9/9 Calc3

Quiz 4

$\bar{u} = \langle 1, 1, 2 \rangle$

$\bar{v} = \langle 1, -1, 2 \rangle$

$$\begin{vmatrix} i & j & k \\ 1 & 1 & 2 \\ 1 & -1 & 2 \end{vmatrix} = 1$$

$4i - 0j + (-2)k$

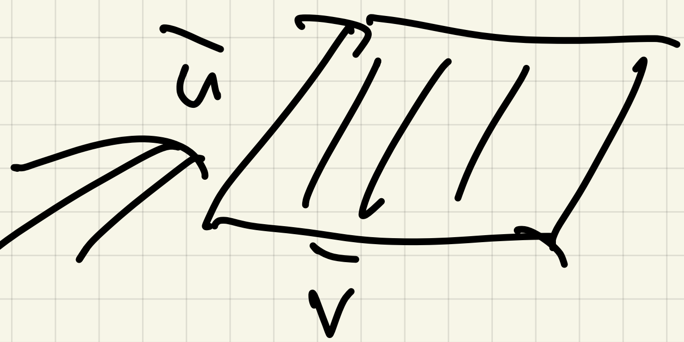
#1

$= 4i - 2k$

$= \boxed{\langle 4, 0, -2 \rangle}$

#2

(a)



$\text{Area} = \| \bar{u} \times \bar{v} \|$

$\| \langle 4, 0, -2 \rangle \| = \sqrt{4^2 + 0^2 + 2^2}$

$= \sqrt{20}$ $\leftarrow$

(b) $\dfrac{\langle 4, 0, -2\rangle}{\sqrt{20}} = \overline{\langle 2/\sqrt{5}, 0, -1/\sqrt{5}\rangle}$

