8/19/Discrete

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Text: E. Scheinermann
3rd edition

Mathematics: a discrete Introduction

Calculator:
TI-83/84 - (STAT)

Grading: HW 10
Q 10
Ex1 15
Ex2 15
Ex3 15
F 35

100

12/9
500-730

A

≥ 90) 88-90

B

≥ 80 78-80

≥ 70

Weekly Planner

Demographics:

What is discrete math?

opposite

$\neq$ discreet

continuous

Excludes all continuous process

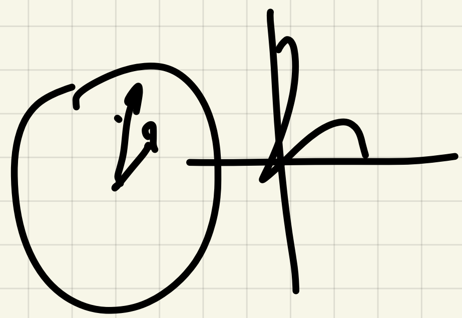
i.e. NO $\left\{ \begin{array}{l} \text{limits} \\ \text{derivative} \\ \text{integral} \end{array} \right.$

Ex Clocks: digital vs analog

discrete

continuous

2:04



Ex $\mathbb{R}$

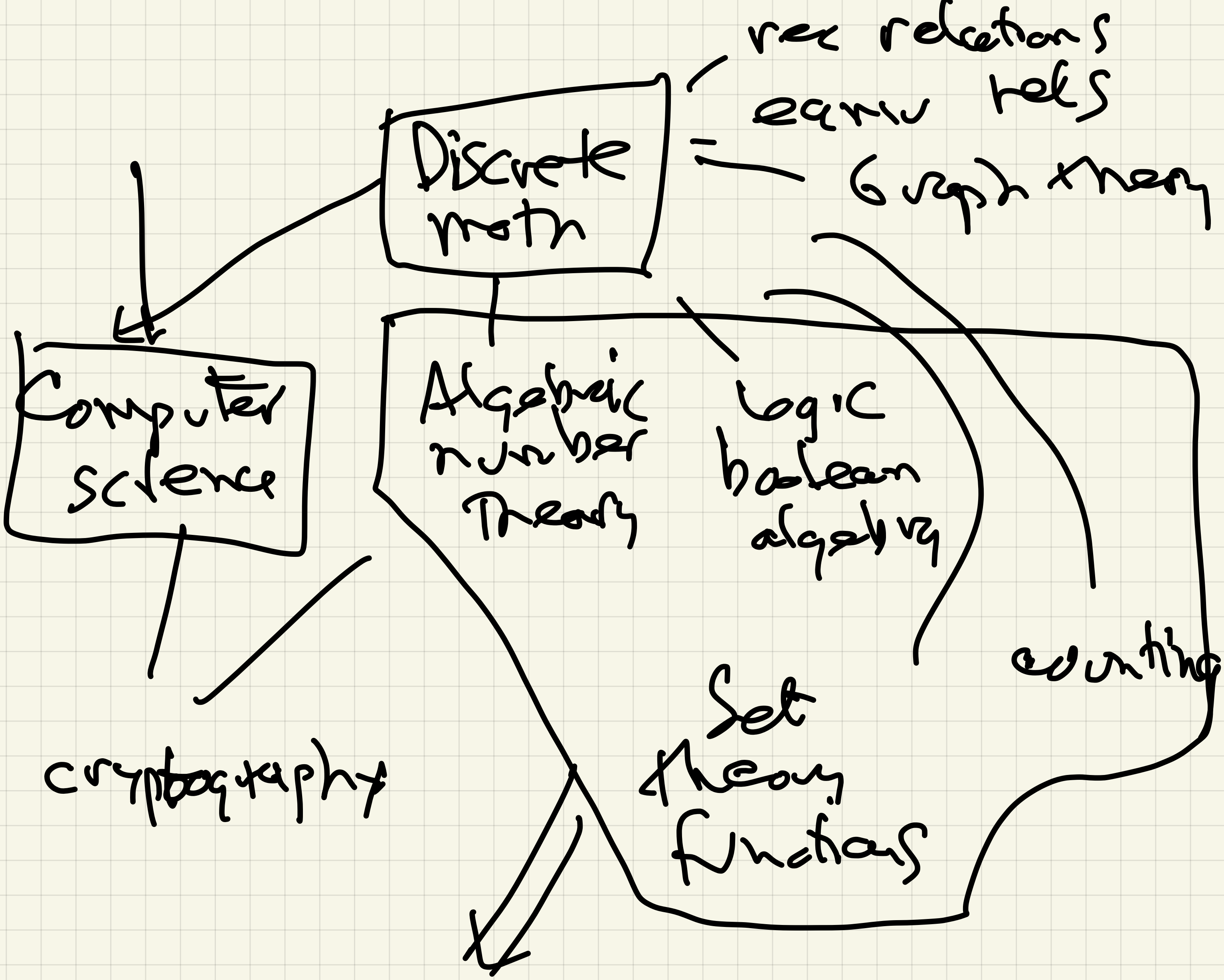
0

continuous

$\mathbb{Z}$ $\cdots$ -3 -2 -1 0 1 2 3 $\cdots$

discrete

What does discrete include?



