9/4/ Calc 8 Quiz 3 avg 90%

$u = \langle 2, 6, 3 \rangle$

$v = \langle 2, -1, 2 \rangle$

(a) $4 + -6 + 6 = 4$

(b) $\cos\theta = \dfrac{\bar{u}\cdot\bar{v}}{|u||v|} = \dfrac{4}{7\cdot 3} = \dfrac{4}{21}$

$|u| = \sqrt{2^2+6^2+3^2}$

$4+36+9$

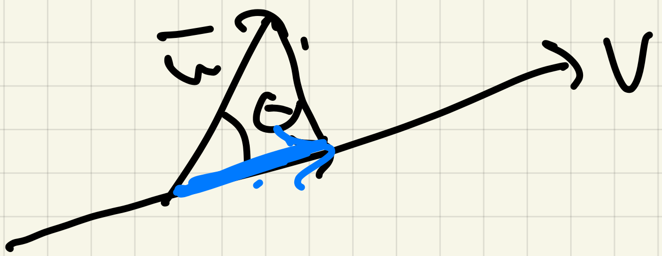
(c) $\bar{w} = \langle 3, -2, 2 \rangle$

$\bar{u}\cdot\bar{w} = 6 - 12 + 6 = 0 \quad \bar{u} \perp \bar{w}$

$\bar{v}\cdot\bar{w} = 6 + 2 + 4 = 12 \neq 0 \quad \bar{v} \not\perp \bar{w}$

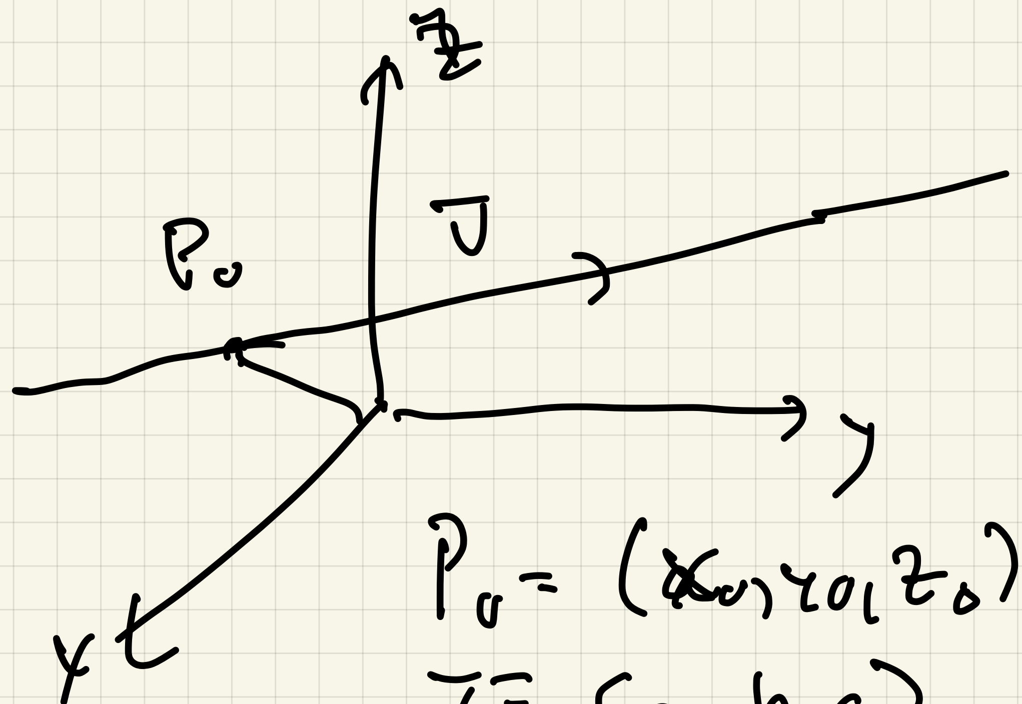
(d) $\text{Proj}_{\bar{v}}\bar{u} = \dfrac{\bar{u}\cdot\bar{v}}{\bar{v}\cdot\bar{v}}\bar{v}$

