Quiz 19

1. $\vec{r} = \langle t, 2t \rangle$, $\vec{r}' = \langle 1, 2 \rangle$, $|r'| = \sqrt{5}$

$$\int_{C_1} (x^2 + y)\, ds = \int_0^2 (t^2 + 2t)\sqrt{5}\, dt =$$

$$\sqrt{5}\left(\frac{t^3}{3} + t^2\right)\Big|_0^2 = \sqrt{5}\left(\frac{8}{3} + 4\right) = \frac{20\sqrt{5}}{3}$$

2. $y = x^2$, so $\vec{r}(t) = \langle t, t^2 \rangle$, $0 \le t \le 2$

$$\vec{r}' = \langle 1, 2t \rangle, \quad |r'| = \sqrt{1 + 4t^2}$$

$$\int_{C_2} (x^2 + y)\, ds = \int_0^2 (t^2 + t^2)\sqrt{1 + 4t^2}\, dt =$$

$$\int_0^2 2t^2 \sqrt{1 + 4t^2}\, dt$$

$t = \frac{1}{2}\tan\theta$, $dt = \frac{1}{2}\sec^2\theta\, d\theta \Rightarrow$

$$\int_0^{\tan^{-1}4} \frac{1}{2}\tan^2\theta \sec\theta \cdot \frac{1}{2}\sec^2\theta\, d\theta =$$

$$\frac{1}{4}\int_0^{\tan^{-1}4} \tan^2\theta \sec^3\theta\, d\theta$$

can be computed with reduction formula