

Quiz 8

$$\vec{r}(t) = \langle 3t^2, t^3, t^2 \rangle \text{ for } 0 \leq t \leq 5$$

$$1. \text{ Arc length} = \int_0^5 |\vec{r}'(t)| dt =$$

$$\int_0^5 |(6t, 3t^2, 2t)| dt = \int_0^5 \sqrt{36t^2 + 9t^4 + 4t^2} dt$$

$$= \int_0^5 \sqrt{40t^2 + 9t^4} dt = \int_0^5 t \sqrt{40 + 9t^2} dt$$

$$u = 40 + 9t^2$$

$$du = 18t dt \Rightarrow \int_{40}^{265} \frac{1}{18} \sqrt{u} du = \frac{2}{54} u^{3/2} \Big|_{40}^{265}$$

$$\frac{1}{27} (265^{3/2} - 40^{3/2})$$

$$2. \quad \vec{r}'' = \langle 6, 6t, 2 \rangle \quad \vec{r}'(1) = \langle 6, 3, 2 \rangle,$$

$$\vec{r}''(1) = \langle 6, 6, 2 \rangle$$

$$\vec{r}' \times \vec{r}'' = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 3 & 2 \\ 6 & 6 & 2 \end{vmatrix} = \langle -6, 0, 18 \rangle$$

$$k = \frac{|\vec{r}' \times \vec{r}''|}{|\vec{r}'|^3} = \frac{6\sqrt{10}}{(\sqrt{49})^3} = \frac{6\sqrt{10}}{7^3}.$$