9/5) Calc 3

$P_0 = (x_0, y_0, z_0)$

Last time $\vec{v} = \langle a, b, c \rangle$

① The line through P_0 with direction $\vec{v}$ has parametric equations

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_0 + at \\ y_0 + bt \\ z_0 + ct \end{pmatrix}$$

$$\vec{r}(t) = P_0 + \vec{v}t$$

② The plane through P_0 with normal vector $\vec{v}$ has ~~se~~ an equation

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

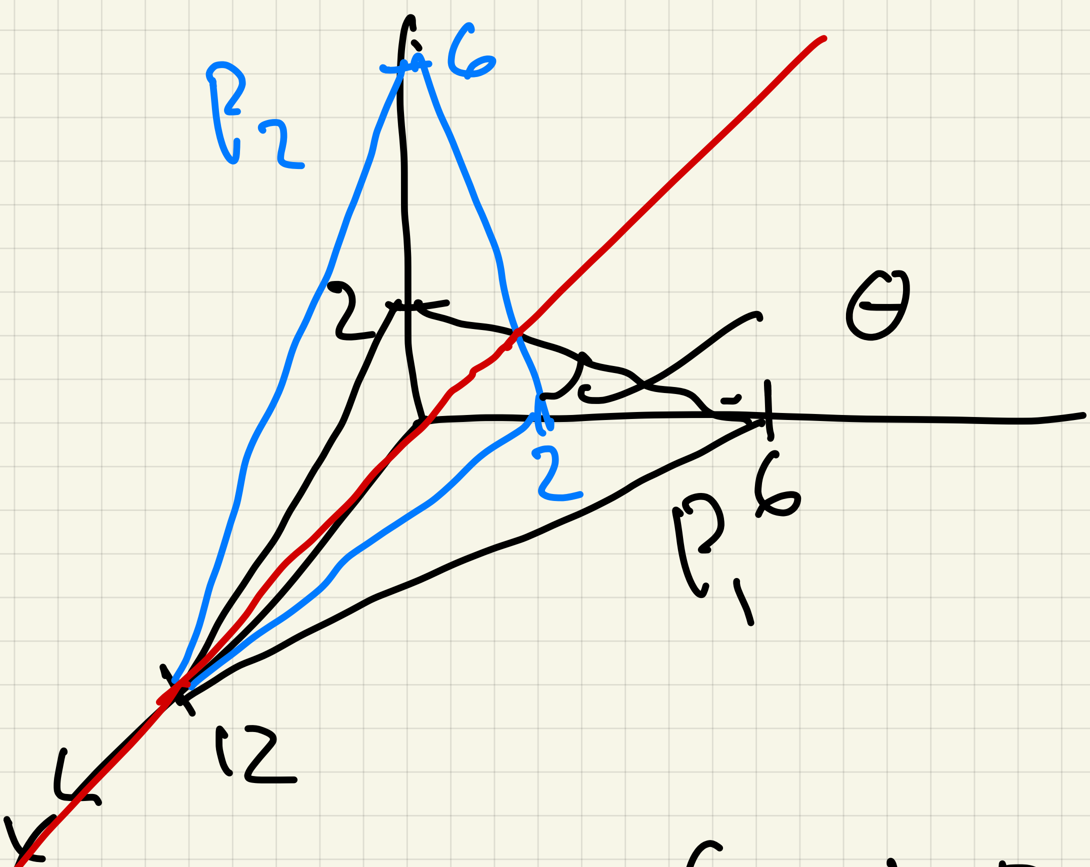
Ex) Consider planes

$P_1 : \quad x + 2y + 6z = 12$

$P_2 : \quad x + 6y + 2z = 12$

$\overline{n_1} = \langle 1, 2, 6 \rangle$

$\overline{n_2} = \langle 1, 6, 2 \rangle$



Last time we found the line $L = P_1 \cap P_2$

(a) $L: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 - 8t \\ 0 + t \\ 0 + t \end{pmatrix}$

(b) The angle θ between planes is given by

$$\cos\theta = \frac{|n_1 \cdot n_2|}{|n_1||n_2|} = \frac{25}{41}$$

(c) Where does L intersect the plane

$$P_3 : \quad 2x + y + 8z = 32 \, ?$$

Easy:

$$L : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 - 8t \\ t \\ t \end{pmatrix}$$

substitute:

