9/30 Calc 3

Exam 2 → October 24

Quiz 8 avg 72%
med 95%

$r(t) = \langle 3t^2, t^3, t^2 \rangle$

$r'(t) = \langle 6t, 3t^2, 2t \rangle$

$$\int_0^5 |r'(t)|\,dt = \int_0^5 \sqrt{36t^2 + 9t^4 + 4t^2}\,dt$$

$$\int_0^5 \sqrt{40t^2 + 9t^4}\,dt \qquad t \geq 0$$

$$\int_{t=0}^{t=5} t\sqrt{40 + 9t^2}\,dt$$

$u = 40 + 9t^2$

$du = 18t\,dt$

$\frac{1}{18}du = t\,dt$

$$\int_{40}^{265} \frac{1}{18}\sqrt{u}\,du = \frac{1}{18}\cdot\frac{2}{3}u^{3/2}\Big|_{40}^{265}$$

$$= \frac{1}{27}\left(265^{3/2} - 40^{3/2}\right)$$

$$\kappa = \frac{|T'(t)|}{|r'(t)|} \qquad T = \text{unit tangent}$$

$$= \frac{|r' \times r''|}{|r'|^3} \leftarrow \underline{\text{easier}}$$

$$r' = \langle 6t, 3t^2, 2t\rangle = \langle 6,3,2\rangle$$

$$r'' = \langle 6, 6t, 2\rangle = \langle 6,6,2\rangle$$

$$\begin{vmatrix} i & j & k \\ 6 & 3 & 2 \\ 6 & 6 & 2 \end{vmatrix} = \langle -6, 0, 18\rangle$$

$$|\langle -6,0,18\rangle| = 6\,|\langle -1,0,3\rangle|$$

$$= 6\sqrt{10}$$

$$|r'| = \sqrt{36+9+4} = \sqrt{49} = 7$$

$k = \frac{6\sqrt{10}}{7^3} =$

13.3 Recall $y = f(x)$

Calc 1: $f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

Calc 3 $z = f(x,y)$

$\lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x} = f_x = z_x = \partial f/\partial x = \partial z/\partial x$

$\lim_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y} = f_y = z_y = \partial f/\partial y = \partial z/\partial y$

Easy to compute:

Ex 1 $z = f(x,y) = x^2 + x^3 y^4 + \frac{x}{y} + y^2$

$f_x = 2x + 3x^2 y^4 + \frac{1}{y}$

$f_y = 0 + 4x^3 y^3 + \frac{-x}{y^2} + 2y$

Ex2 $z = f(x,y) = x \ln y + e^{x^2 y} + 7\sin(y^2+2x)$

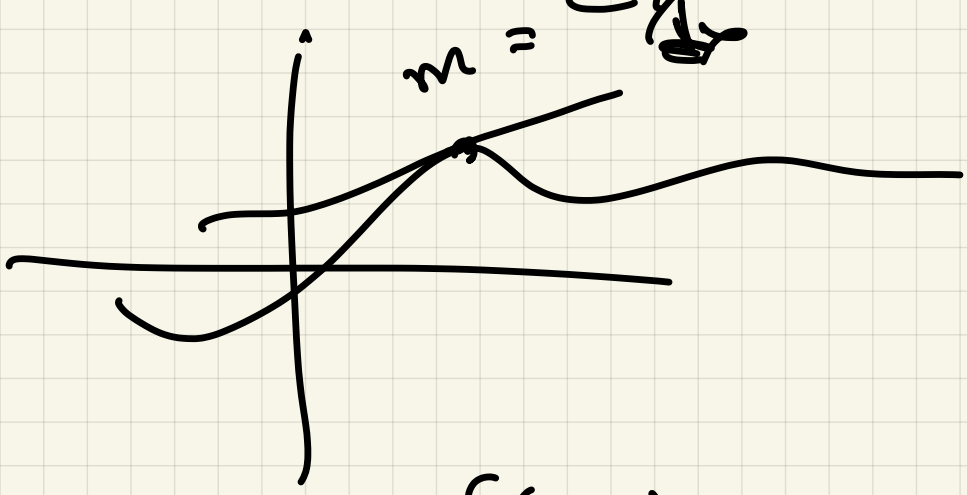
$\partial z/\partial x = \ln y + 2xy e^{x^2 y} + 14\cos(y^2+2x)$

$\partial z/\partial y = \frac{x}{y} + x^2 e^{x^2 y} + 14\cos(y^2+2x)\cdot y$

$= \frac{x}{y} + x^2 e^{x^2 y} + 14y\cos(y^2+2x)$

Interpretation:

Calc 1: $\frac{dy}{dx}$ = slope of tangent line



Calc 3: $z = f(x,y)$

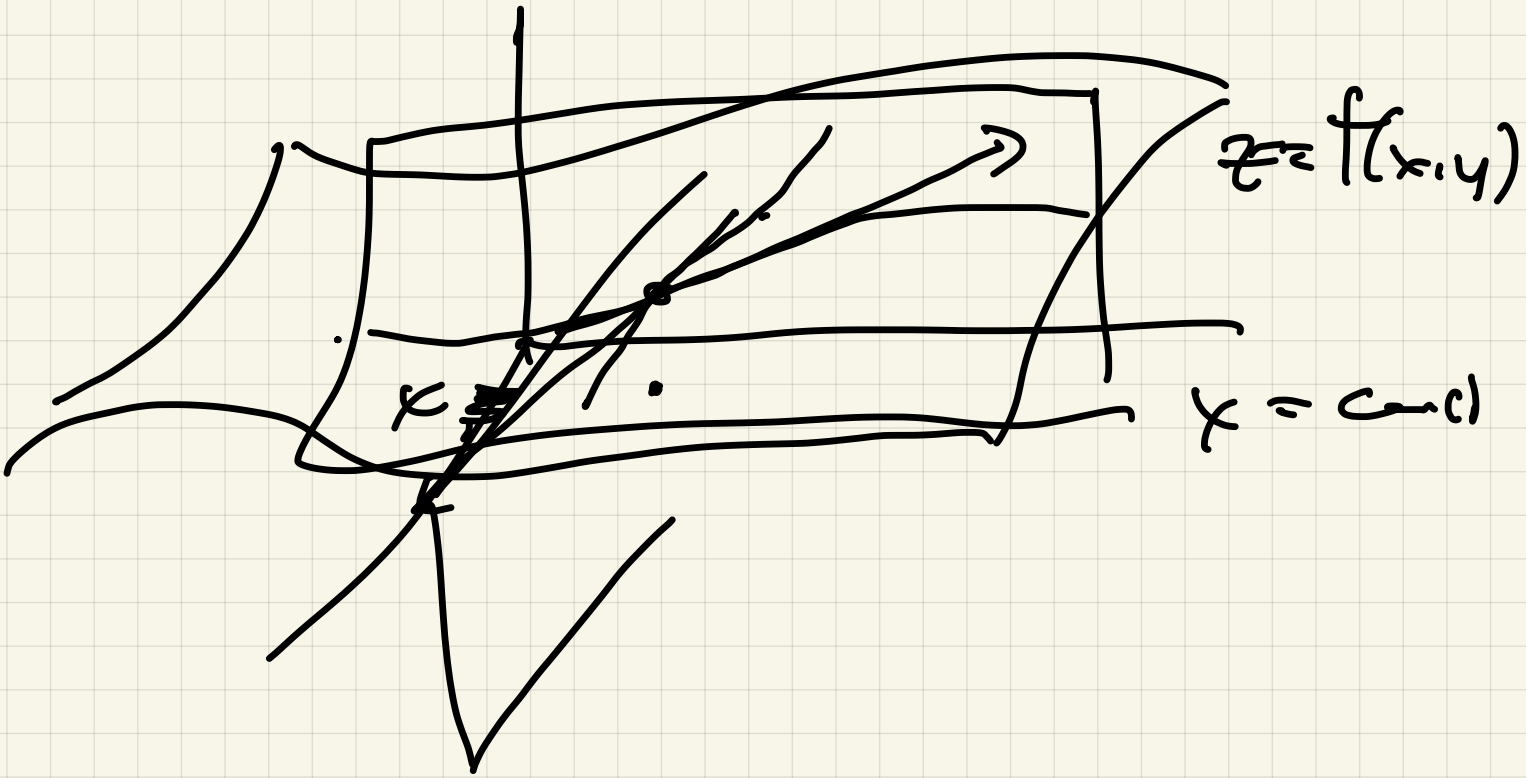
$\partial z/\partial x$ = slope of tangent line

to y-trace of graph
(set y = constant)