$\| \quad |v|^2$



$$\frac{4}{9}\langle 2,-1,2\rangle = \left\langle \frac{8}{9}, -\frac{4}{9}, \frac{8}{9} \right\rangle$$

Last time



$P_0 = (x_0, y_0, z_0)$

$\bar{v} = \langle a, b, c \rangle$

$$\begin{aligned} x &= x_0 + at \\ y &= y_0 + bt \\ z &= z_0 + ct \end{aligned} \qquad t \in \mathbb{R}$$

$\bar{P}_0 + t\bar{v}$

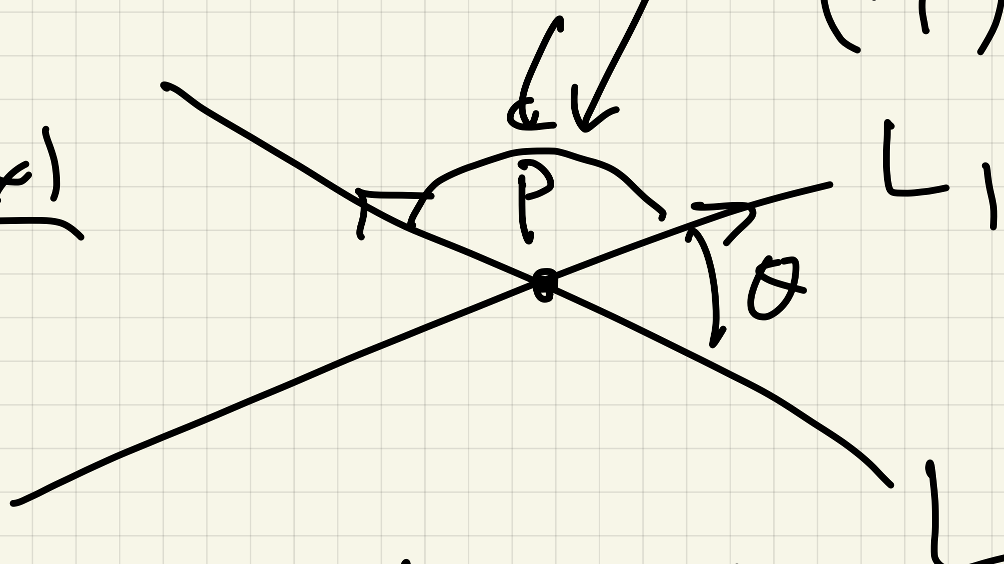
t = parameter

Ex0 $L_1: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1+2t \\ 2+3t \\ 4-5t \end{pmatrix} \quad t \in \mathbb{R}$

$L_3: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3+3s \\ 13-4s \\ -3+s \end{pmatrix} \quad s \in \mathbb{R}$

Intersect at $P = \begin{pmatrix} 3 \\ 5 \\ -1 \end{pmatrix}$ $t=1$ and $s=2$

Ex1



What is the $\underline{\underline{\text{angle}}}$ θ between L_1 and L_3?

$\underline{\underline{\text{smallest}}}$

direction of L_1 $\langle 2,3,-5\rangle = \vec{u}$

direction of L_3 $\langle 3,-4,1\rangle = \vec{v}$

$$\cos\theta = \frac{u \cdot v}{|u||v|} = \frac{6-12-5}{\sqrt{38}\sqrt{26}} = \sqrt{988}$$

$$\frac{-11}{\sqrt{38}\sqrt{26}} < 0 \Rightarrow \theta > 90^\circ$$

want smaller angle:

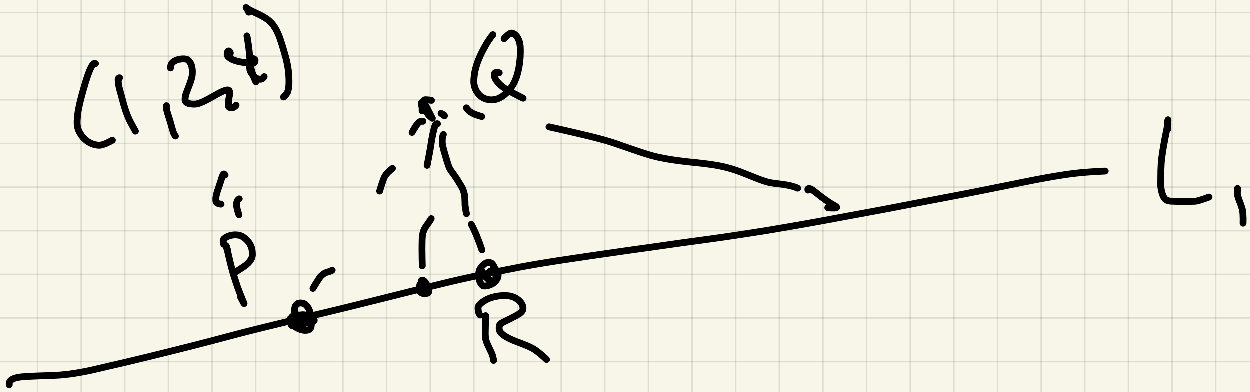
use $\vec{u}, -\vec{v}$ directions

$$\cos\theta = \frac{+11}{\sqrt{988}} \Rightarrow \theta = 1.213 \text{ rad} = 69.52^\circ$$

Ex2 Find distance from

$Q = (1,2,3)$ to line

$$L_1: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1+2t \\ 2+3t \\ 4-5t \end{pmatrix}$$



Lots of approaches:

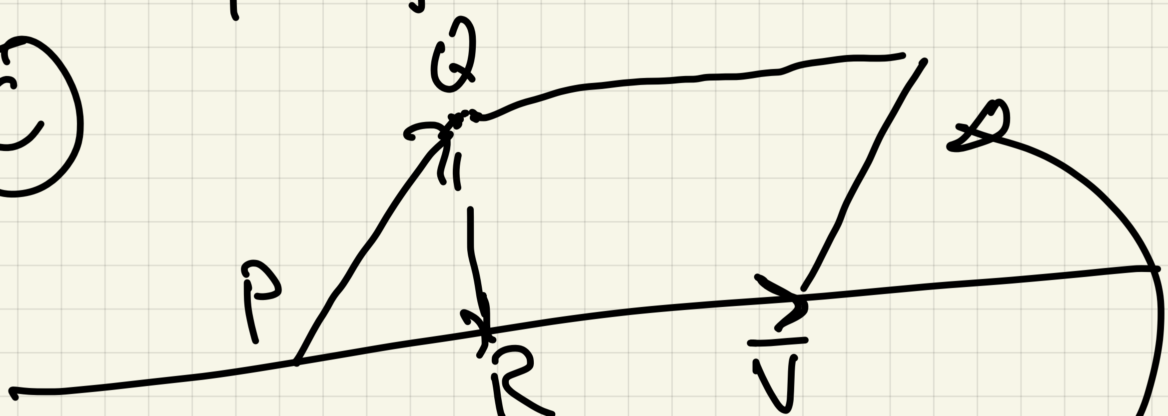
(A) **Calc**: minimize function $d(t) = \text{dist}(Q, (1+2t, 2+3t, 4-5t))$

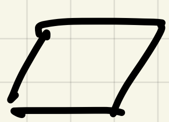
(B) **better**: $\overrightarrow{PR} = \text{Proj}_{\vec{v}} \overrightarrow{PQ}$, $\vec{v} = (2,3,-5)$

so $\overrightarrow{RQ} = \overrightarrow{PQ} - \overrightarrow{PR}$

$|\overrightarrow{QR}| = $ dist.