$\langle -\frac{2}{\sqrt{5}}, 0, \frac{1}{\sqrt{5}}\rangle$

Last time surfaces in $\mathbb{R}^3$

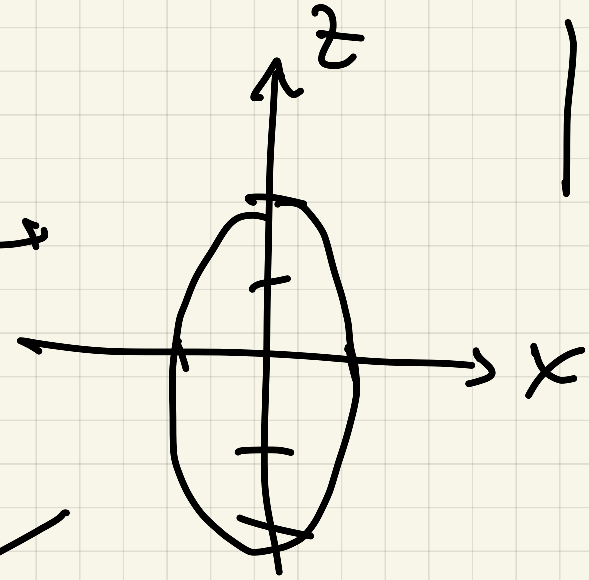
Planes

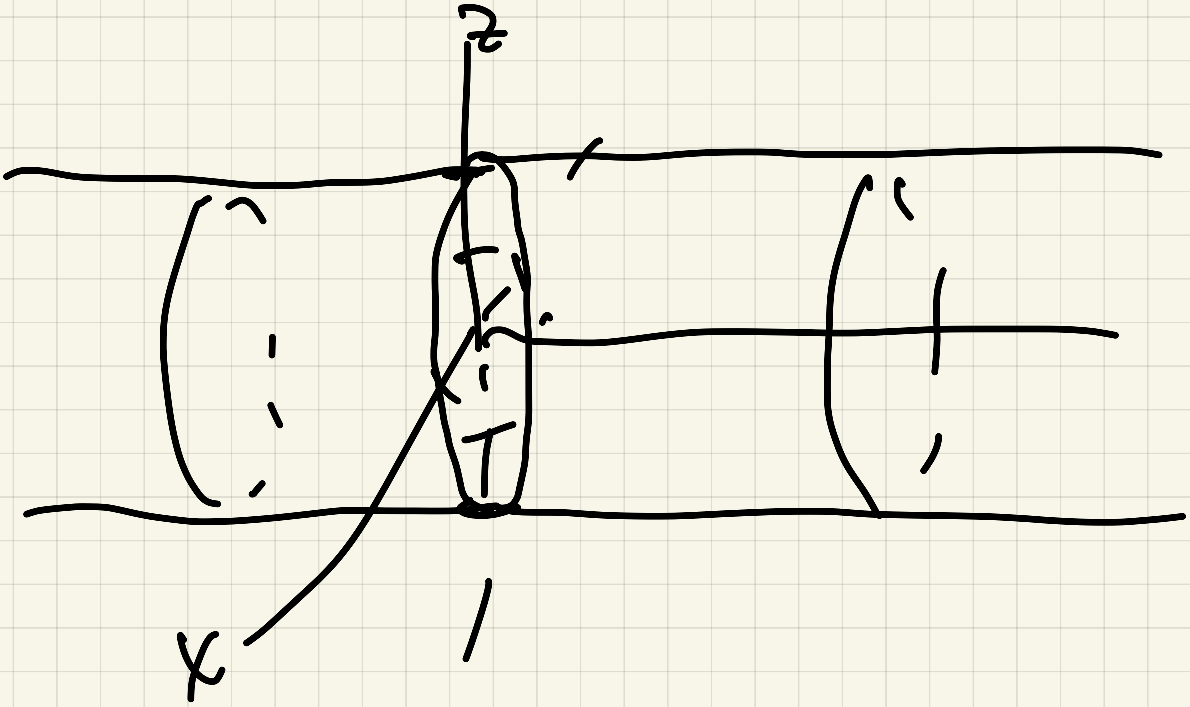
spheres