<u>Goals</u>: I. Basic proof techniques/
Logic

II. Sets & functions

III. Counting

IV. Number theory

Ch 1, 2, 3, 4, 5, 7

§1 Pep talk

§2. Read this

Communicating math $\boxed{\text{precisely}}$

(a) use complete sentences

(b) avoid nonsense / mismatch categories

(c) avoid pronouns

(d) rewrite if necessary

§3 Fundamentals:

IR

When writing proofs, need to start from somewhere

Our starting point: (App D)

You know integers

$$\mathbb{Z} = \{0, \pm 1, \pm 2, \ldots \}$$

German: Zahl = number

can use axioms freely

① If x, y in $\mathbb{Z}$, then so are

$x+y$, ~~x~~ and $x \cdot y$
(closure)

② If x, y in $\mathbb{Z}$ then (commutative)

$x+y = y+x$, $x \cdot y = y \cdot x$

③ If x, y, z in $\mathbb{Z}$, then

$(x+y)+z = x+(y+z)$

$(xy)z = x(yz)$

(associate)

④ If x in $\mathbb{Z}$,

$x+0 = x$ (identity)

$x \cdot 1 = x$

⑤ If x in $\mathbb{Z}$, there is y in $\mathbb{Z}$

so that $x+y=0$
(add inverse)

(6) If x,y,z in $\mathbb{Z}$, then
$$x(y+z) = (xy)+(xz)$$
(distributive rule)

<u>Rmk</u>: Other number systems satisfy (1) - (6)

$\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$, $\mathbb{R}[x]$

$\mathbb{Z}[\sqrt{2}] = \{a+b\sqrt{2} \mid a,b \text{ in } \mathbb{Z}\}$

$\mathbb{N} = \{0, 1, 2, 3, \ldots \}$

~~$\{1, 2, 3, \ldots\}$~~

<u>convention</u> in our book

<u>Defn</u> Let a,b in $\mathbb{Z}$, then b <u>divides</u> a if

/ there's an integer c in $\mathbb{Z}$ so that

Notation $a, b \in \mathbb{Z}$ $bc = a$

Notation: $b|a$

Ex T/F

$5|5$ ✓ T

$1|5$ ✓ T

$5|1$ F

$5|200$ T

$3|50$ F

$2|79$ F

~~2|10~~ $7|1001$ T

$7 \cdot 143 = 1001$

Defn For $a \in \mathbb{Z}$, a is <u>even</u> if a is divisible by 2

$(2 = 1+1)$

<u>Defn</u> For $a \in \mathbb{Z}$, a is <u>odd</u> if there's an x in $\mathbb{Z}$ so that $\boxed{a = 2x+1}$

<u>Rmk</u> Later we'll <u>prove</u> that $a \in \mathbb{Z}$ ~~⇒~~ a even or odd, not both.

<u>Ex 3</u>

$$278 = 2 \cdot 139$$

$$543 = 2 \cdot \underline{271} + 1$$

<u>Defn</u> An ~~integer~~ p ∈ ~~ℤ~~ n, no is prime if $p > 1$ and the only positive divisors of p are 1 and p

Ex 4 $p=2$ prime

$p=3$ prime

$p=4$ not prime $1, 2, 4$

$p=0$

$p=57$ not $\boxed{1, \underline{3}, \underline{19}, 57}$

Defn A positive ~~int~~ $a \in \mathbb{Z}$ is composite if there's an $b \in \mathbb{Z}$: $b \mid a$ and $1 < b < a$

$a=19$ not composite

$a=87$ is composite

$3 \cdot 29$

Ex 5 How many positive divisors 12^3

$a = 12$, $1, 2, 3, 4, 6, 12$ $\quad 6$

$a = 13$, $1, 13$ $\quad 2$

$a = 18$, $1, 2, 3, 6, 9, 18$ $\quad 6$

$\uparrow$ 18^3

$a = 81 = 3^4$, $1, 3, 9, 27, 81$ $\quad 5$

$a = 360 = 2^3 \cdot 3^2 \cdot 5$

pos divs: $2^a \cdot 3^b \cdot 5^c$

$0 \le a \le 3$
$0 \le b \le 2$
$0 \le c \le 1$

$1, 5 \quad 2, 10, \quad 4, 20, \quad 8, 40 \quad 8$

$3, 15, \quad 6, 30, \quad 12, 60 \quad 24, 120 \quad 8$

$9, 45$ – —— – $\quad 8$

$= 24$