x^2$



$\operatorname{Im} f = [0, \infty)$

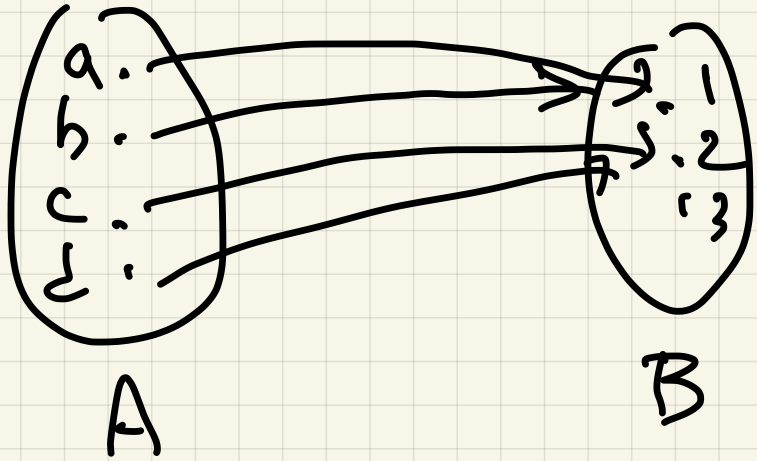
(c) $f : \mathbb{R} \to \mathbb{R}$

$f(x) = x^3$



$\operatorname{Im} f = \mathbb{R}$

(d)



$A = \{a, b, c, d\}$, $B = \{1, 2, 3\}$

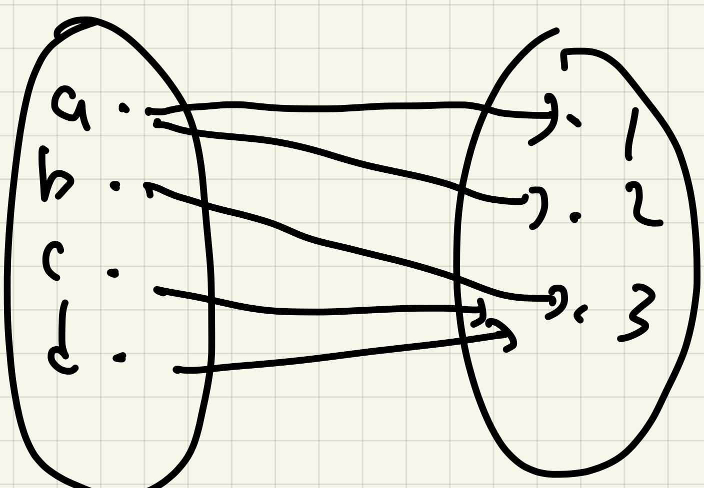
$f(a) = 1 = f(b)$

$f(c) = 2 = f(d)$

<u>Graph</u> $\{(a,1), (b,1)$
$(c,2)\ (d,2)\}$

$\mathrm{Im} f = \{1, 2\}$

(e)



<u>Not a function</u>

Ex1 $f: \mathbb{N} \to \{0, 1, 2, \ldots, 9\}$

$f(n) = n^{th}$ digit of π

$\pi = 3{,}14159\ldots$

$f(0) = 3$

$f(1) = 1$

$f(2) = 4$

$f(3) = 1$

$f(4) = 5$

$1 \neq 3$

Defn: $f: A \to B$ is one-to-one

($1-1$, injective)

if $f(a_1) = f(a_2) \Rightarrow a_1 = a_2$

(Contrapositive $a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2)$)

Ex2 Which of Ex'0 / Ex1 are

1-1?

1 (a) $f(x) = 2x-1$ is <u>1-1</u>

$$\left(f(a_1) = f(a_2) \Rightarrow 2a_1 - 1 = 2a_2 - 1 \Rightarrow 2a_1 = 2a_2 \Rightarrow a_1 = a_2 \right)$$

(b) NO $f(1) = f(-1)$

$1 \neq -1$

(c) $f(x) = x^3$ is 1-1

(d) NO

(e) ______

<u>Ex</u> NOT 1-1

<u>Defn</u>: A function $f: A \to B$ is <u>onto</u> (surjective) if $\operatorname{Im} f = B$

$\left(\forall b \in B \; \exists a \in A ; \; f(a) = b \right)$

Ex 3 Which of Ex 0/1 are onto

0 (a) onto

0 (b) NOT $\left(\begin{array}{l} x^2 = -1 \\ f(x) = -1 \\ \text{no solution} \end{array} \right)$

0 (c) onto

$f(x) = x^3$

0 (d) NO $f(x) = 3$

false all x in A

Ex 1 ~~onto~~ onto ✓

Defn A function $f : A \to B$ is 1-1 correspondence (bijection) if f is 1-1 and onto