Cylinders

$x^2 + \frac{z^2}{4} = 1$

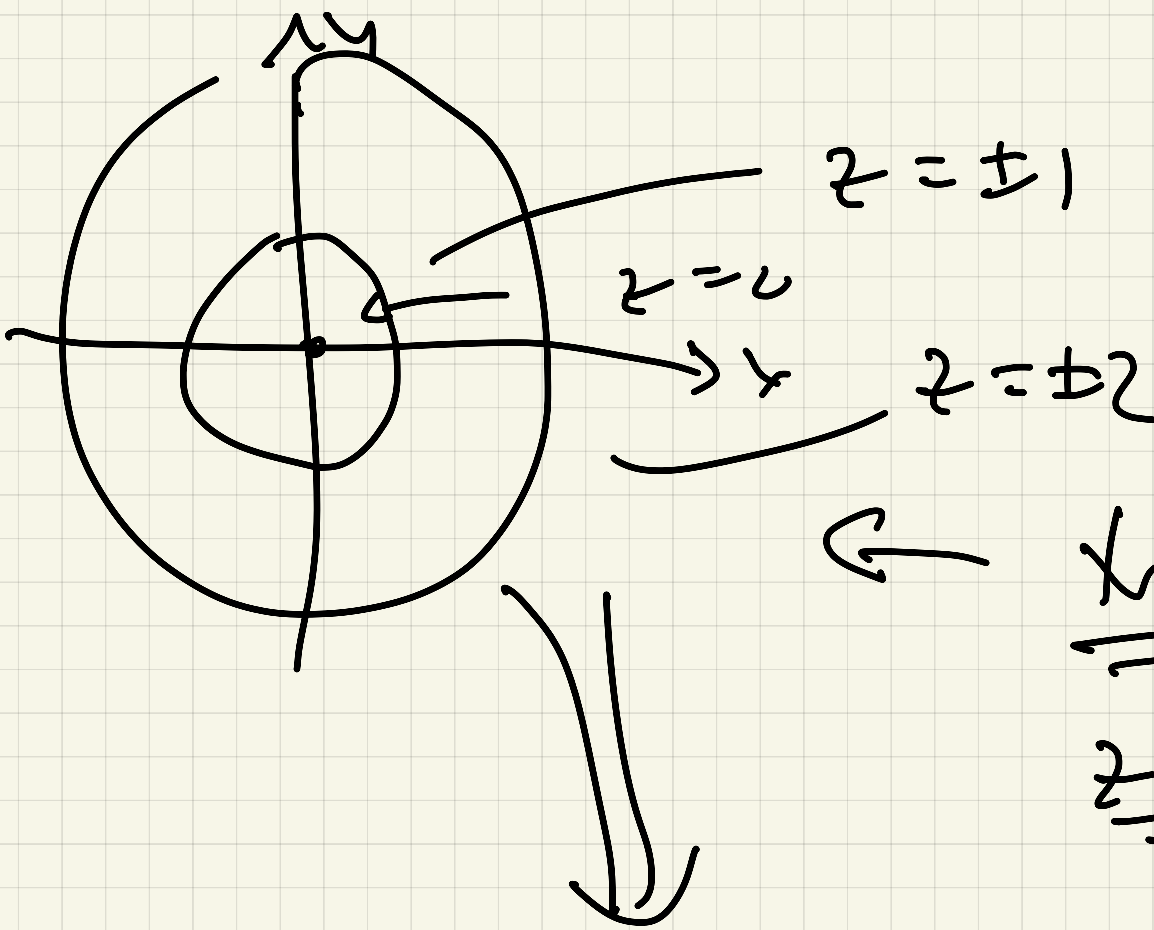
$x^2 + \frac{z^2}{2^2} = 1$





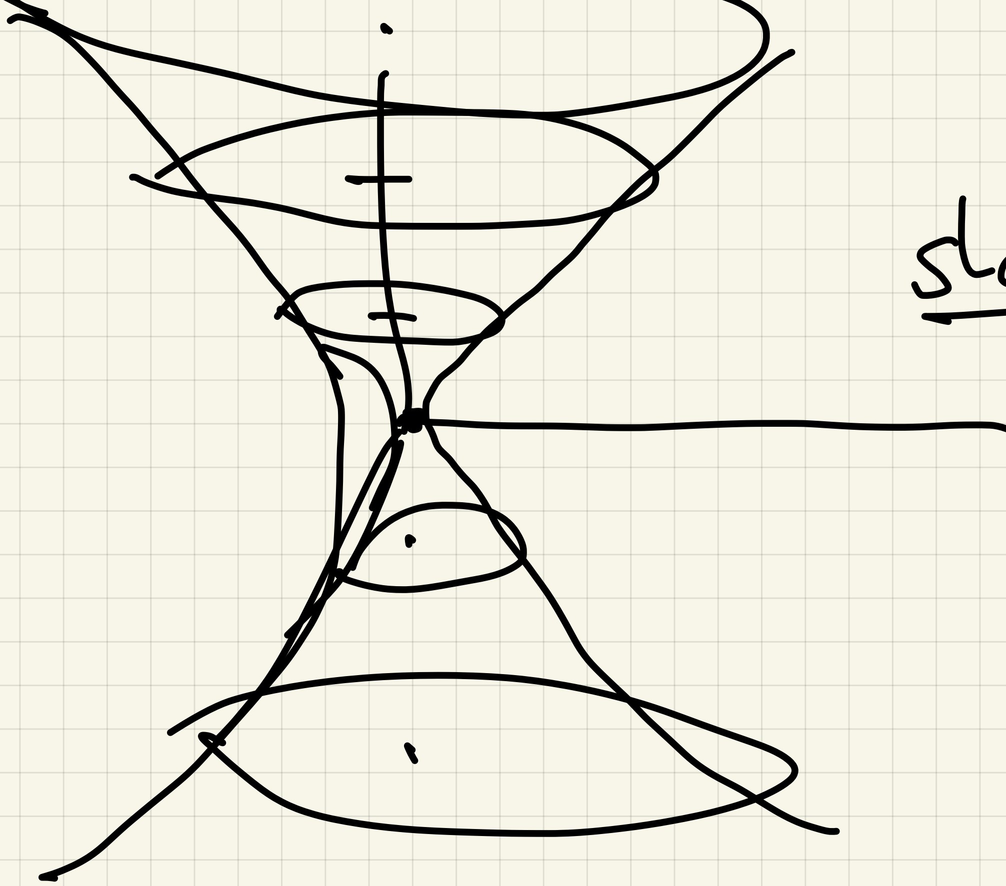
Quadric surfaces

Ex] (a) $x^2+y^2=z^2$



traces