rate of change of z in positive x direction

$\partial z/\partial y$ = Slope of tangent line to x-trace of graph

rate of change of z in positive y-direction

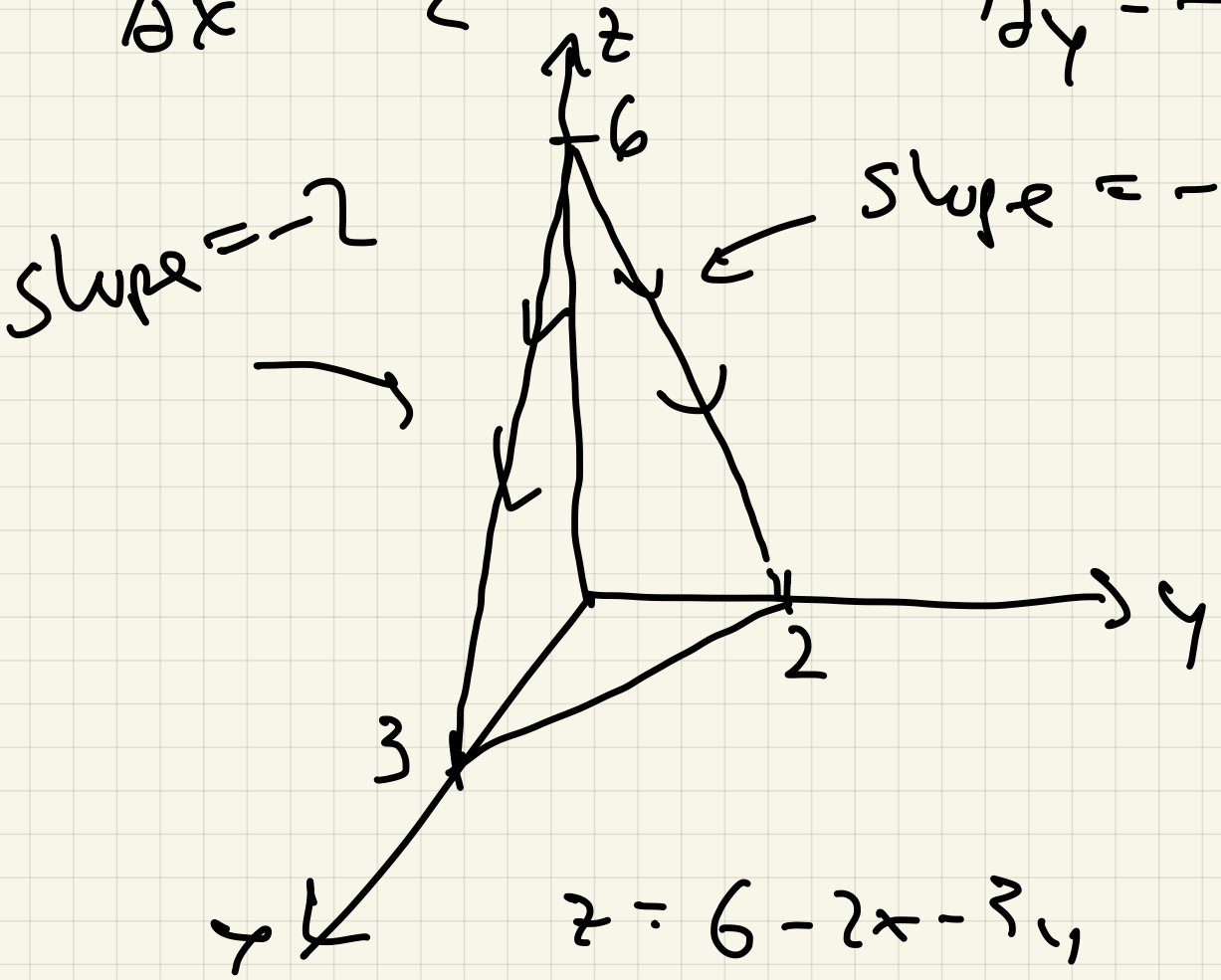


Ex) $z = f(x,y) = 6 - 2x - 3y$

Sketch surface and

Compute $\partial z/\partial x$ and $\partial z/\partial y$ at $(0,1,0)$

$\partial z/\partial x = -2$ $\quad$ $\partial z/\partial y = -3$



$z = 6 - 2x - 3y$

$2x + 3y + z = 6$

Ex² Same for

$$z = f(x,y) = \sqrt{9 - (x+1)^2 - (y-2)^2}$$

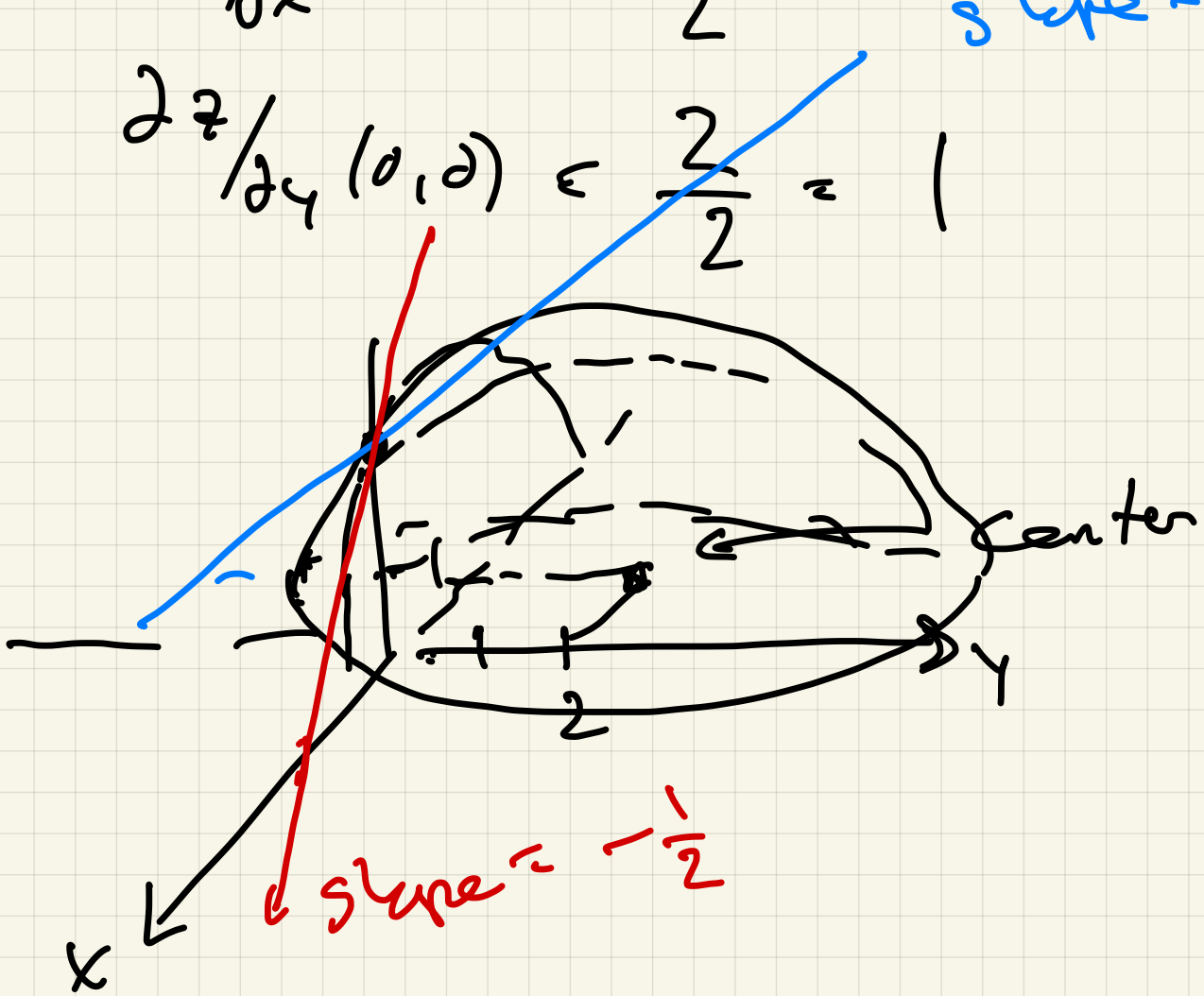
$$\partial z/\partial x = \frac{1}{2}\left(9 - (x+1)^2 - (y-2)^2\right)^{-1/2} \cdot (-2(x+1))$$

$$= \frac{-(x+1)}{\sqrt{9-(x+1)^2-(y-2)^2}}$$

$$\partial z/\partial y = \frac{-(y-2)}{\sqrt{9-(x+1)^2-(y-2)^2}}$$

so $\partial z/\partial x(0,0) = \frac{-1}{2}$

$\partial z/\partial y(0,0) = \frac{2}{2} = 1$



Similarly for 3 variables

Ex5 Find the first partial derivatives of

$w = h(x,y,z) =$ $\ln x + \ln y$

$= 3x^2y - 5y\cos z + ze^{xy} + \ln xy$

$\partial h/\partial x$ ← hold y/z const take wrt x

$$\partial h/\partial x = 6xy + yze^{xy} + \frac{1}{x}$$

$$\frac{\partial h}{\partial y} = 3x^2 - 5\cos z + xze^{xy} + \frac{1}{y}$$

$$\partial h/\partial z = 5y\sin z + e^{xy}$$

(was a y in class)