(C)



$|QR| = $ ht of  =

$$\frac{\text{Area}}{\text{base}} \leftarrow \frac{|\overrightarrow{PQ} \times \bar{v}|}{|\bar{v}|}$$ book formula

$P = (1,2,4) \qquad Q = (1,2,3)$

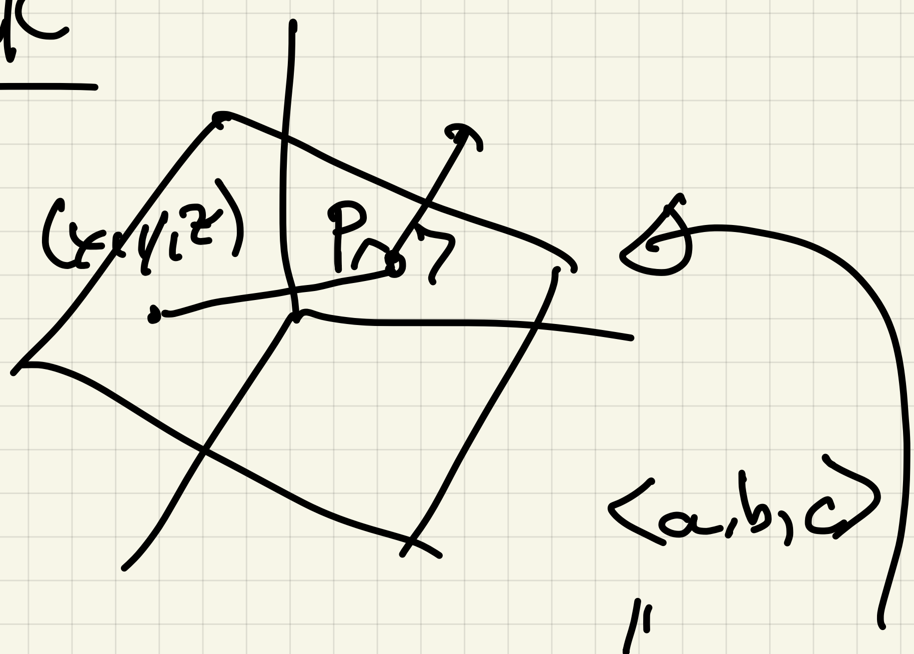
$\overrightarrow{PQ} = \langle 0,0,-1 \rangle \qquad v = \langle 2,3,-5 \rangle$

$$\overrightarrow{PQ} \times \bar{v} = \begin{vmatrix} i & j & k \\ 0 & 0 & -1 \\ 2 & 3 & -5 \end{vmatrix} = \langle 3,-2,0 \rangle$$

so $$|\overrightarrow{RQ}| = \frac{|\langle 3,-2,0\rangle|}{|\langle 2,3,-5\rangle|} = \frac{\sqrt{13}}{\sqrt{38}} = \sqrt{\frac{13}{38}}$$

Planes in $\mathbb{R}^3$

A plane has no directions but it does have



a perpendicular / normal $\bar{n}$

$P_0 = (x_0, y_0, z_0)$ pt in plane

$P = (x, y, z)$ is on the plane

$\Leftrightarrow$ $\overrightarrow{P_0P} \perp \bar{n} \Leftrightarrow \langle P - P_0 \rangle \cdot \bar{n} = 0$

$\langle x - x_0, y - y_0, z - z_0 \rangle \cdot \langle a, b, c \rangle = 0$

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

standard form

Ex

The plane containing
$P = (3, 5, -1)$ normal to
$\bar{n} = \langle 2, 3, 5 \rangle$ has equation

$2(x - 3) + 3(y - 5) + 5(z + 1) = 0$

i.e. $2x + 3y + 5z - 6 - 15 + 5 = 0$
$-16 = 0$

$\underline{2}x+\underline{3}y+\underline{5}z=16$

General form

<u>Note</u> ① easy to read off normal vector

② Easy to sketch:

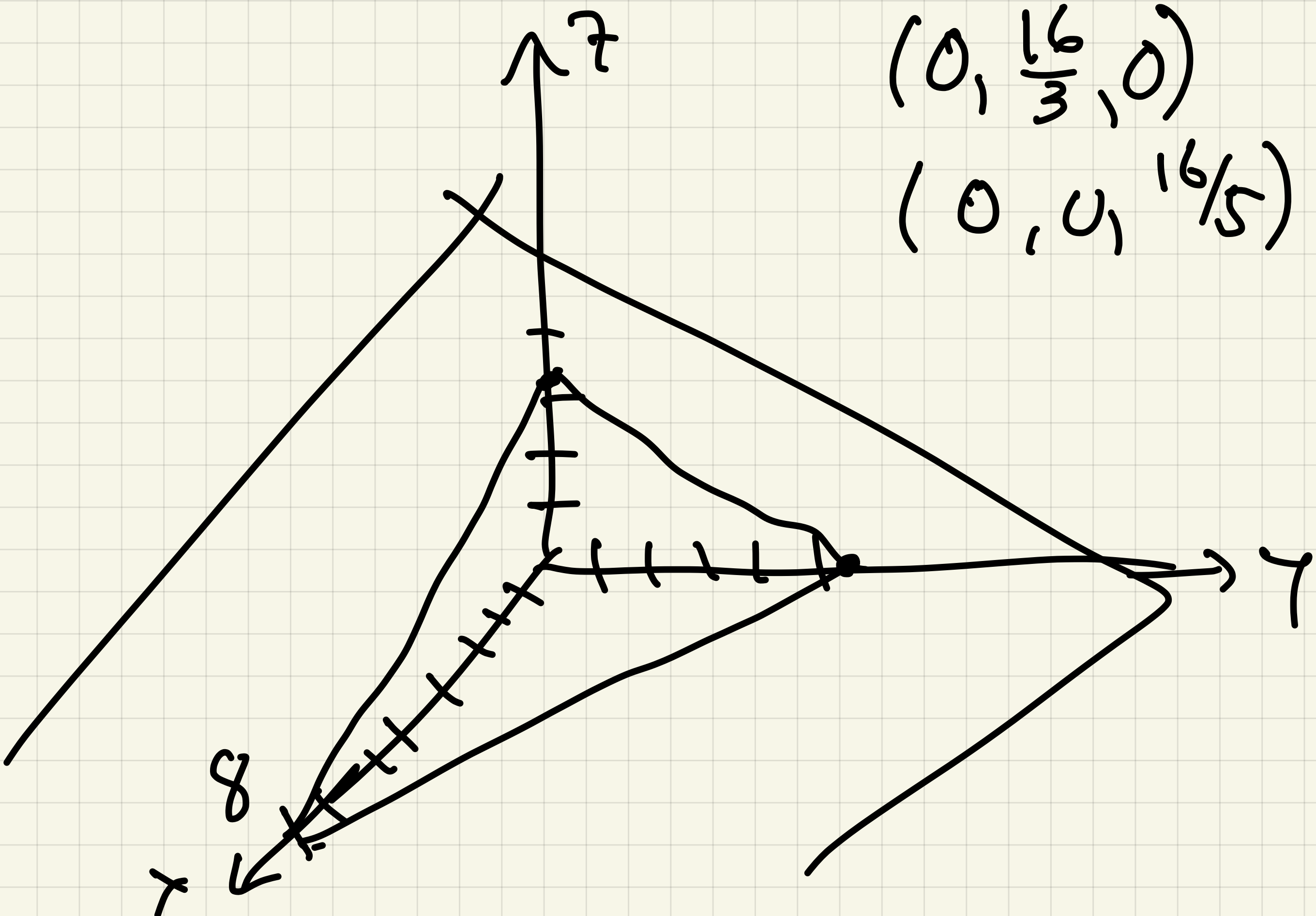
intersect with x-axis : $\underline{\underline{y=z=0}}$