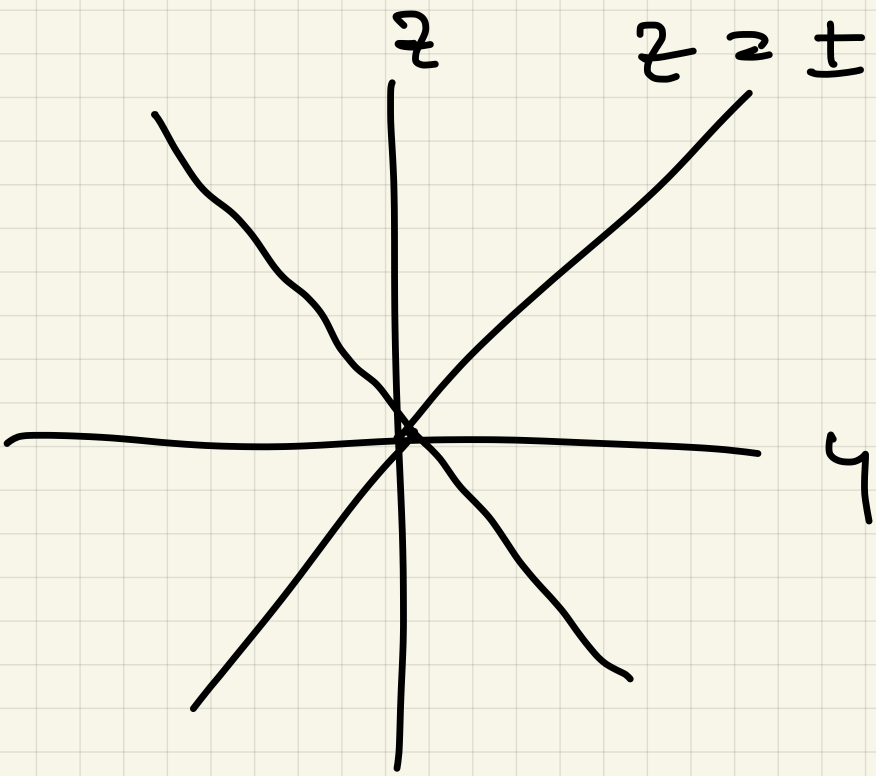
$z=\text{const}$

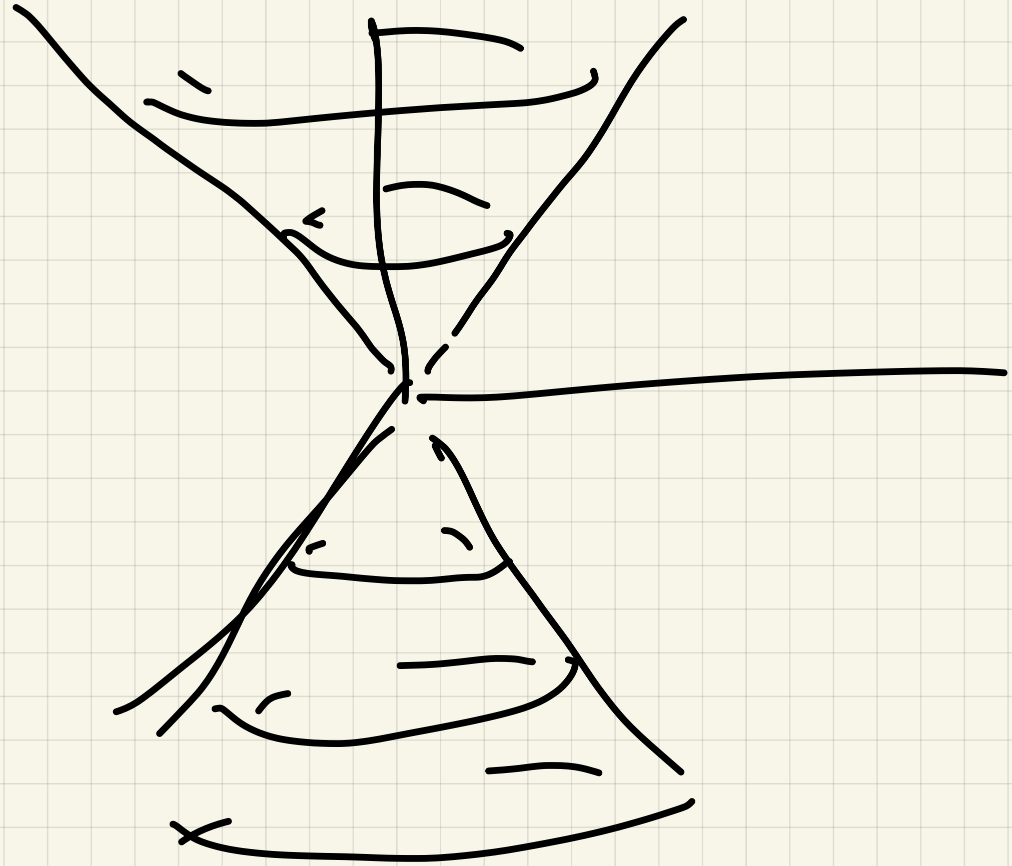


sloppy

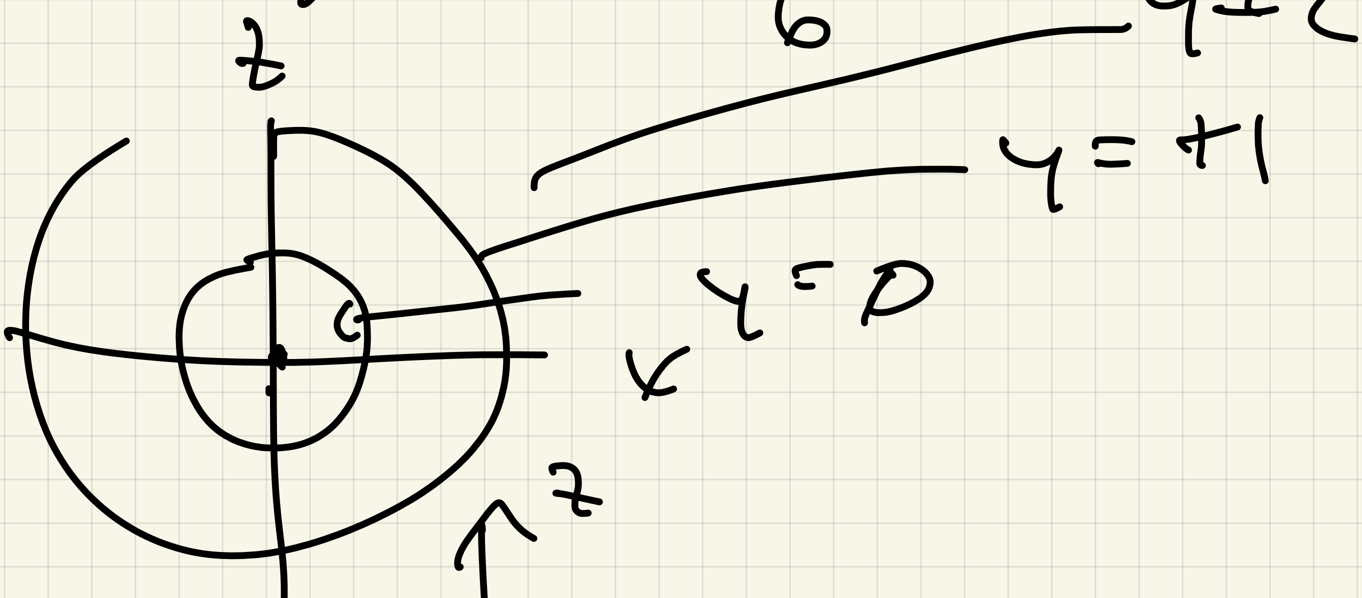
Set $x=0$ $\quad y^2 = z^2$

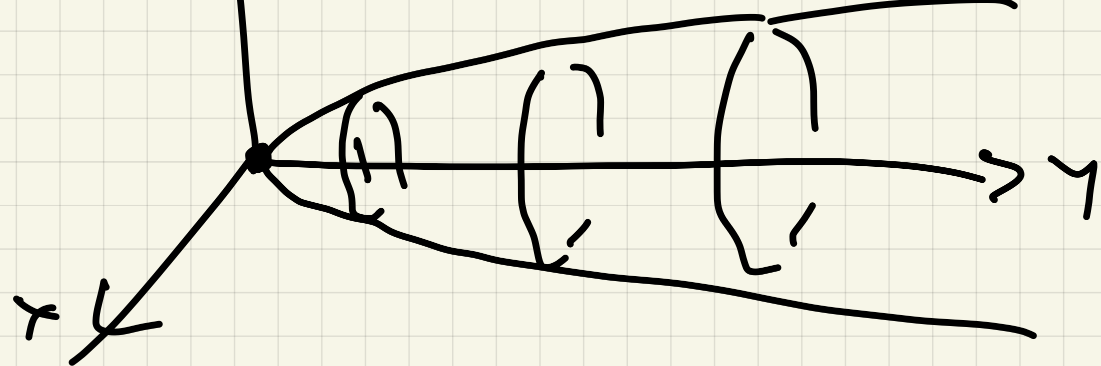