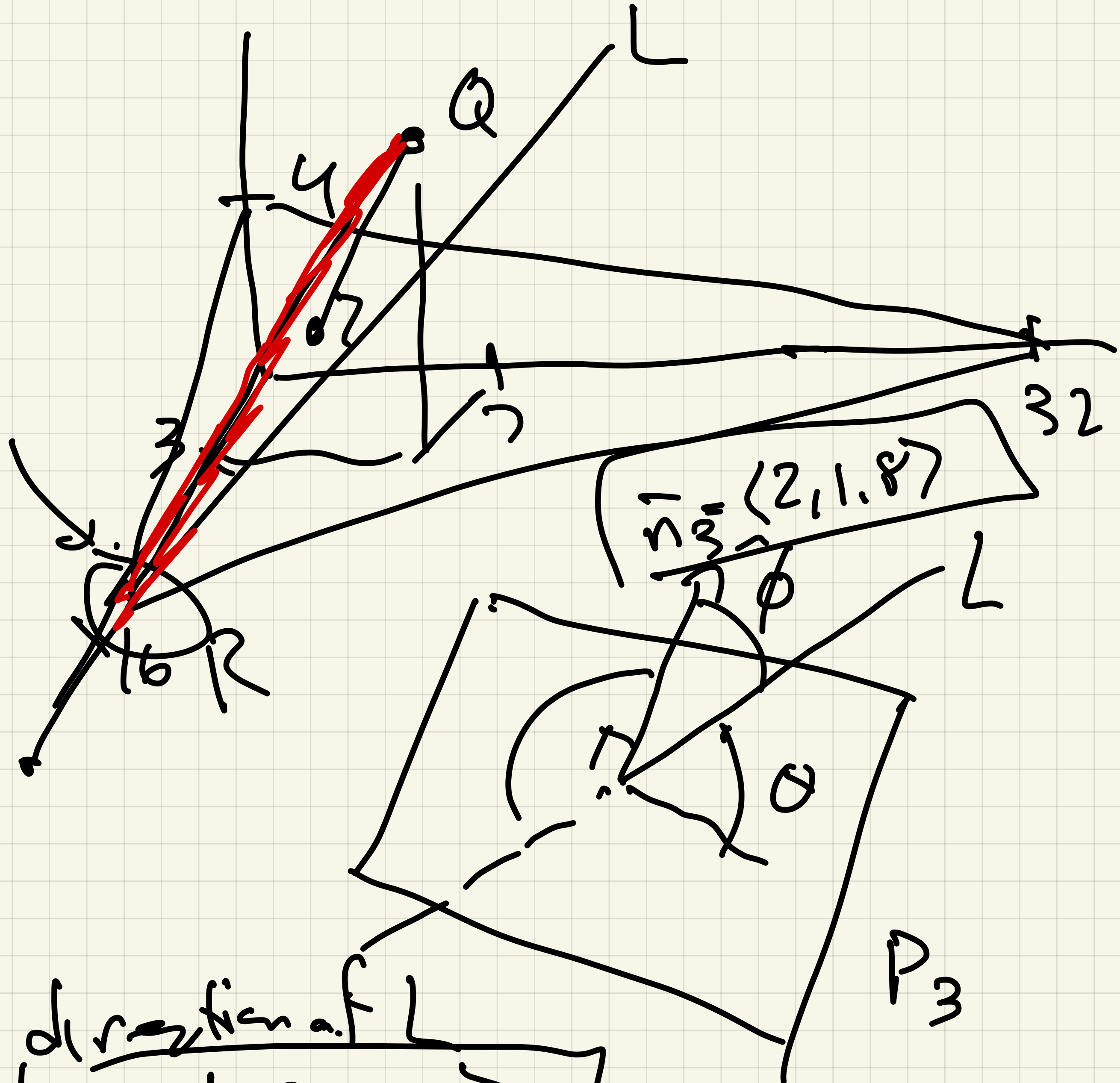
$$2(12-8t) + t + 8(t) = 32$$

$$-16t + t + 8t + 24 = 32$$

$$-7t = 8 \quad t = -8/7$$

$$\text{so } P = \begin{pmatrix} 12 + 64/7 \\ -8/7 \\ -8/7 \end{pmatrix} = \begin{pmatrix} 148/7 \\ -8/7 \\ -8/7 \end{pmatrix}$$

(c) Find the angle between L and P_3



direction of L

$$\boxed{\bar{v} = \langle -8, 1, 1 \rangle}$$

So $\cos\phi = \dfrac{|\bar{n}_3 \cdot \bar{v}|}{|n_3||\bar{v}|} =$

$$\frac{|-16+1+8|}{\sqrt{69}\,\sqrt{66}} = \frac{7}{\sqrt{69}\sqrt{66}} = \cos\phi$$

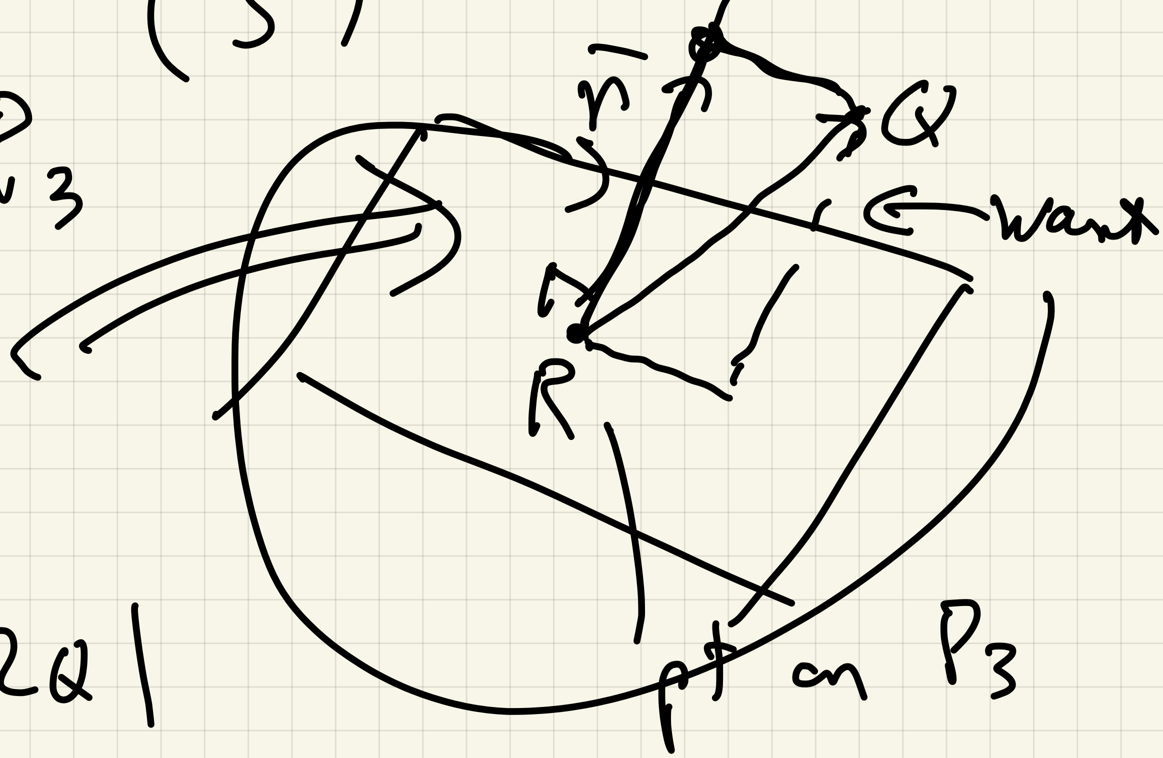
so ϕ

$\theta = 90^\circ - \phi = 5.954^\circ$

(e) Find distance from

$Q = \begin{pmatrix} 3 \\ 7 \\ 5 \end{pmatrix} = (3, 7, 5)$

to P_3



dist
$=$
$|\mathrm{Proj}_{\bar{n}_3} RQ|$

R = pt on P_3;

Take $R = \begin{pmatrix} 16 \\ 0 \\ 0 \end{pmatrix}$

$$Q = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix} \quad \text{so}$$

$$\overrightarrow{RQ} = \begin{pmatrix} -13 \\ 7 \\ 5 \end{pmatrix} \qquad \bar{n} = \langle 2, 1, 8 \rangle$$

$$\parallel$$

$$\langle -13, 7, 5 \rangle$$

$$\boxed{\text{Proj}_{\bar{v}} \bar{u} = \frac{\bar{u} \cdot \bar{v}}{|v|^2} \cdot \bar{v}}$$

