Exam 1

$\boxed{1}$ (a) An integer x divides an integer y if there exists an integer c with $y = cx$.

(b) An integer x is composite if there exists an integer b with $1 < b < x$ and $b | x$.

(c) 1, 2, 3, 6, 9, 18

$\boxed{2}$ (a) $B \Rightarrow A$ is true

(b) $A \Rightarrow B$ is true, but $B \Rightarrow A$ is false (take $x = 1, y = 2$)

(c) Neither are true:

$A \Rightarrow B$ false: $x = -1$

$B \Rightarrow A$ false: $x = 0$

(d) Both are true

(e) $B \Rightarrow A$ vacuously true as B false

$\boxed{3}$ Truth table:

(a)

x	y	$\neg x$	$\neg y$	$\neg x \vee y$	$x \vee \neg y$	$(\neg x \vee y) \wedge (x \vee \neg y)$
T	T	F	F	T	T	T
T	F	F	T	F	T	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

(b) Expression is logically equiv. to both $x \leftrightarrow y$ and $(x \wedge y) \vee (\neg x \wedge \neg y)$

$\boxed{4}$ If x, y are even, then $4 \mid xy$

Proof: Let x, y be even.

Then there exist integers c, d with $x = 2c$ and $y = 2d$

Therefore $xy = (2c)(2d) = 4(cd)$ is divisible by 4 because c, d integers $\Rightarrow$ cd is integer.

$\boxed{5}$ ($\Rightarrow$) Let x, y be integers, $x \mid y$.

Then there exists an integer c with $y = cx$.

Since c is an integer, so is

$c+5$, therefore $x \mid y+5x$ because

$$y+5x = cx+5x = (c+5)x$$

($\Leftarrow$) Let x, y be integers, $x \mid y+5x$. Then there is an integer d with

$y+5x = dx$ so that $y = dx - 5x = x(d-5)$. Now d an integer $\Rightarrow$ $d-5$ is an integer, so $y = x(d-5)$ implies $x \mid y$.

$\boxed{6}$ (a) 9, 15, 21, ... many examples.

(b) Only example is $n = 1$

(c) Take $c = 1/2$, $x = 0$

(d) $x = 1 > 0 = y$, $z = -1$, then

$$\frac{x}{z} = -1, \quad \frac{y}{z} = 0, \quad \frac{x}{z} > \frac{y}{z} \text{ false.}$$

(e) $x = \frac{1}{10} > 3 \cdot \left(\frac{1}{10}\right)^2 = .03$

Any $x < 1/3$ works

$\boxed{7}$ (a) 26^5 (b) 25^5

(c) $5 \cdot 25^4$ because 5 places to put H, 25^4 possible for rest.

(d) $26^5 - 25^5$ (avoid those in part (b))

(e) $26_5 = 26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 = \dfrac{26!}{21!}$

$\boxed{8}$ $\displaystyle\prod_{k=1}^{3} \frac{(2k+1)}{(2k-1)} = \frac{3}{1} \cdot \frac{5}{3} \cdot \frac{7}{5} = 7.$