# 8/26/Discrete

## Quiz 1

(a) $-5 \mid 30$ $\qquad -5c = 30$

T $\qquad c = -6$

(b) $6 \mid 15$ $\qquad 6c = 15$

no integer c

F

(c) $6 \mid 0$ $\qquad 6c = 0$

T $\quad (c = 0)$

(d) $0 \mid 6$

$0c = 6$

F $\qquad$ no solution

2. $54 = \underline{3^3 \cdot 2^1}$ $\qquad 0 \le a \le 3$

$3^a \cdot 2^b$ $\quad 0 \le b \le 1$

$$\begin{array}{c|cccc} & 3^0 & 3^1 & 3^2 & 3^3 \\ \hline b=0 & 1 & 3 & 9 & 27 \\ b=1 & 2 & 6 & 18 & 54 \end{array}$$

$1, 2, 3, 6, 9, 18, 27, 54$

<u>Last time</u> §4

If - Then $\quad A \Rightarrow B$

hypothesis $\quad$ conclusion

| $A$ | $B$ |
|---|---|
| T | T |
| T | F ← impossible |
| F | T |
| F | F |

Compound conditions

<u>not</u> $\quad \neg A$

**and** $A \wedge B$

**or** $A \vee B$

**vacuous truth**

if $A$ then $B$

when $A$ is false, then $A \Rightarrow B$ is true (no matter what $B$ says)

## §5 **Proofs**

Argument that shows a (math) is true always

**Proposition 1** The sum of two integers is even.

**Proof:**

(a) If $\underbrace{\boxed{\text{x and y are odd integers}}}_{A}$ then $\underbrace{\boxed{x+y \text{ is an even integer}}}_{B}$

(b) Let x and y be two odd integers

$c \in \mathbb{Z}$

(c) Since x is odd, There's an integer c so that $x = 2c+1$

Since y is odd, there's an integer d so that $y = 2d+1$

Therefore

(d)
$$\begin{aligned} x+y &= (2c+1)+(2d+1) \\ &= 2c+1+2d+1 \\ &= 2c+2d+2 \\ &= 2(c+d+1) \end{aligned}$$

Is divisible by 2 because

Since $c, d$ are integers, so is $(c+d+1)$

(e) Therefore $x+y$ is even. $\square$

end of proof QED

<u>Notes</u>: <u>Look at format</u>:

(a) Converted statement to if $\boxed{A}$ then $\boxed{B}$

(b) State hypothesis A

(c) Start with, unwind hypothesis to see exactly what A delivers

(d) <u>Creative step</u>: did some algebra to see conclusion

(e) State conclusion

Ex2 Prove that the cube of an odd number is odd.

Proof: If $\overbrace{\boxed{x \text{ is an odd integer}}}^{A}$ then $\underbrace{\boxed{x^3 \text{ is an odd integer}}}_{B}$,

Let $x$ be an odd integer,

Since $x$ is odd, there's an integer $c$ so that $x = 2c+1$

Therefore $x^3 = (2c+1)^3 =$

$$\underbrace{(2c+1)(2c+1)}(2c+1) =$$

$$(4c^2 + 2c + 2c + 1)(2c+1)$$

$$(4c^2+4c+1)(2c+1) =$$

$$2c(4c^2+4c+1) + 1(4c^2+4c+1)$$

$$= 8c^3 + 8c^2 + 2c + 4c^2 + 4c + 1 =$$

$$8c^3 + 12c^2 + 6c + 1$$

$$= 2\underbrace{(4c^3 + 6c^2 + 3c)} + 1$$

Since $c$ is an integer, so $4c^3 + 6c^2 + 3c$, so

$x^3$ is odd ✓

<u>Ex 3</u> If an integer is divisible by a second integer, then so is any integer multiple of <u>it</u>.

We must show that if

A $a, b, c$ are integers, and $b \mid \underline{a}$ A then $\boxed{b \mid \underline{ac}}$ B

integer mult of $a$

Let $a, b, c$ be integers so that $b \mid a$.

Since $b \mid a$, there's an integer $d$ so that $bd = a$

Therefore $ac = (bd)c$
$= b(dc)$

$dc$ is an integer ($b$, $c$, $d$, $c$ are), so
$b \mid ac$ by definition.

Ex/ Question: Are differences of squares composite? NO

I.e. if $x, y$ are integers, is $x^2 - y^2$ composite?

Need $x^2 > y^2$ for this to make

i.e. $x = 2,\ y = 4$

$x^2 - y^2 = 4 - 16 = -12 < 0$

not composite

suppose $x^2 > y^2$

$2^2 - 1^2 = 3$ not composite

$3^2 - 2^2 = 5$ not composite

$4^2 - 2^2 = 12$ composite

$3^2 - 1^2,\ 4^2 - 1^2,\ 5^2 - 2^2 =$

Suggests: the difference of non consecutive squares is composite.

Also false:

$5^2 - (-4)^2$ ~~exmp~~

$4^2 - (-3)^2 =$ not com

$5^2 - (-5)^2 = 0$

fails

Conjecture: differences of squares of nonconsecutive positive integers is composite

"Proof" If $x, y$ are positive integers and $\underbrace{x+1 < y}_{\text{nonconsecutive}}$

then $y^2 - x^2$ is composite

$$y^2 - x^2 = \underset{a}{(y-x)}\underset{b}{(y+x)}$$

$$1 < a < y^2 - x^2$$