$$\text{Proj}_{\langle 2,1,8 \rangle} \langle -13, 7, 5 \rangle =$$

$$\frac{-26 + 7 + 40}{69} \langle 2, 1, 8 \rangle =$$

$$\left| \frac{21}{69} \langle 2, 1, 8 \rangle \right| =$$

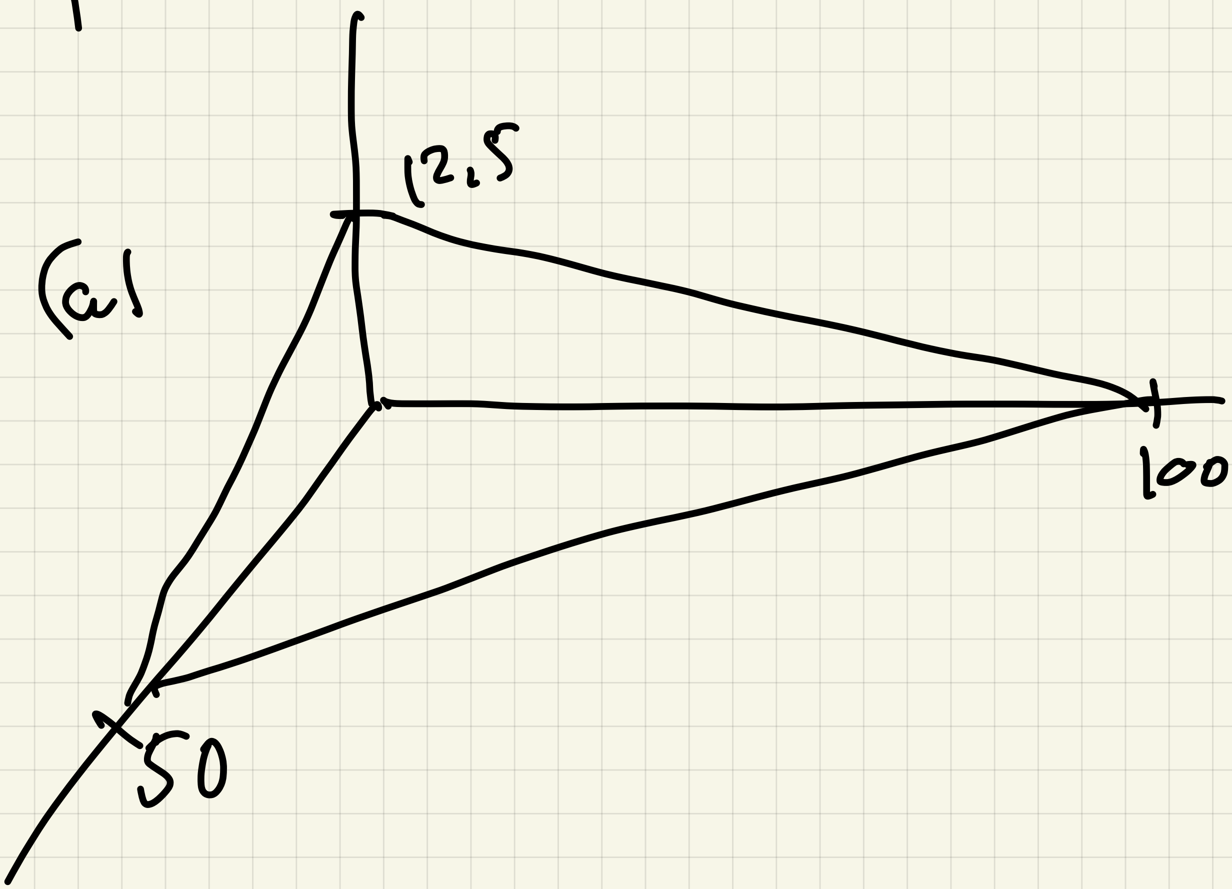
$$\frac{21}{69} \sqrt{69} = \frac{21}{\sqrt{69}} = 2.528$$

§ 11.6 Surfaces in $\mathbb{R}^3$

Exo (a) planes $2x+4y+8z=100$

(b) $(x-1)^2+(y-7)^2+(z+3)^2=144$

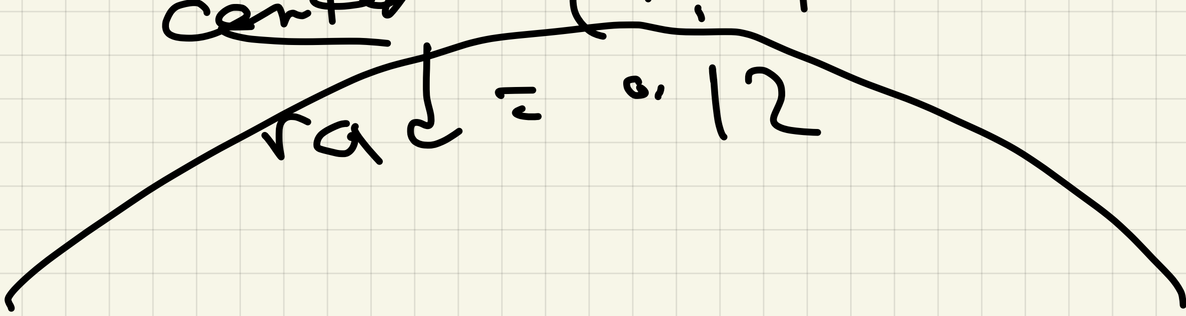
(a)

