

Quadratic

$$z = f = x^3 + 3x^2 + y^3 - 3y$$

1. $\nabla F = \langle 3x^2 + 6x, 3y^2 - 3 \rangle = \langle 0, 0 \rangle$

$$\begin{matrix} 3x^2 + 6x & 3y^2 - 3 \\ \overset{\text{"}}{3x(x+2)} & \overset{\text{"}}{3(y-1)(y+1)} \\ \overset{\text{"}}{0} & \overset{\text{"}}{0} \end{matrix}$$

\Rightarrow critical pts $(0, 1), (0, -1), (-2, 1), (-2, -1)$

2. $f_{xx} = 6x + 6, f_{xy} = 0, f_{yy} = 6y$

$$d = \det \begin{pmatrix} 6x+6 & 0 \\ 0 & 6y \end{pmatrix}$$

(0, 1) $d = 36 > 0, f_{xx} = 6 > 0, \frac{\text{local min}}{\text{min}}$

(0, -1) $d = -36 < 0, \underline{\text{saddle}}$

(-2, 1) $d = -36 < 0, \underline{\text{saddle}}$

(-2, -1) $d = 36 > 0, f_{xx} = -6 < 0, \frac{\text{local max}}{\text{max}}$