

8/21/ Discrete

Definitions:

Last time

divisible $b|a$

even, odd,
prime, composite,
positive divisors

Today: §4

Theorem
(statements)

Ex 1: T/F

mostly?
T

(a) The sky is blue

(b) Human body temperature

is $98.6^{\circ}F$

mostly T ?

(c) The pope is Catholic.

T ?

(d) $p = 3$ is prime, $\top!$

Higher standard of truth:

Defn: A Theorem is a mathematical statement that has a proof.

An argument showing the statement is always true

Ex 2 Theorem

(a) $a = -10$ is even Thm!
(b) $p = 3$ is prime Thm!

(c) $p = 21$ is prime FALSE

(d) If x, y are odd, then
so is xy Thm

(e) If a right triangle
has legs a & b and
hypotenuse c , then

$$a^2 + b^2 = c^2$$

of length

of length

NONSENSE!

Pythagorean Theorem

(f) If a, b, c, n are integers
and $n > 2$, and

$$a^n + b^n = c^n$$

then $a = 0$ or $b = 0$,

True: Fermat's last Theorem

Andrew Wiles 1994

Not all theorems are
created equal!

Variations

Fact: very minor theorem
($a = -10$ even)
($p = 3$ prime)

Proposition: minor theorem

Lemma: Proposition used
to prove a stronger
one,

Corollary Consequence of
a theorem

Claim Same as lemma

Result neutral

Most results have statements of form

If $\frac{A}{\text{hypothesis}}$ then $\frac{B}{\text{conclusion}}$

Means: Whenever A is true, then B must be true

A	B	
T	T	possible
T	F	impossible ←
F	T	possible
F	F	possible

Notation: $A \Rightarrow B$
 $A \rightarrow B$

Ex 3 "If you don't do your chores, then you won't get your allowance"

A

B

What is meant: threat

But more is implied:

If you do your chores,
then you get your allowance,

Expert:

A	B
T	T
T	F
F	T
F	F

OK
impossible
impossible
OK

(not $A \Rightarrow$ not B understood)

(b) If you work hard, A
then you pass your classes, B

agrees with

mathematical if-then

Synonyms: if A then B :

A implies B ,

A is sufficient for B ,

Whenever A , then also B ,

B necessary for A , $B \Leftarrow A$

In Ex 3(a), what's really

meant is

$A \Rightarrow B$ also $B \Rightarrow A$

A	B	
T	T	OK
T	F	impossible
F	T	impossible
F	F	OK

In math, this is

A if and only if B

A, iff B, $A \iff B$
 abbr. \nearrow

Ex 4 Which of $A \Rightarrow B$,

$B \Rightarrow A$, or $A \iff B$

(a) A: x is even

B: $x+1$ is odd

all
are
OK

(b) A: x is even

B: $x^2 + x$ is even

Only $A \Rightarrow B$

(c) $A: x \geq 0$ all are
 $B: x^2 > 0$ false

Three combinations

If A, B are statements

① NOT

Not A is true $\Leftrightarrow A$ is false

Symbol: $\neg A$

A

A	$\neg A$
T	F
F	T

② AND

A and B means both A and B are true

Symbol: $A \wedge B$

Ex 5 is even and 7 is prime

(a) False

(b) 5 is prime and 7 is odd

True

A	B	$A \wedge B$
T	T	T pos
T	F	F
F	T	F
F	F	F

③ OR

A or B means

A is true or

B is true or

both

Symbol

$A \vee B$

5 is even and 7 is prime
true

5 is even and 7 composite
false

A	B	$A \vee B$
T	T	T
T	F	T
F	T	T
F	F	F

Ex (a) Car shopper says
" I'll buy the Chevy or
I'll buy Ford" A
B

In English, means

A true or B true, but
not both.

(h) "Give me liberty or
give me death"

Vacuous truth:

Ex If $\frac{x^2 < 0}{A}$, then $\frac{x = 20}{B}$.

A	B		
T	T	OK	<u>True</u>
T	F	impossible	
F	T	OK	
F	F	OK	

(h) If 5 is composite,
then pigs can fly.