Ex 4 Ex 0 (a) 1-1 corresp

Ex 0 (b) not

Ex 0 (c) 1-1 corresp

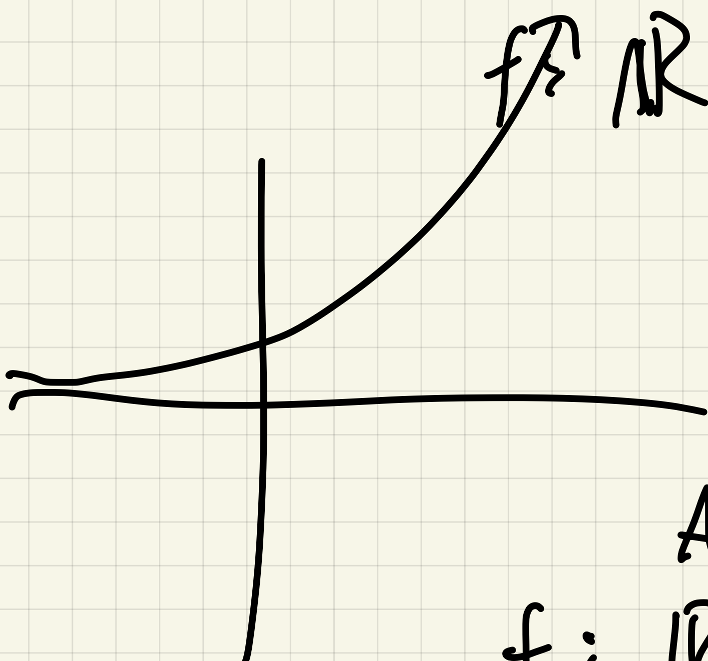
Ex 0 (d) NO

Ex 1 NO

Ex 5 these concepts depend on A + B

(a) (a) $f(x) = e^x$

$f: \mathbb{R} \to \mathbb{R}$



f 1-1

not onto

$$\begin{array}{ccc} & A & & B \\ f: & \mathbb{R} & \longrightarrow & (0, \infty) \end{array}$$

$f(x) = e^x$

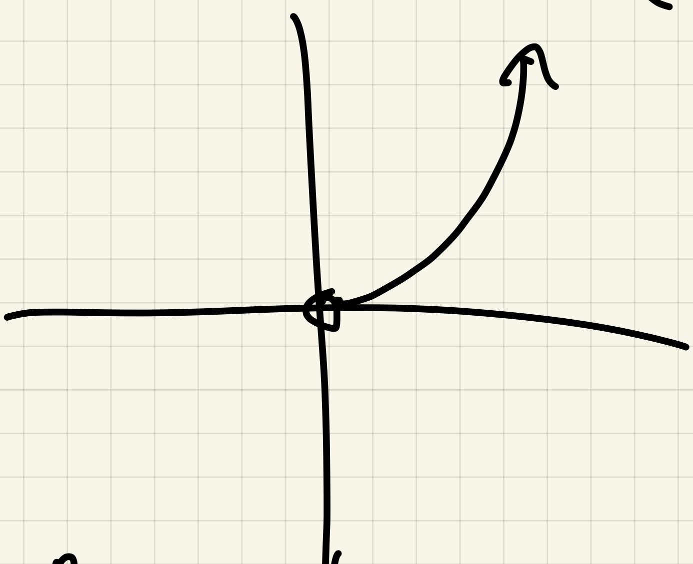
f is 1-1 correspondence

(b) $f(x) = x^2$ not 1-1

$f: \mathbb{R} \to \mathbb{R}$ not onto

If

<u>But</u> $f: (0,\infty) \to (0,\infty)$



now $f(x) = x^2$ is 1-1 corresp

<u>Remark</u>:

<u>Thm</u>: $f: A \to B$ is 1-1 corresp.

$\iff \exists\, g: B \to A$ function

such that $f(g(b)) = b \ \forall b \in B$

and $g(f(a)) = a \quad \forall a \in A$

$$\left[\, g = f^{-1} \,\right]$$

<u>Ex</u>: (a) $f: \mathbb{R} \to \mathbb{R}$

$f(x) = 2x - 1$

$y = 2x - 1$

$x = 2y - 1$

$x + 1 = 2y$

$\frac{x+1}{2} = y$

$g(x) = \frac{x+1}{2}$

(c) $f: \mathbb{R} \to \mathbb{R}$

$$\left.\begin{array}{l} f(x) = x^3 \\ g(x) = \sqrt[3]{x} \end{array}\right\} \text{inverses}$$