$z = \pm y$



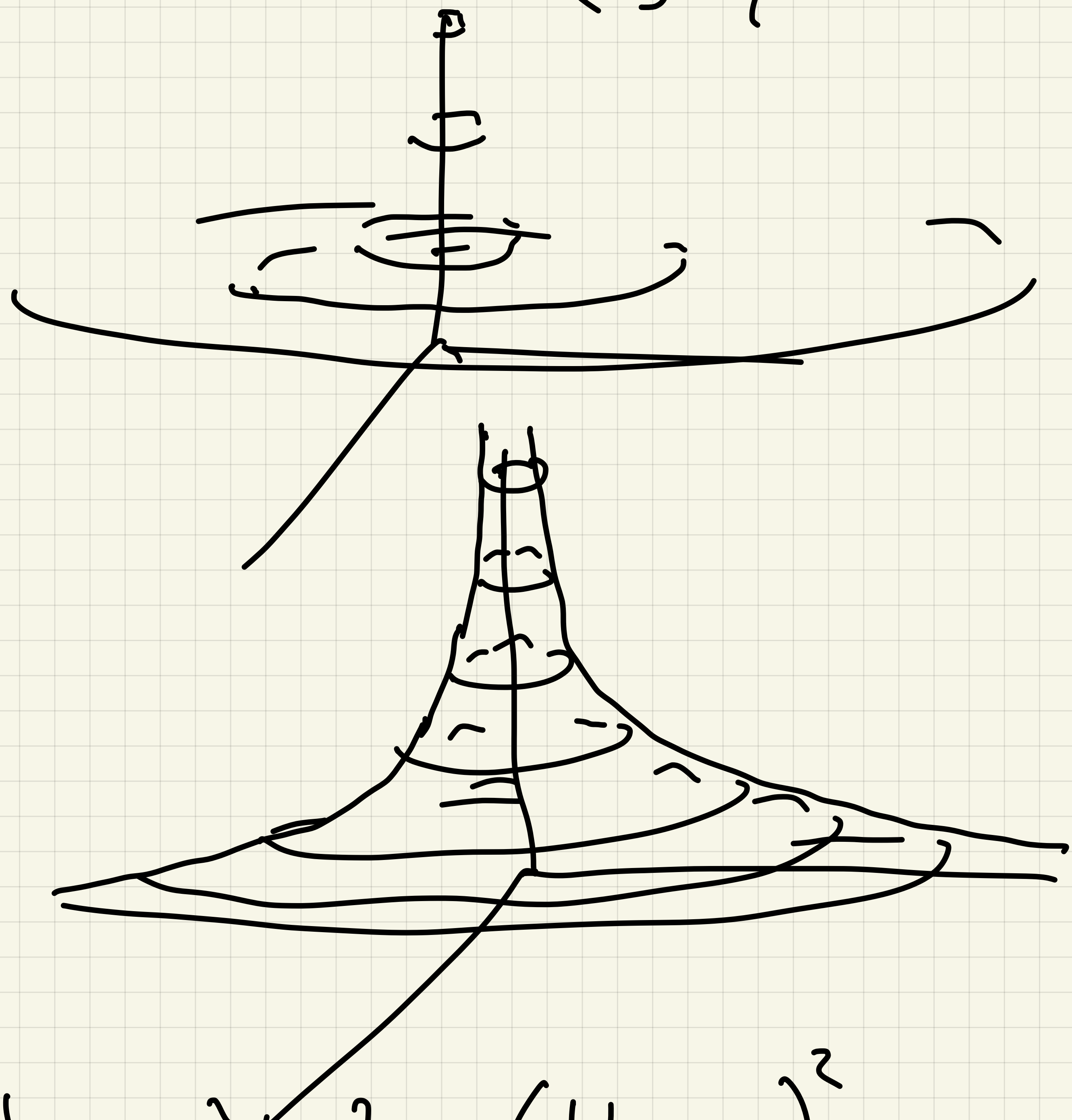


(b) $x^2+z^2 = \frac{y}{6}$

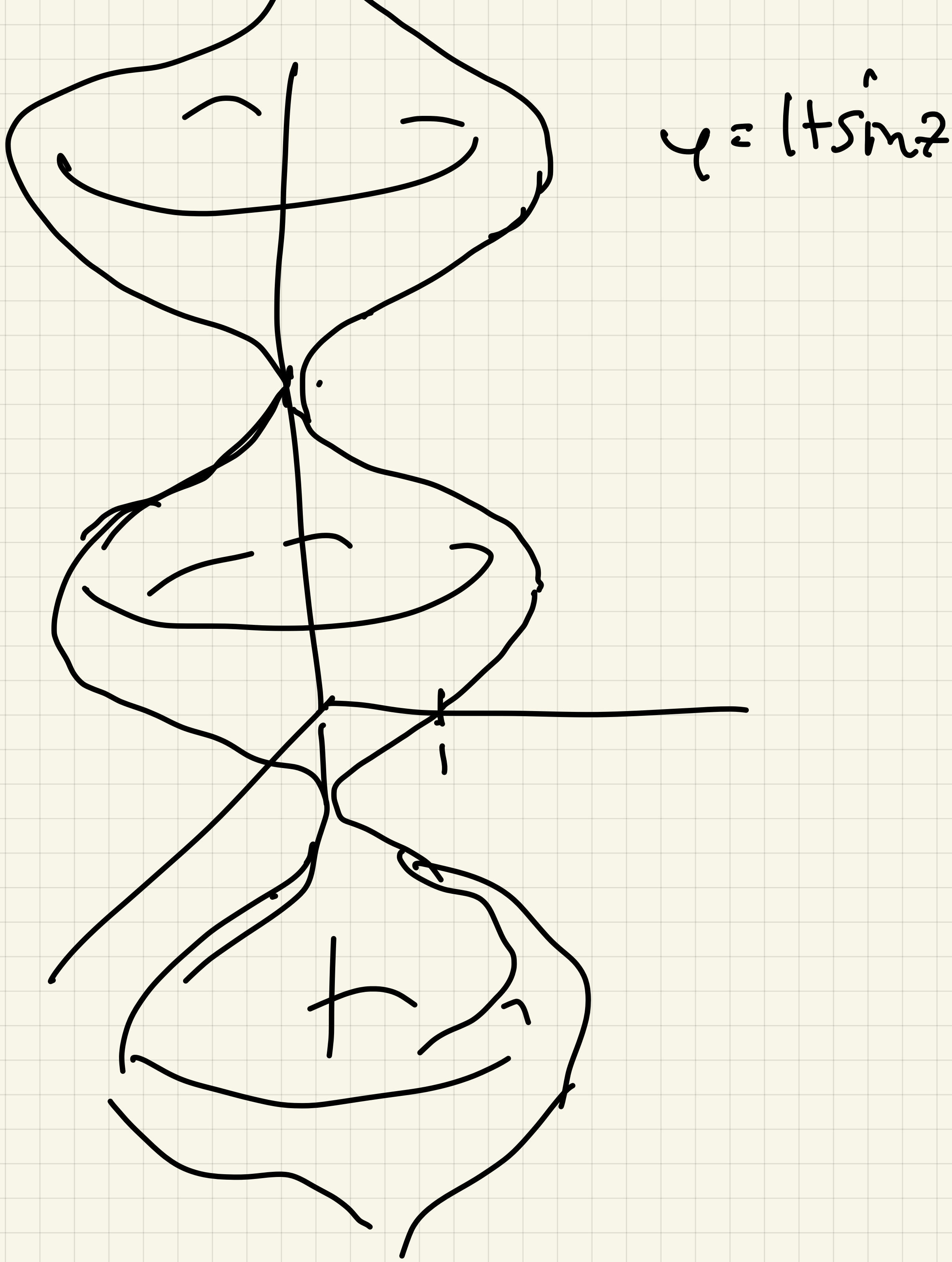




(c) $x^2+y^2 = \left(\frac{1}{z}\right)^2, \ z > 0$



