8/28/ Discrete

Proofs

Last time ✓

format ⇝ ① write statement if A then B.

② State A.

③ unwind what A actually says

④ **Creative step**:

③ $\Rightarrow$ B ?? A $\longrightarrow$ B ??

⑤ State B

Ex1 "$x \neq y \Rightarrow y^2 - x^2$ composite"

$\Downarrow$ modify

If x, y are positive integers and $\boxed{x+1 < y}$, then $y^2 - x^2$

composite.

Proof: Scratch:

Given x, y need to show is $y^2 - x^2$ composite

Idea: $y^2 - x^2 = (y-x)(y+x)$

Need $y^2 - x^2 > 1$, and has divisor b: $1 < b < y^2 - x^2$

Try: $b = y - x$ or $b = y + x$

Need $1 < y - x < y^2 - x^2$

Know $x + 1 < y$
subtract x from both: $1 < y - x$

$y - x < (y-x)(y+x)$

Idea: $1 < y + x$ multiply both

sides by y

$y - x > 0$

OK

Why $1 < y + x$?!

x positive, so $x > 0 \Rightarrow$

~~$y \geq 1 + x$~~

$x \geq 1 \Rightarrow y > 1 + x \Rightarrow y \geq 3$

$\Rightarrow x + y \geq y > 1$ ✓

If x, y positive integers and $x + 1 < y$, then $y^2 - x^2$ is composite.

Proof: ①

② Let x, y be positive ints with $x + 1 < y$.

(3) Since x positive, $\Rightarrow x > 0$ & $\boxed{x \geq 1}$. Also $\underline{x+1} < y$

≥ 2 (b/c $x \geq 1$)

so $\boxed{y \geq 3}$

in particular $x+y \geq 4$, so

(a) $\boxed{x+y > 1}$

Also, $x+1 < y \Rightarrow$ (b) $\boxed{1 < y-x}$

Consider $b = y-x$.

Notice $b \mid y^2 - x^2$ b/c

(1) $y^2 - x^2 = (y-x)(y+x)$ where $y-x = b$ and $y+x$ is an integer

(2) $1 < b = y-x$ ok (b)

(3) $1 < x+y$ ok (a)

multiply both sides by $y-x > 0$ to get

$b = y - x < (y-x)(y+x) = y^2 - x^2$

$\therefore$ $y^2 - x^2$ is composite.

<u>Ex2</u> · An integer is odd <u>if and only if</u> its negation is odd.

<u>Note</u> $A \iff B$ is equivalent to <u>$A \Rightarrow B$</u> and <u>$B \Rightarrow A$</u>

<u>Proof</u>: For x an integer,

$\underbrace{x \text{ is odd}}_{A} \iff \underbrace{-x \text{ is odd}}_{B}$

<u>$A \Rightarrow B$</u>: Let x be an odd integer. Therefore there's an integer t so that

$x = 2t+1,$

$$-x = -2t-1 = \underbrace{-2t-1-1}_{-2t-2} + 1 = 2(-t-1)+1$$

Since t is an integer, so $s = -t-1$ and

$-x = 2s+1,$

$-x$ is odd. ✓

<u>$B \Rightarrow A$</u>: Let $-x$ be an odd integer. Therefore there's an integer t so that $-x = 2t+1$

Therefore $x = -2t-1$ ⊜

$$2(-t-1)+1$$

since $s = -t-1$ is an integer, $-x = 2s+1$ is odd,

<u>Notice</u>: $x = -(-x)$, so can just apply $A \Rightarrow B$ argument to $(-x)$

<u>§6</u> We've seen what's involved in proving if A then B.

How about <u>disproving</u> one?

A	B
T	T
T	F ← impossible
F	T
F	F

if A then B

So to disprove, just give <u>one example</u> where A is true and B is false,

<u>counterexample</u>

<u>Ex)</u> <u>Disprove</u>:

(a) If p and q are prime, then $p+q$ is composite.

<u>False</u>: $p=2,\ q=3$

(b) If a, b are integers

and $a \mid b$ and $b \mid a$,
then $a = b$.

False $a = 2$, $b = -2$

(c) For x an integer,
x is positive $\Longleftrightarrow$ $x+1$ positive.

($\Longrightarrow$) always true

($\Longleftarrow$) false, when $x = 0$

(d) If p is prime, then
$2^p - 1$ is prime.

$p = 2, 3, 5, 7, 11$

$2^{11} - 1 = 2047 = 23 \times 89$

(e) A positive integer is a palindrome is it reads the same forwards as backwards

1111 = $11 \cdot 101$ 22 = $2 \cdot 11$ 2772 = $2 \cdot 252$

1331 = $11 \cdot 121$

All palindromes with at least 2 digits are multiples of 11.

<u>False</u> : $x = 111 = 3 * 37$