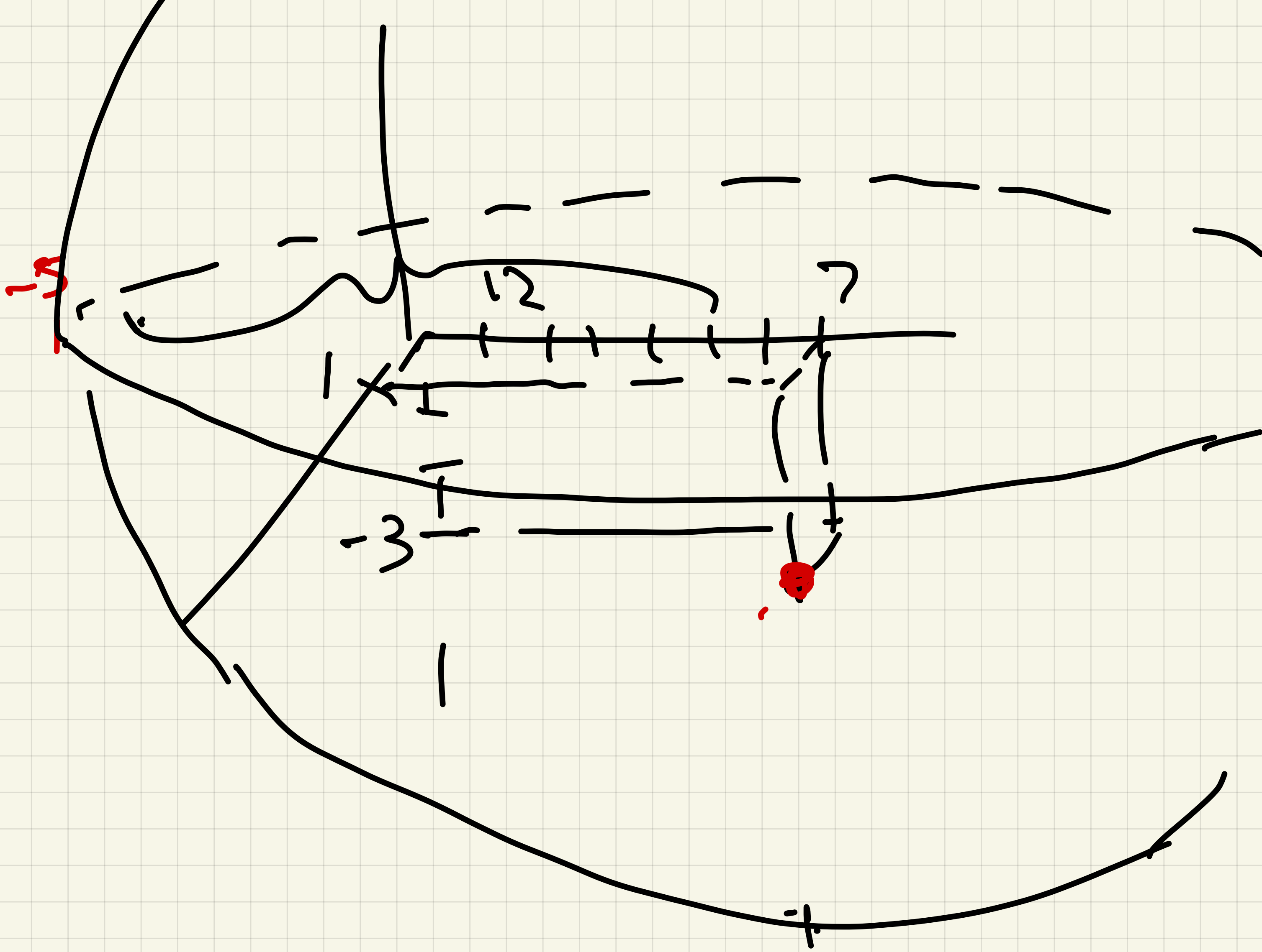


(b) sphere

center $(1,7,-3)$

rad $= 12$

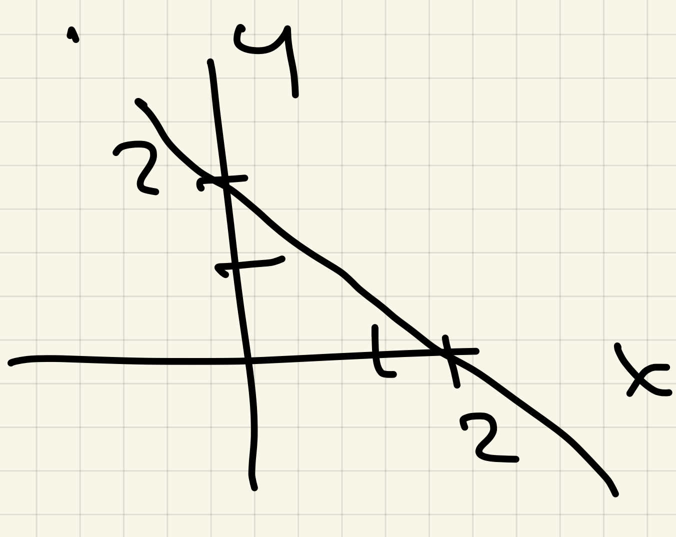




I. Cylinders

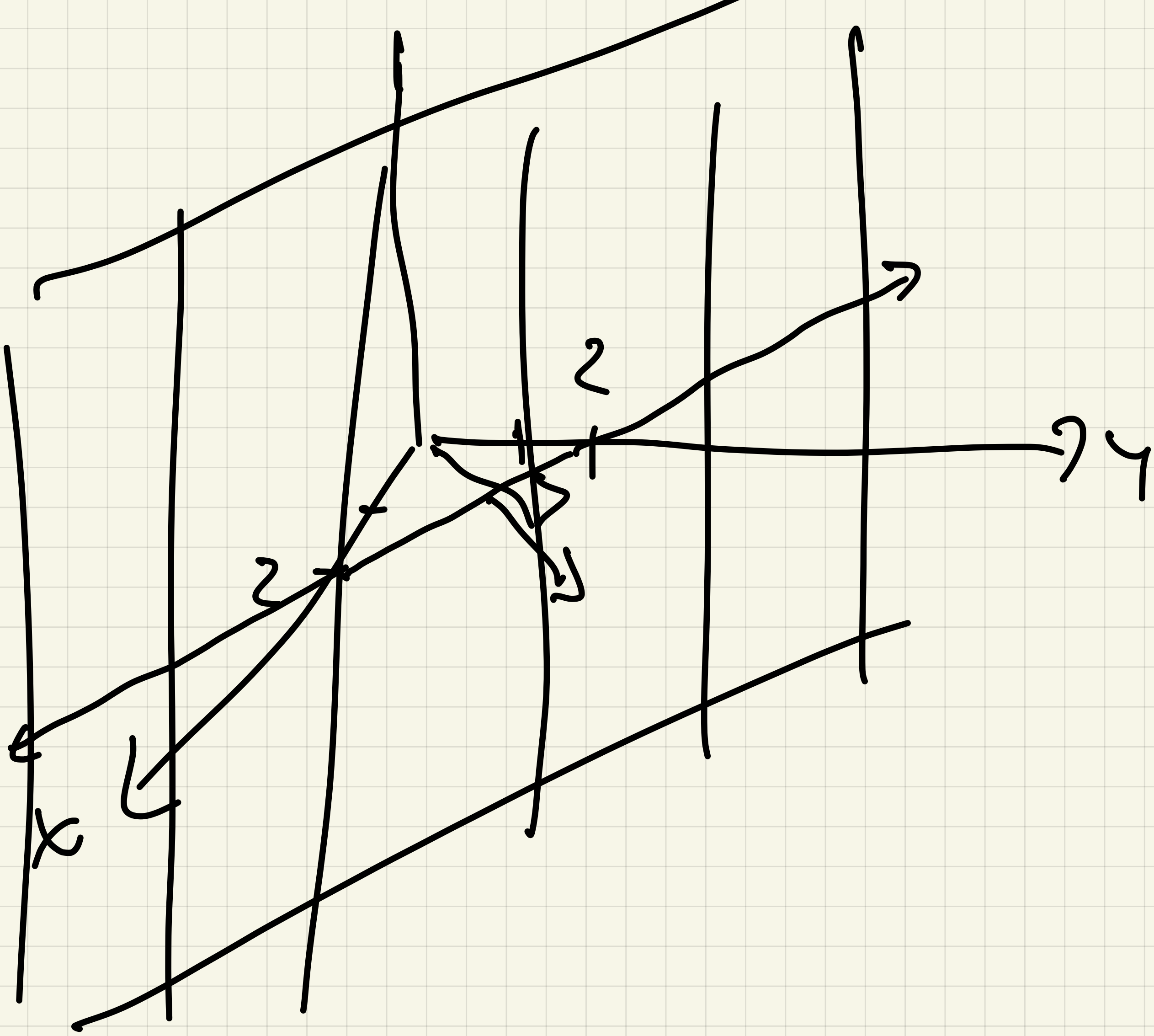
Equations use only 2 of 3 variables.