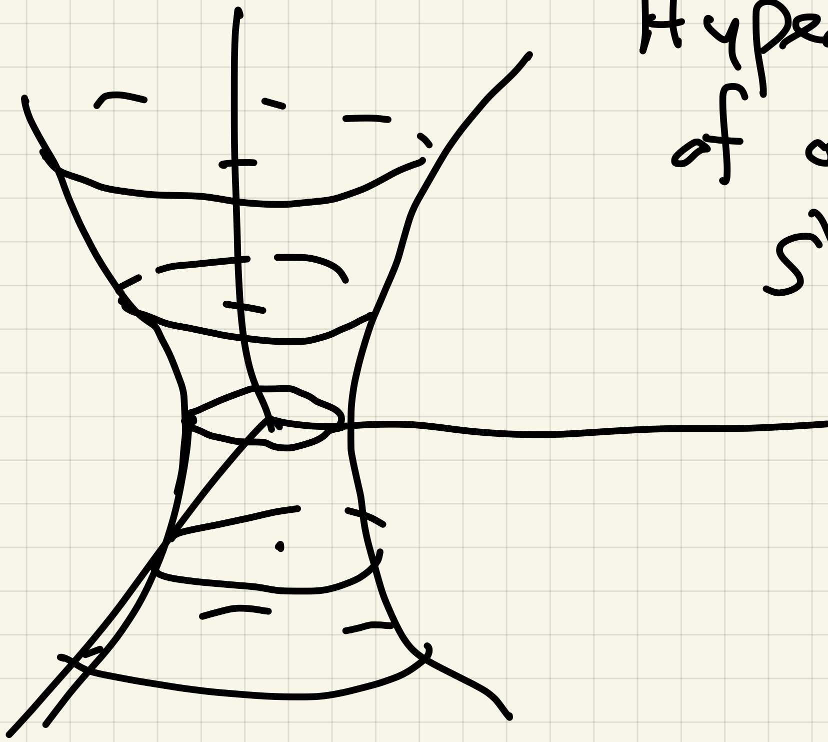
(d) $x^2+y^2 = (1+\sin z)^2$



More quadrics

(a) $x^2+y^2-\frac{z^2}{4}=1$

$$x^2+y^2 = 1 + \frac{z^2}{4}$$



Hyperboloid of one sheet

