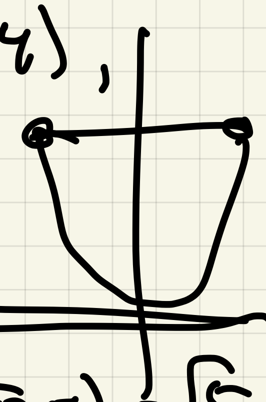
Exam 3

1. $\int_0^1\int_0^{y^2}(2x+y)\,dx\,dy = \int_0^1 x^2+xy\Big|_0^{y^2} =$

$\int_0^1 y^4+y^3\,dy = \frac{y^5}{5}+\frac{y^4}{4}\Big|_0^1 = \frac{1}{5}+\frac{1}{4} = \boxed{\frac{9}{20}}$

2. $\nabla f = \langle y-4, x\rangle = \langle 0,0\rangle$ at $(0,4)$,

$\boxed{f(0,4)=0}$



boundary: top $y=5$, $-\sqrt{5}\le x\le\sqrt{5}$

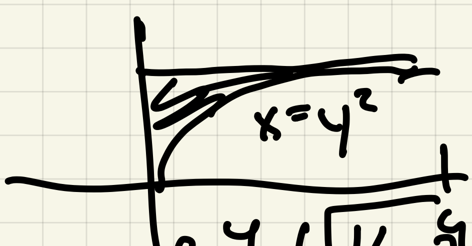
$f(x,5)=x$, $\boxed{f(-\sqrt{5},5)=-\sqrt{5},\ f(\sqrt{5},5)=\sqrt{5}}$

bottom $f(x,x^2)=x^3-4x$, $f'=3x^2-4=0$

at $x=\pm\frac{2}{\sqrt{3}}$, $\boxed{f\left(\frac{2}{\sqrt{3}},\frac{4}{3}\right)=\frac{8}{3\sqrt{3}}-\frac{8}{\sqrt{3}}=\frac{-16}{3\sqrt{3}}}$ ← min

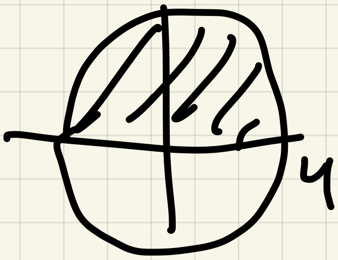
$\boxed{f\left(-\frac{2}{\sqrt{3}},\frac{4}{3}\right)=\frac{16}{3\sqrt{3}}}$ ← max

3. $\int_0^4\int_{\sqrt{x}}^2 e^{y^3}\,dy\,dx = \int_0^2\int_0^{y^2} e^{y^3}\,dy =$



$\int_0^2 y^2e^{y^3}\,dy = \frac{1}{3}e^{y^3}\Big|_0^2 = \boxed{\frac{e^8-1}{3}}$

4. $\int_{-4}^{4}\int_0^{\sqrt{16-y^2}}\sqrt{25-x^2-y^2}\,dx\,dy =$



$$\int_0^{\pi}\int_0^{4}\sqrt{25-r^2}\cdot r\,dr\,d\theta =$$

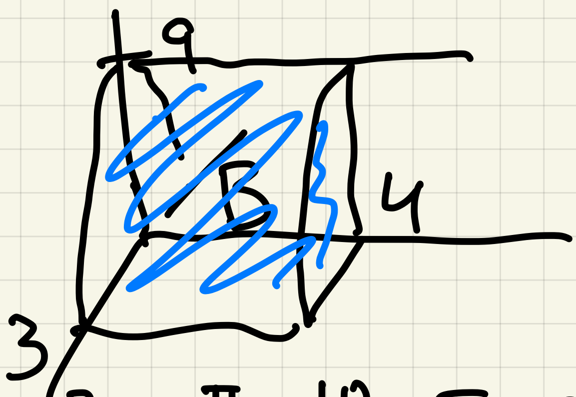
$$\int_0^{\pi} -\frac{1}{3}(25-r^2)^{3/2}\Big|_0^4 = \int_0^{\pi} -\frac{1}{3}(27)+\frac{1}{3}(125) =$$

$$\int_0^{\pi}\frac{98}{3}\,d\theta = \boxed{\frac{98\pi}{3}}$$

$\boxed{5.}$ (a) $$\int_0^4\int_0^3\int_0^{9-x^2} x\,dz\,dx\,dy = \int_0^4\int_0^3 9x-x^3\,dx\,dy$$

$$= \int_0^4 \frac{9}{2}x^2-\frac{x^4}{4}\Big|_0^3 dy = \int_0^4\left(\frac{81}{2}-\frac{81}{4}\right)dy = \int_0^4\frac{81}{4}dy = \boxed{81}$$

(b)



(c) $$\int_0^9\int_0^4\int_0^{\sqrt{9-z}} x\,dx\,dy\,dz$$

$\boxed{6.}$ $$\int_0^{2\pi}\int_0^{\pi}\int_7^{10}\frac{5}{\cancel{\rho^2}}\cancel{\rho^2}\sin\phi\,d\rho\,d\phi\,d\theta =$$

$$\int_0^{2\pi}\int_0^{\pi} 5\rho\Big|_7^{10}\sin\phi = \int_0^{2\pi}\int_0^{\pi} 35\sin\phi\,d\phi =$$

$$\int_0^{2\pi} -35\cos\phi\Big|_0^{2\pi} = \int_0^{2\pi} 70\,d\theta = \boxed{140\pi}$$

$\boxed{7.}$ $\displaystyle\int_0^{\pi}\int_0^{2}\int_0^{\sqrt{16-r^2}} \underbrace{2}zr\,dz\,dr\,d\theta = z^2\Big|_0^{\sqrt{16-r^2}}$

$\displaystyle = \int_0^{\pi}\int_0^{2} 16r - r^3\,dr\,d\theta = \int_0^{\pi} 8r^2 - \frac{r^4}{4}\Big|_0^2 =$

$\displaystyle \int_0^{\pi}(32-4)\,d\theta = 28\theta\Big|_0^{\pi} = \boxed{28\pi}$