Ex) $\underline{\underline{x+y=2}}$



$\vec{n} = \langle 1,1,0 \rangle$

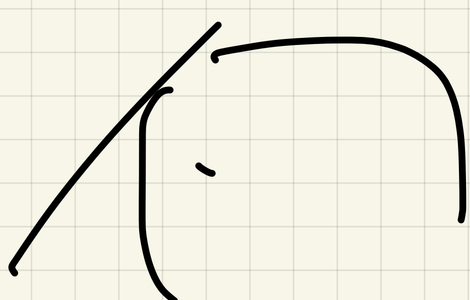
↑ ?

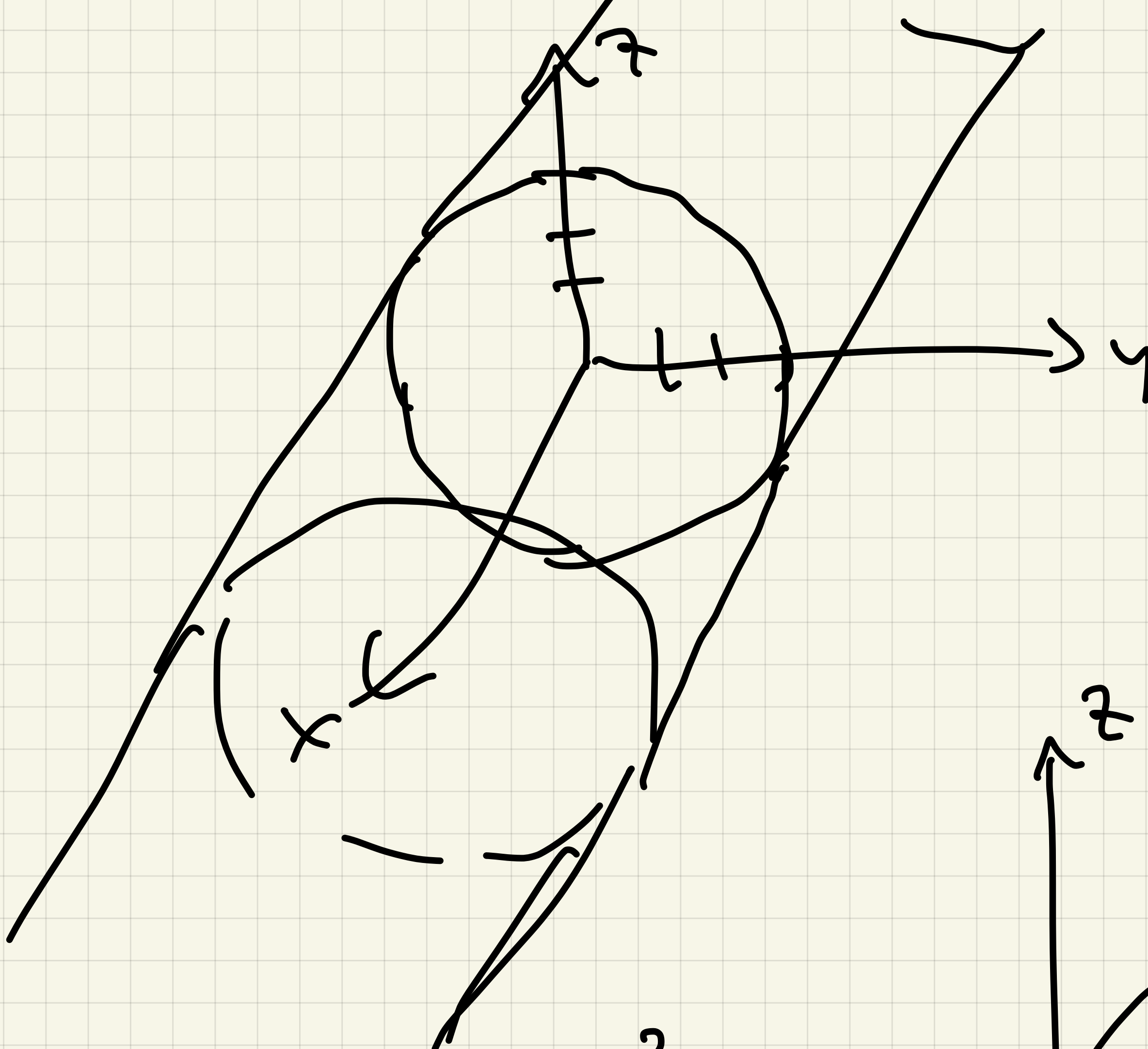


