Exam 2

1. (a) F   (b) T   (c) T   (d) T (always)

(e) F (because $\phi \notin A$)

(f) T (because $\{1,2\} \in P(A)$)

(g) $\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$

Note $\phi \neq \{\phi\}$!

2. $A = \{n \in \mathbb{Z} : -12 \leq n \leq 4, 2 | n\} = \{-12, -10, -8 \ldots, 0, 2, 4\}$

$B = \{n \in \mathbb{Z} : -3 \leq n \leq 7\} = \{-3, -2, -1 \ldots, 5, 6, 7\}$

(a) $A \cap B = \{-2, 0, 2, 4\}$
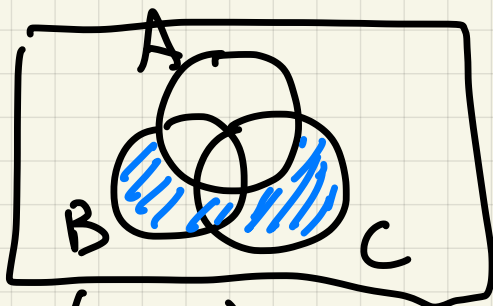
(b) $A - B = \{-12, -10, -8, -6, -4\}$

(c) $B - A = \{-3, -1, 1, 3, 5, 6, 7\}$
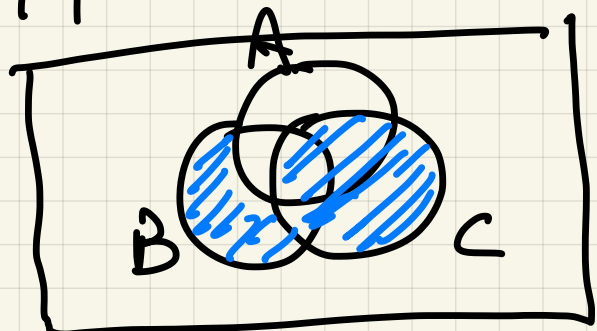
(d) $|A \triangle B| = 5 + 7 = 12$

(e) $|A \cup B| = |A| + |B| - |A \cap B| = 9 + 11 - 4 = 16$

(f) $|A \times B| = 9 \times 11 = 99$

3.

(a)

$(B \cup C) - A$

$(B - A) \cup C$

(b) Just need $A \cap C \neq \phi$: $A = B = C = \{1\}$

(c) Need $A \cap C = \phi$ $A = B = \{1\}$, $C = \{2\}$

4. (a) $\forall x \in \mathbb{Z}$ $\exists y \in \mathbb{Z}$: $x < y$

(b) $\exists x \in \mathbb{Z}$ $\forall y \in \mathbb{Z}$: $x \geq y$

(c) There is an integer which is greater than or equal to every integer.

5. (a) $\exists x \in \mathbb{N}$ $\forall y \in \mathbb{N}$ $x > y$ T

(b) $\exists x \in \mathbb{N}$ $\exists y \in \mathbb{N}$ $x > y$ F

(c) $\forall x \in \mathbb{N}$ $\exists y \in \mathbb{N}$ $x \geq y$ T
                                                        $(x = 0)$

(d) $\forall y \in \mathbb{N}$ $\exists x \in \mathbb{N}$ $x > y$ F

6. (a) $\binom{52}{4}$ 4-card hands from deck of 52

(b) $\binom{13}{4}$ (take from clubs)

(c) $\binom{13}{4} \cdot 4$ $\left(\begin{array}{l} 4 \text{ choices of suit,} \\ \binom{13}{4} \text{ choice of cards} \end{array} \right)$

(d) $13 \cdot \binom{4}{3} \cdot 12 \cdot \binom{4}{1}$

   pip $\nearrow$ $\uparrow$ $\uparrow$ $\searrow$

             suits   pip   suits

7. (a) 26^6

   (b) $\binom{6}{3} \cdot 25^3$ $\left(\begin{array}{l} \binom{6}{3} \text{ ways to place } H \\ 25^3 \text{ for rest} \end{array} \right)$

(c) 25^6

(d) $25^6 + 25^6 - 24^6$

(e) $25^6 + 25^6 + 25^6 - 24^6 - 24^6 - 24^6 + 23^6$