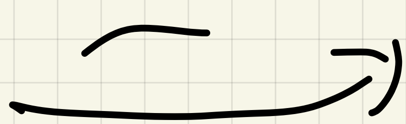
(b) $x^2+y^2 - \frac{z^2}{4} = -1$

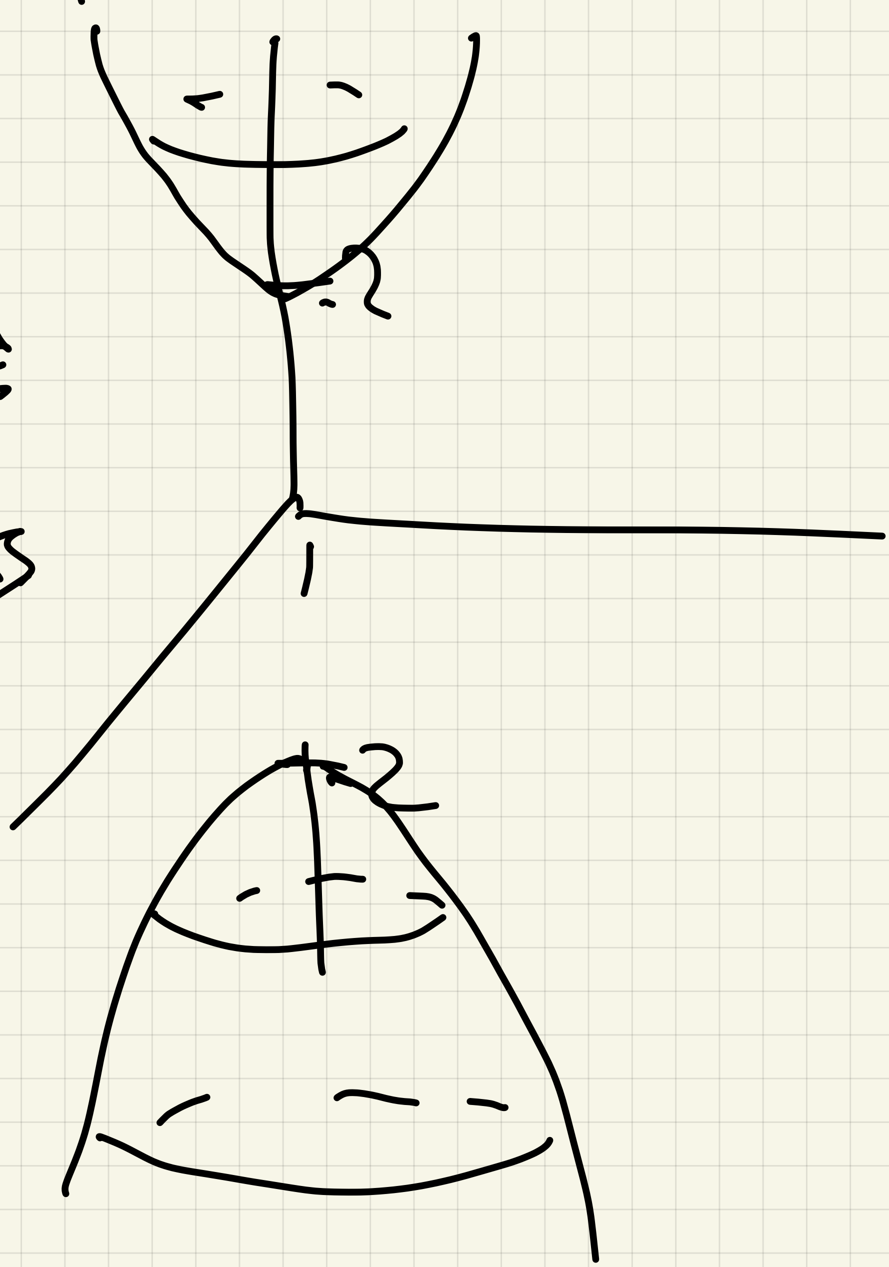
$$x^2+y^2 = -1 + \frac{z^2}{4}$$

for $-2<z<2$, $-1+\frac{z^2}{4}<0$

$\Rightarrow$ nothing on graph.