lines // to z-axis are called <u>rulings</u>

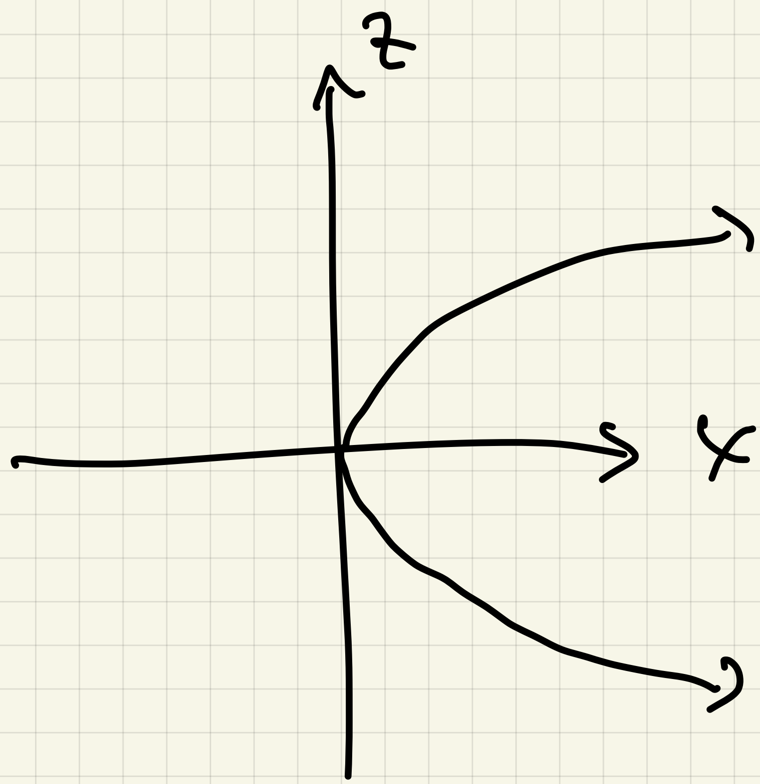
(b) $y^2+z^2=9$

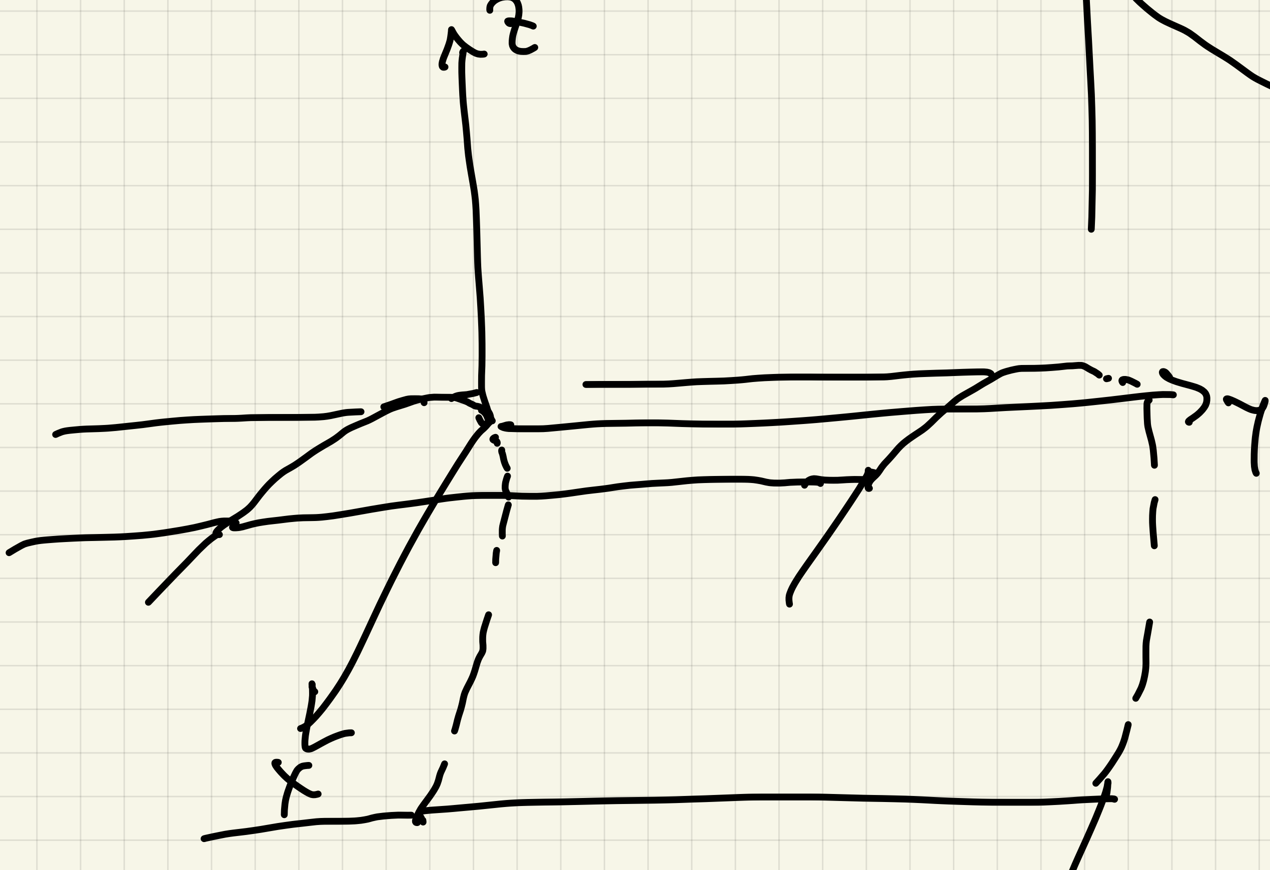
