Exam 2

$\boxed{\#1}$ $r = \langle t^2, t^3+t^2, t^3-t^2 \rangle$

(a) $v = \langle 2t, 3t^2+2t, 3t^2-2t \rangle$

$a = \langle 2, 6t+2, 6t-2 \rangle$

(b) $v(1) = \langle 2, 5, 1 \rangle$, $a(1) = \langle 2, 8, 4 \rangle$

$$v \times a = \begin{vmatrix} i & j & k \\ 2 & 5 & 1 \\ 2 & 8 & 4 \end{vmatrix} = \langle 12, -6, 6 \rangle$$

$$K = \frac{|v \times a|}{|v|^3} = \frac{6|\langle 2,-1,1 \rangle|}{|\langle 2,5,1 \rangle|^3} = \frac{6\sqrt{6}}{(\sqrt{30})^3}$$

$\boxed{\#2}$ accel $= a = \langle 0, -6 \rangle$ $\downarrow 6$

(a) $v = \int a\,dt = \langle 0, -6t \rangle + \bar{C}$,

$v(0) = \langle 20, 9 \rangle \Rightarrow v(t) = \langle 20, 9-6t \rangle$

(b) $r = \int v = \langle 20t, 9t - 3t^2 \rangle + D$

$r(0) = \langle 0, 12 \rangle \Rightarrow r(t) = \langle \overset{x}{20t}, \overset{y}{12+9t-3t^2} \rangle$

(c) $x = 70 \Rightarrow t = 7/2 \Rightarrow y = 27/4 = 6.75 > 6$,

so ball misses man (by 9 inches)

$\boxed{\#3}$ $f = ye^{xy}$

(a) $f_x = y^2e^{xy}$, $f_y = e^{xy} + xye^{xy}$

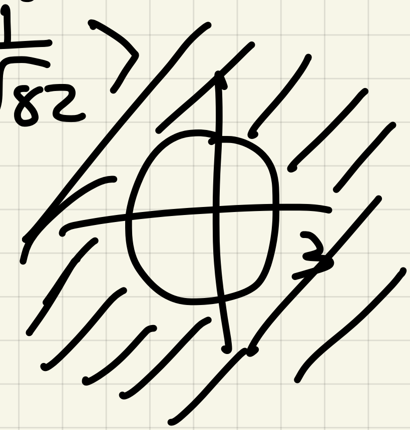
$f_{xx} = y^3e^{xy}$, $f_{xxy} = 3y^2e^{xy} + xy^3e^{xy}$

(b) $\nabla f = \langle y^2e^{xy}, e^{xy} + xye^{xy} \rangle$, $\nabla f(0,3) = \langle 9, 1 \rangle$

(c) $\nabla f(0,3) \cdot \vec{u} = \langle 9,1 \rangle \cdot \frac{\langle -4,3 \rangle}{5} = -\frac{33}{5}$

(d) $-|\nabla f(0,3)| = -|\langle 9,1 \rangle| = -\sqrt{82}$

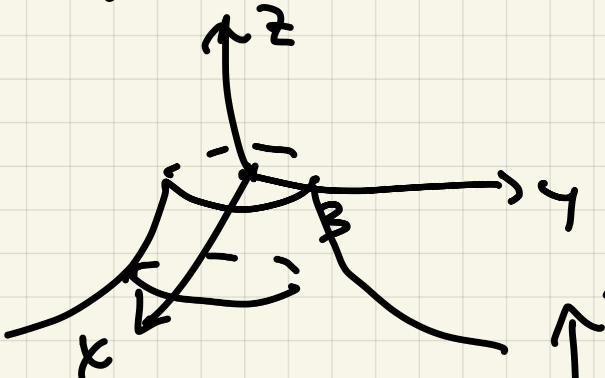
(e) $\frac{\nabla f}{|\nabla f|} = \frac{\langle 9,1 \rangle}{\sqrt{82}} = \left\langle \frac{9}{\sqrt{82}}, \frac{1}{\sqrt{82}} \right\rangle$



$\boxed{\#4}$ (a) Domain: $x^2+y^2 \geq 9$

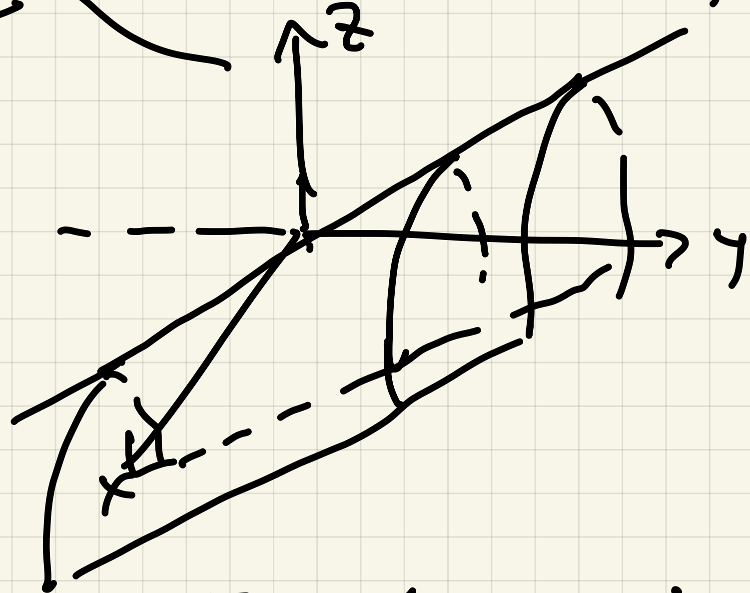
(b) $z = \sqrt{x^2+y^2-9} \Rightarrow$

$z^2 = x^2+y^2-9 \Rightarrow z^2+9 = x^2+y^2, \; z \leq 0$



(c) Range $= (-\infty, 0]$

$\boxed{\#5}$ (a)



(b) $\overbrace{z+x^2-y}^{g} = 0$; $\nabla g = \langle 2x, -1, 1 \rangle$

level surface

$\nabla g(2,+1,-3) = \langle 4,-1,1 \rangle$

$\langle 4,-1,1 \rangle \cdot \langle x-2, y-1, z+3 \rangle = 0 \Rightarrow$

$4x - y + z = 4$

$\boxed{\#6}$

(a) $\lim\limits_{(x,y)\to(1,3)} \dfrac{9x^2-4y^2}{3x-2y} = \dfrac{9-36}{3-6} = \dfrac{-27}{-3} = 9$

(b) $\lim\limits_{(x,y)\to(2,3)} \dfrac{\cancel{(3x-2y)}(3x+2y)}{\cancel{3x-2y}} = \lim(3x+2y) = 12$

$\boxed{\#7}$ $z - xz^3 + xy = 5$

(a) $\partial/\partial x$: $z_x - z^3 - 3z^2 x z_x + y = 0 \Rightarrow$

$$z_x = \frac{z^3 - y}{1-3z^2x}\Bigg|_{(2,3,1)} = \frac{-2}{-5} = \frac{2}{5}.$$

$\partial/\partial y$: $z_y - 3z^2 x z_y + x = 0 \Rightarrow$

$$z_y = \frac{-x}{1-3z^2x}\Bigg|_{(2,3,1)} = \frac{-2}{-5} = 2/5$$

(b)

$\partial/\partial y$: $z_{yy} - 6zx(z_y)^2 - 3z^2x z_{yy} = 0 \Rightarrow$

$$z_{yy} = \frac{+6zx(z_y)^2}{1-3z^2x}\Bigg|_{(2,3,1)} = \frac{12\left(\frac{2}{5}\right)^2}{-5} = \frac{-48}{125}.$$