for $|z|>2$, get circles



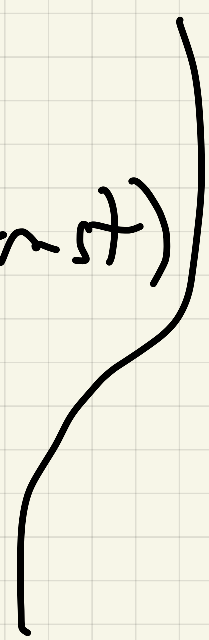


Hyperboloid of two sheets

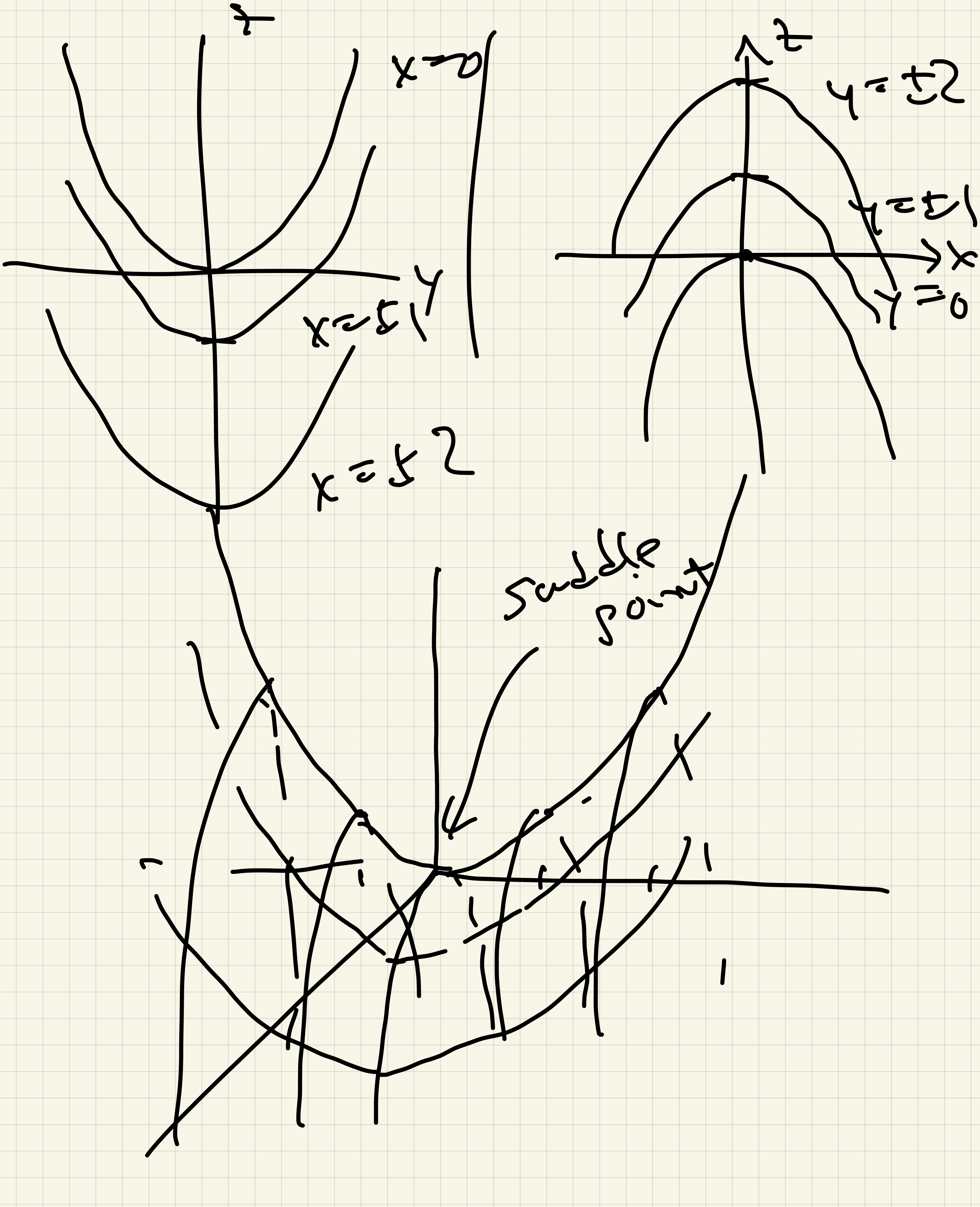
(c) $z = y^2 - x^2$

<u>Idea</u>:

yz-traces (x const)



$y = \text{const}$


z
x=0
y
x=±1
x=±2
z
y=±2
y=±1
x
y=0
saddle point

§12.1

Vector-valued functions

have the form

$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$$

$$= f(t)\,\vec{i} + g(t)\,\vec{j} + h(t)\,\vec{k}$$

Can visualize as motion of a particle in $\mathbb{R}^3$

Ex) $r(t) = \langle 0, 2t, 2-t \rangle$

$r(t)$.

$x = 0$

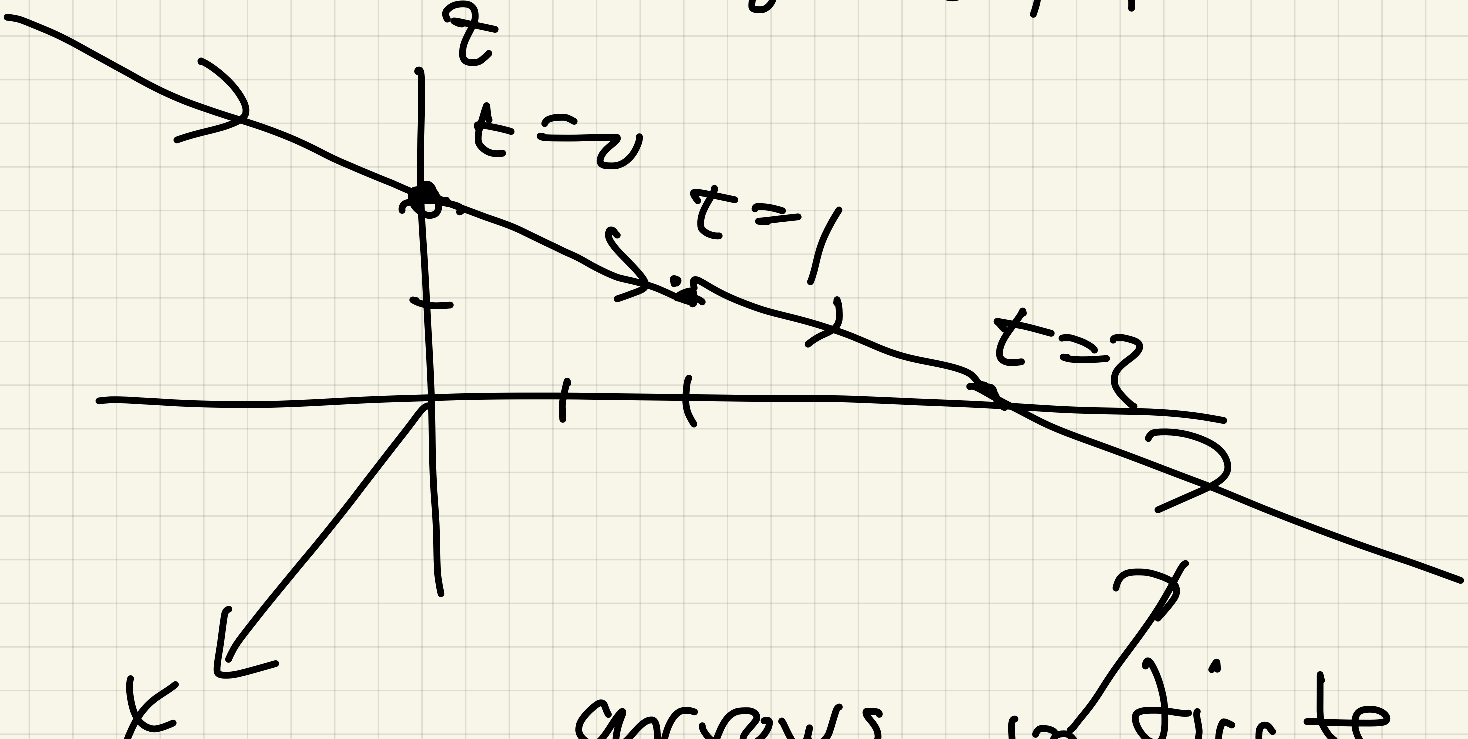
$y = 2t$

$z = 2-t$

$r(t)$ gives parametric for

line with direction $\langle 0,2,-1\rangle$

through $P_0 = (0,0,2)$



arrows indicate orientation