$2x=16, \quad x=8$

at $(8,0,0)$

$(0,\frac{16}{3},0)$

$(0,0,{}^{16}/_{5})$

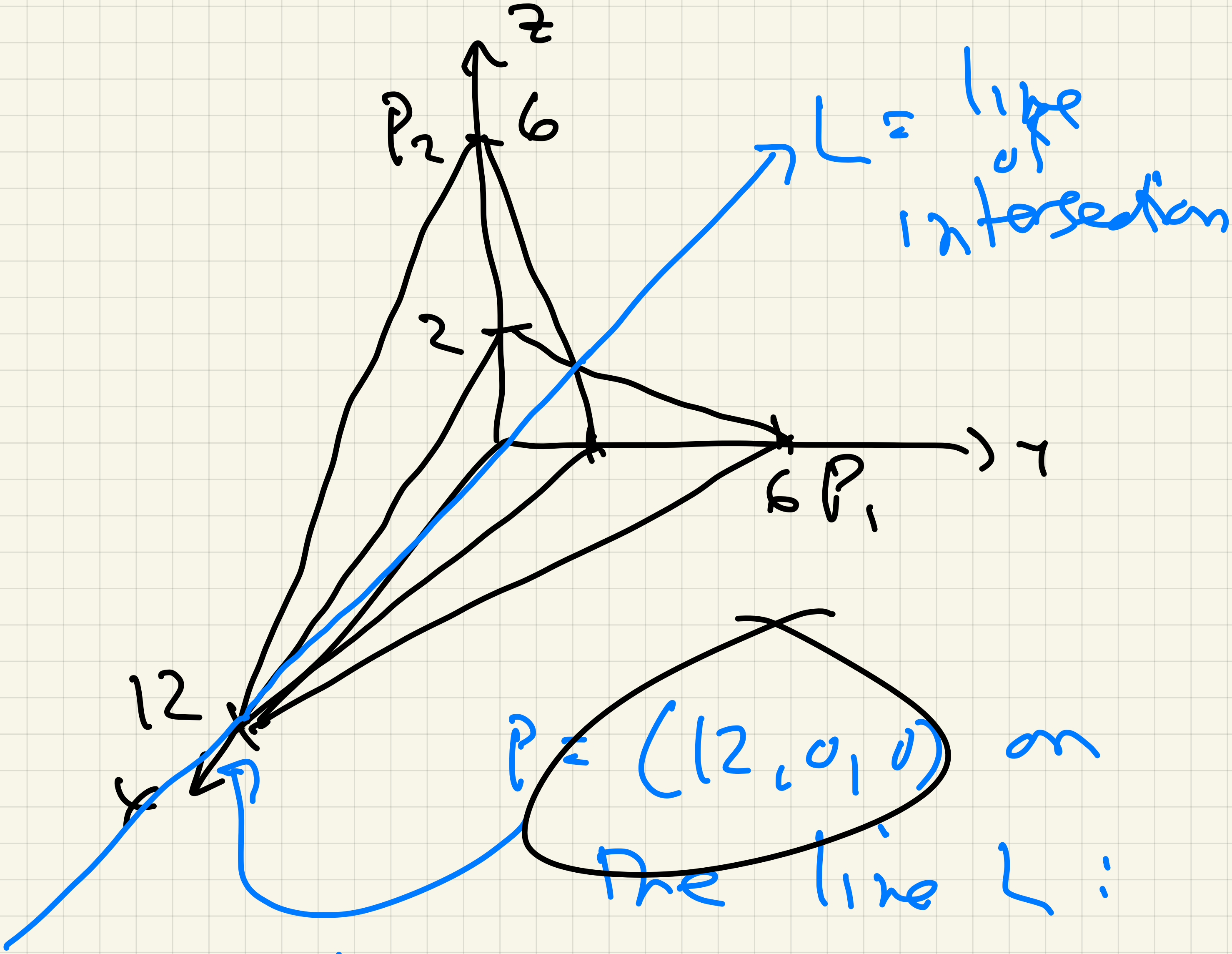


Ex 2 Consider planes

$P_1: \; x + 2y + 6z = 12$

$P_2: \; x + 6y + 2z = 12$

$n_1 = \langle 1, 2, 6 \rangle \qquad n_2 = \langle 1, 6, 2 \rangle$



$P = (12, 0, 0)$ on the line L:

direction $\bar{v}$ for L ???