no x-variable





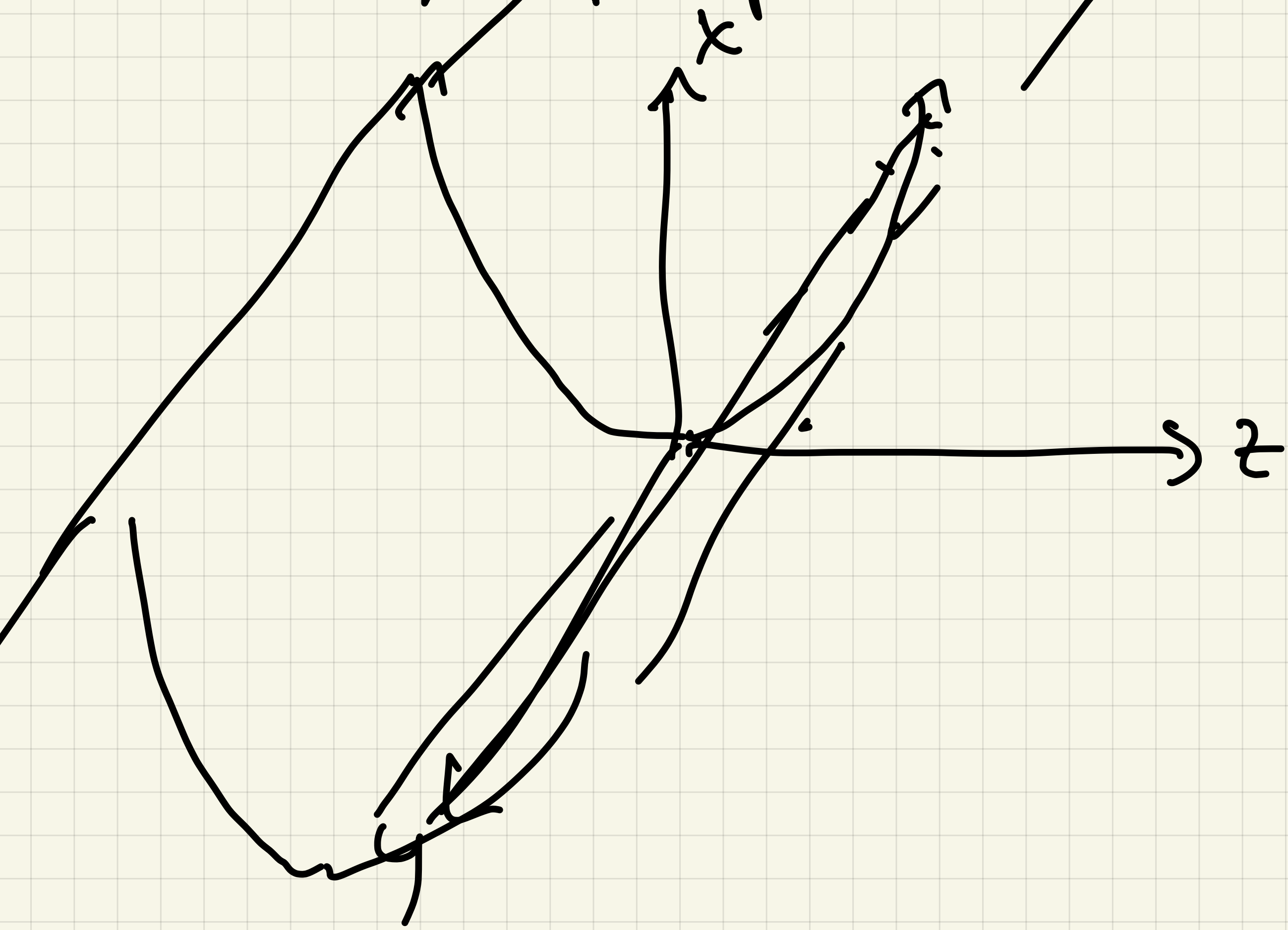
(c) $x = z^2$



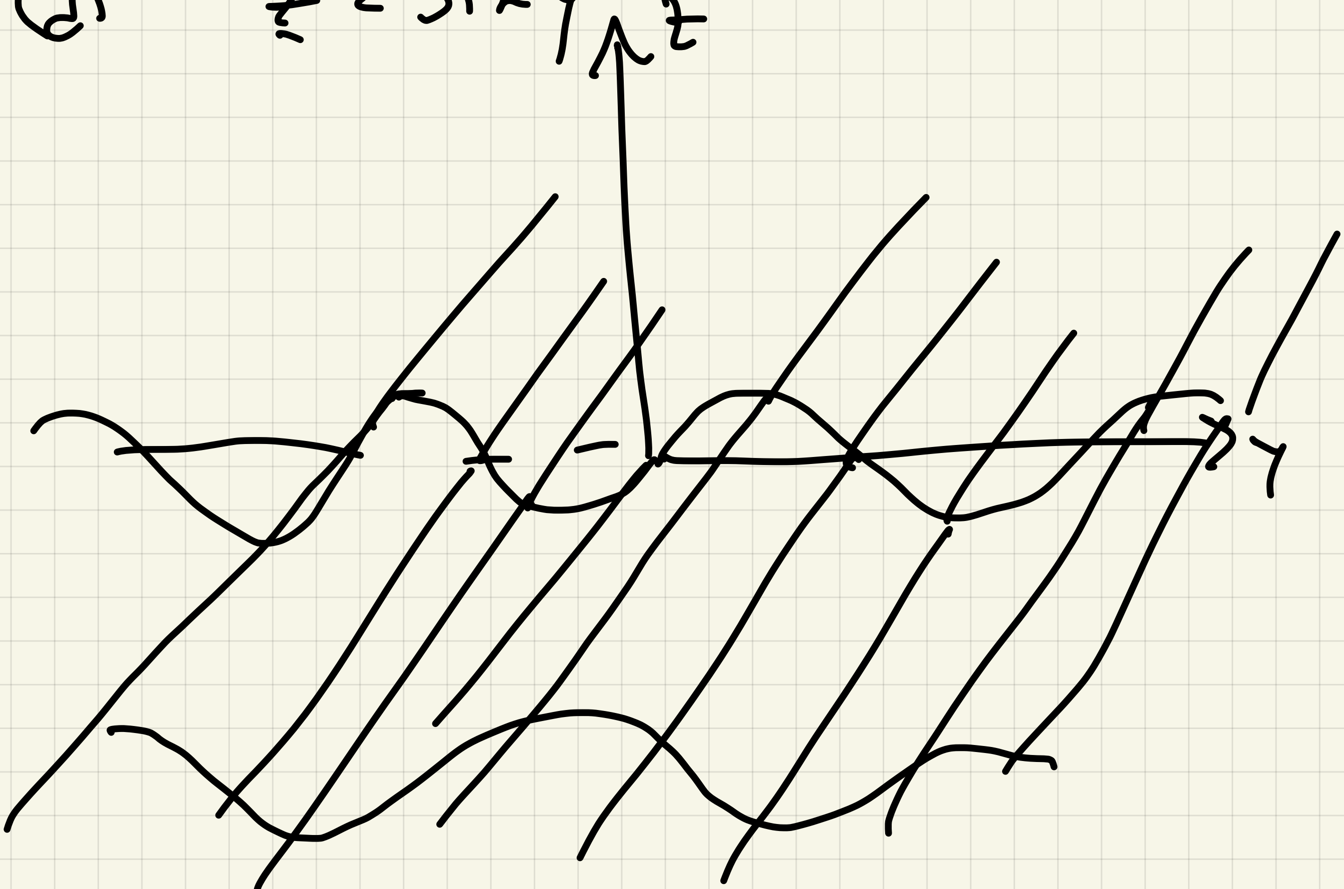


tough to draw

change perspective:



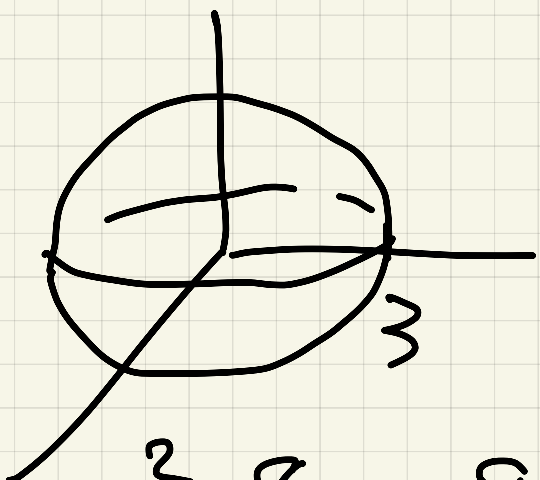
(d) $z = \sin y$



II. Quadric surfaces

Equations of degree ≤ 2 in x, y, z

Ex 2. (a) $x^2+y^2+z^2=9$

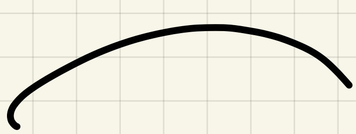


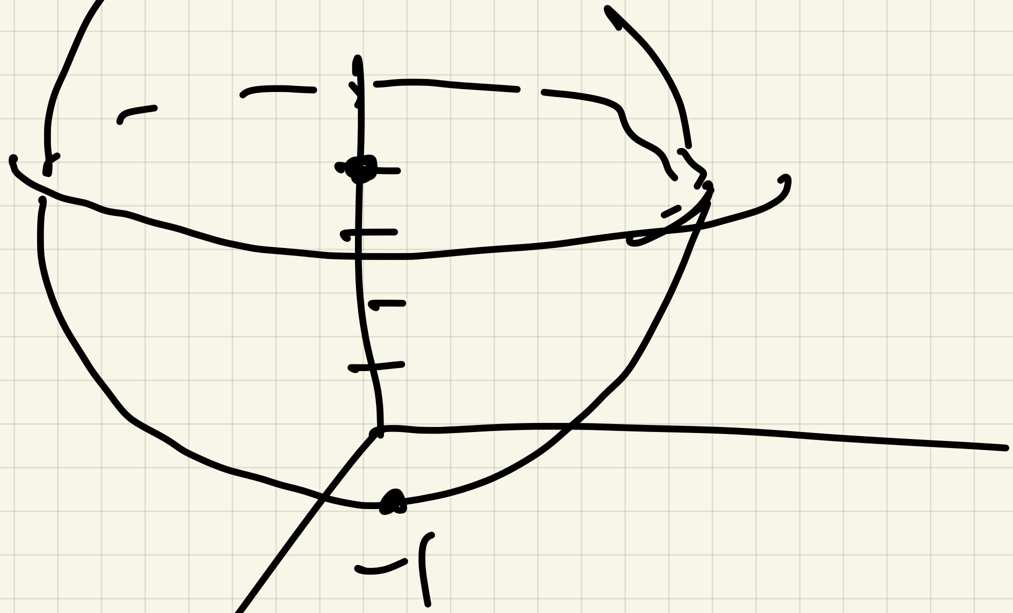
(b) $x^2+y^2+z^2-8z=9$

$$\Downarrow$$

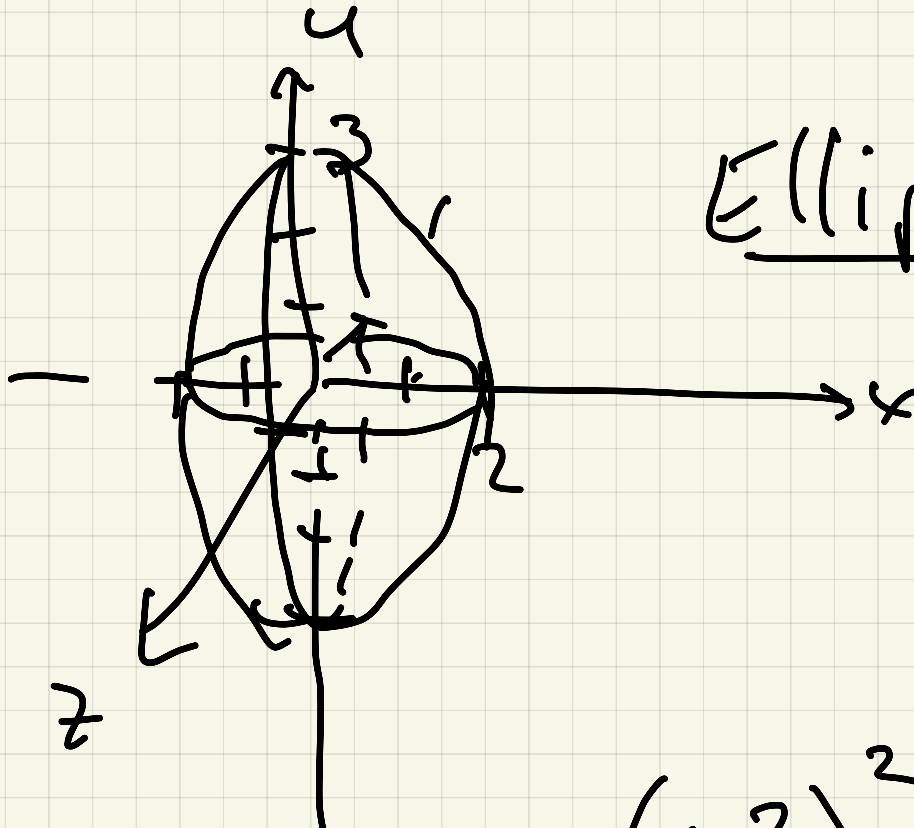
$$x^2+y^2+\underline{z^2-8z+16}=9+16$$

$$x^2+y^2+(z-4)^2=25$$





(c) $\frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1$



Ellipsoid

(d) $x^2 + z^2 = \left(\frac{y-2}{2}\right)^2$

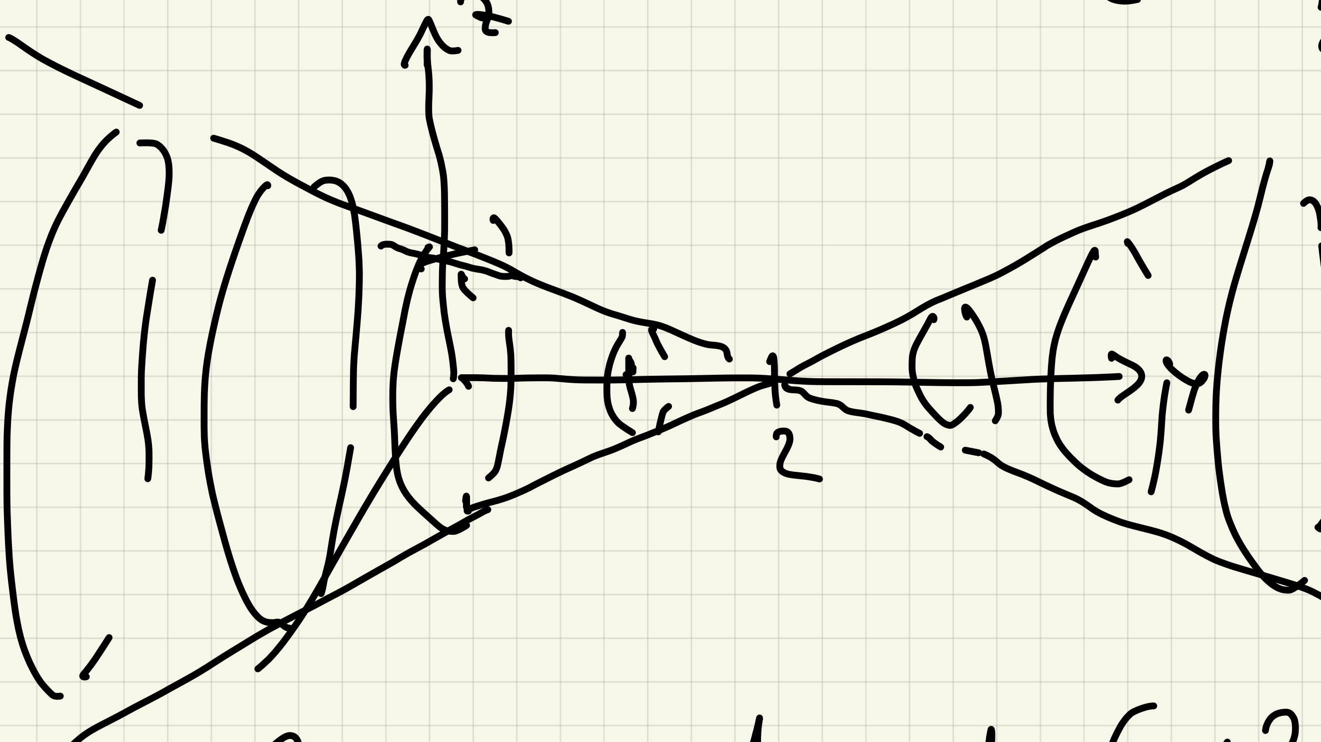
Idea: for y fixed,

$x^2+z^2 = \left(\frac{y-2}{2}\right)^2$ is

a circle of radius

$\left|\frac{y-2}{2}\right|$

$z = \frac{y-2}{2}$



Cone vertex at $(0,2,0)$