Observe $\bar{v} \perp n_1 = \langle 1,2,6\rangle$

$\bar{v} \perp n_2 = \langle 1,6,2\rangle$

So $\bar{v} \parallel n_1 \times n_2$

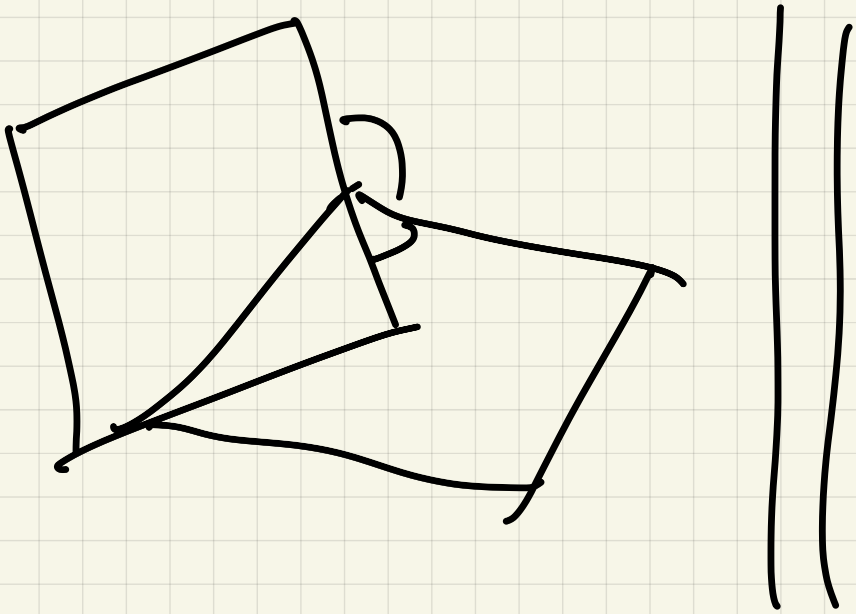
$$\bar{n}_1 \times \bar{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & 2 & 6 \\ 1 & 6 & 2 \end{vmatrix} = \langle -32, 4, 4\rangle$$

direction is $\langle -32, 4, 4\rangle \parallel \langle -8, 1, 1\rangle$

So L

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{matrix} = \\ = \\ = \end{matrix} \begin{pmatrix} 12-8t \\ t \\ t \end{pmatrix}$$

(b) What is the smallest angle of intersection between planes P_1 & P_2?



angle between the normal vectors

$n_1 = \langle 1, 2, 6 \rangle$

$n_2 = \langle 1, 6, 2 \rangle$

$$\cos\theta = \frac{n_1 \cdot n_2}{|n_1| \, |n_2|} = \frac{1 + 12 + 12}{\sqrt{41}\,\sqrt{41}} = \frac{25}{41}$$

so $\theta = \cos^{-1}\left(\frac{25}{41}\right)$

(c) where does L intersect

the plane $P_3: 2x+y+8z=32$

$$L: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12-8t \\ t \\ t \end{pmatrix}$$

Easy: substitute:

$$2(12-8t) + 1(t) + 8(t) = 32$$

$$24 - \underbrace{16t + t + 8t}_{-7t} = 32$$

$$-7t = 32-24 = 8$$

$$t = -8/7 \Rightarrow$$

so $P = \begin{pmatrix} 12-8(-8/7) \\ -8/7 \\ -8/7 \end{pmatrix} = \begin{pmatrix} \frac{148}{7} \\ -8/7 \\ -8/7 \end{pmatrix}$